САМАРСКИЙ УНИВЕРСИТЕТ
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Tether system technologies and nanosatellites

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Tether technologies make it possible to create lightweight and at the same time extended structures in orbit, which can have many useful applications.

For example:

1. Measuring systems for monitoring gravitational, magnetic fields, etc.
2. Long range optical interferometer for deep space surveillance.
3. Atmospheric monitoring systems.
4. Surface monitoring systems for planets with higher resolution.
5. Long geostationary systems.
6. Gravitational, aerodynamic and magnetic stabilization of NS.
7. Changes in the orbital parameters of NS using electrodynamic tether systems.
8. Debris of satellites from orbit using electrodynamic tether systems.
9. Deployment and use of large solar sails.
10. Creation and use of power plants in orbit.

The lecture the issues of dynamics and motion control during the deployment of tether groupings of NS of various configurations is considered

Lecture plan:

1. Analysis of dynamics and control during the deployment of a tether system consisting of two satellites.
2. Deployment of a vertical (radial) orbital grouping consisting of several satellites.
3. Using an electrodynamic tether system to change the orbital parameters of satellites, including removing them from orbit.
4. Dynamics of the formation of a triangular tether grouping of satellites.

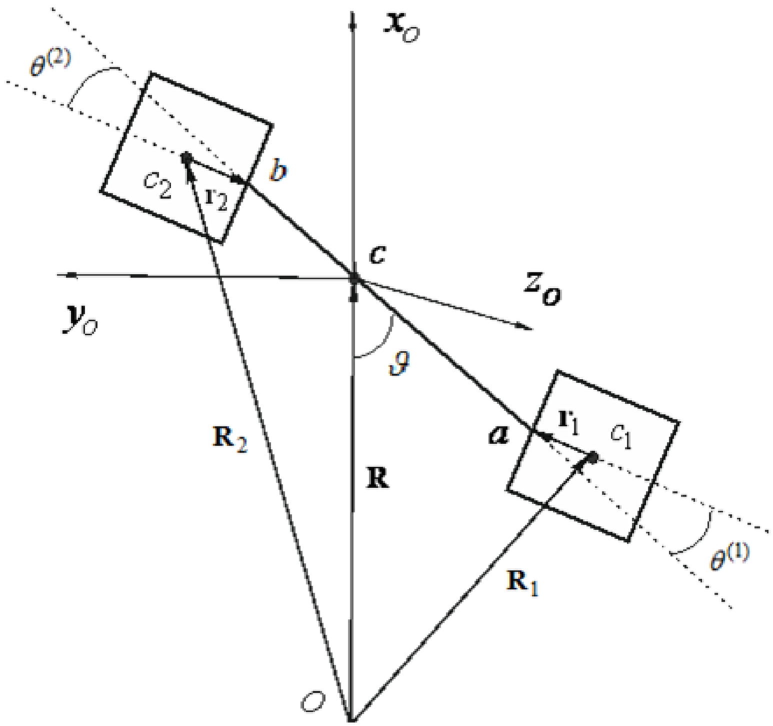

Figure 1

The formation of the satellite tether system will allow:

1. To reduce the angular speed of the system
2. Provide gravitational stabilization of the system
3. Provide stabilization of angular motion of satellites relative to their centers of mass

Main stages of deployment:

1. Separation from the base spacecraft (SC)
2. Separation of nanosatellites
3. Controlled system deployment
4. Stabilization of the system with respect to the vertical

MATHEMATICAL MODEL OF THE MOTION OF THE TWO NS CONNECTED BY A TETHER

The equations of motion for the mass centers of the NS in the geocentric fixed coordinate system:

$$m_k \mathbf{R}_k = \mathbf{G}_k + \mathbf{T}_k \,, \quad k = 1,2 \qquad (1)$$

where $\mathbf{G}_k = -Km\mathbf{R}_k / R_k^3$ - gravitational force, $\mathbf{T}_k$ - tension force of the tether,

$\mathbf{R}_k$ - vectors for the center of mass of NS, $\mathbf{T}_1 = -\mathbf{T}_2$

The tension force is calculated Hooke's law :

$$T = \begin{cases} C\dfrac{R_т - l}{l}, & if \;\; R_т - l \geq 0 \\ 0 \;, & if \;\; R_т - l < 0 \end{cases} \qquad (2)$$

where $C = EA$ - the stiffness of the tether, E - elastic modulus,

A - tether cross-sectional area, l - undeformed length of the tether,

$R_т = |\mathbf{R}_б - \mathbf{R}_a|, \quad \mathbf{R}_a = \mathbf{R}_1 + \mathbf{r}_1, \; \mathbf{R}_б = \mathbf{R}_2 + \mathbf{r}_2,$

$\mathbf{r}_{1,2}$ - the vectors of attachment points of the tether relative to the centers of mass of the NS (Figure 1).

Dynamic Euler equations :

$$J_x^{(k)} \frac{d\omega_x^{(k)}}{dt} + \omega_y^{(k)} \omega_z^{(k)} \left(J_z^{(k)} - J_y^{(k)} \right) = M_x^{(k)}$$

$$J_y^{(k)} \frac{d\omega_y^{(k)}}{dt} + \omega_x^{(k)} \omega_z^{(k)} \left(J_x^{(k)} - J_z^{(k)} \right) = M_y^{(k)} \qquad (3)$$

$$J_z^{(k)} \frac{d\omega_z^{(k)}}{dt} + \omega_x^{(k)} \omega_y^{(k)} \left(J_y^{(k)} - J_x^{(k)} \right) = M_z^{(k)}$$

where $\omega_x^{(k)}, \omega_y^{(k)}, \omega_z^{(k)}$ - angular velocities of the NS,

$J_x^{(k)}, J_y^{(k)}, J_z^{(k)}$ and $M_x^{(k)}, M_y^{(k)}, M_z^{(k)}$ - moments of inertia and forces of NS in the main related coordinate systems.

Kinematic Poisson equations:

$$\dot{\mathbf{e}}_{xk} = \boldsymbol{\omega}_k \times \mathbf{e}_{xk}, \quad \dot{\mathbf{e}}_{yk} = \boldsymbol{\omega}_k \times \mathbf{e}_{yk}, \quad \dot{\mathbf{e}}_{zk} = \boldsymbol{\omega}_k \times \mathbf{e}_{zk} \qquad (4)$$

where $\boldsymbol{\omega}_k = \left(\omega_x^{(k)}, \omega_y^{(k)}, \omega_z^{(k)} \right)$, $\mathbf{e}_{xk}, \mathbf{e}_{yk}, \mathbf{e}_{zk}$ - the unit vectors directed along the main axes of the related coordinate systems.

The calculation of the speeds of the NS after their separation with a relative speed $\mathbf{V}_r$

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{V}_r, \quad \mathbf{V}_2 = \mathbf{V}_c + \frac{m_1}{m_1 + m_2}\mathbf{V}_r \qquad (7)$$

where $\mathbf{V}_c$ - the speed of the center of mass of the system to the separation

The calculation of the angular speeds of the satellites after their separation

$$\omega_{x,y,z}^{(1,2)} = \omega_{x,y,z}^{(0)} + \frac{1}{J_{x,y,z}^{(1,2)}}\left(\mathbf{r}_{1,2} \times \mathbf{S}_{1,2}\right)_{x,y,z} \qquad (8)$$

$\omega_{x,y,z}^{(0)}$ and $\omega_{x,y,z}^{(1,2)}$ - angular velocities of satellites before and after their separation,

$|\mathbf{S}_{1,2}| = S$ - the impulse in the separation, $\mathbf{S}_2 = -\mathbf{S}_1$, $\mathbf{S}_2 = -\frac{m_1 m_2}{m_1 + m_2}\mathbf{V}_r$.

<u>When calculating the linear and angular velocities of nanosatellites, the laws of conservation of impulse and angular impulse are used</u>

The equations that model the release tether:

$$m_u \dot{V}_l = T - F_u, \quad \dot{l} = V_l. \qquad (9)$$

where $F_u = k_v V_l$ - force in the control mechanism,
k_v - program parameter, V_l - speed of tether

For smooth braking of the tether the program is used:

$$k_v = \begin{cases} k_{v1} & \text{if } \quad l \le l_1 \\ k_{v1} + \dfrac{l - l_1}{l_{end} - l_1}(k_{v2} - k_{v1}) & \text{if } \quad l_1 < l \le l_{end} \end{cases} \qquad (10)$$

k_{v12} - the parameters of the control law.

if $l \le l_1$ - fast system deployment

if $l_1 < l \le l_{end}$ - braking and provision at the end of deployment $V_l \approx 0 \ (V_l > 0)$.

Data: $m_1 = m_2 = 2\,\mathrm{kg}$, $L_{end} = 1\,\mathrm{km}$, $C = 7000\,\mathrm{H}$, $H = 400\,\mathrm{km}$, $V_r = 0.25\,\mathrm{m/s}$,

$\omega_x = \omega_y = \omega_z = 0.05\,\mathrm{s}^{-1}$

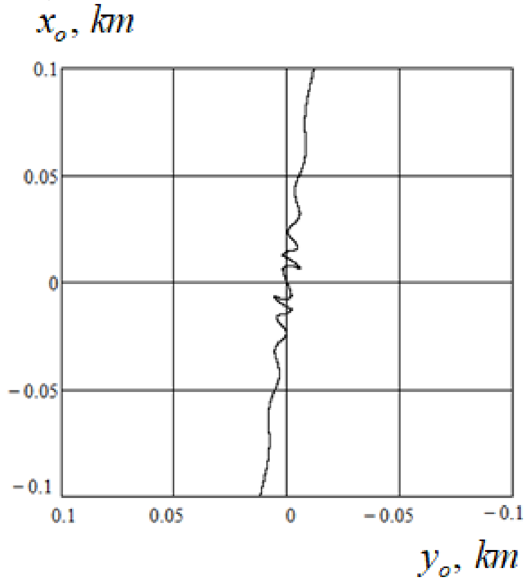

Fig. 2 The trajectories of the satellites with respect to the vertical

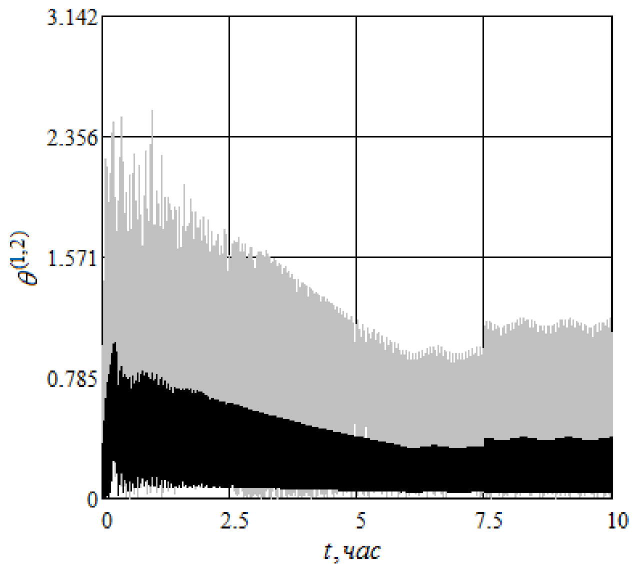

Fig. 3 The deflection angles of the satellites relative to the tether

One satellite with an offset center of mass relative to its longitudinal axis

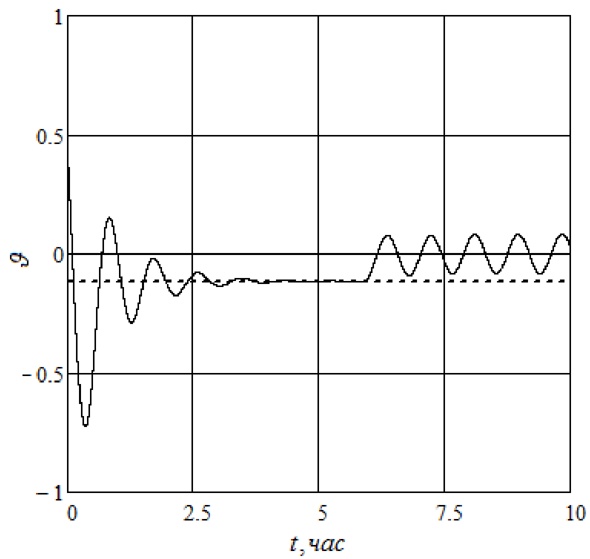

Fig. 3 The deviation angle of tether from vertical

Analytical formula for the angle of deviation of the tether from the vertical

$$\operatorname{tg} \vartheta_s = -2 m_e \frac{\omega_h}{k_{v1}} \qquad (11)$$

The main problems of the tether system formation:

1. The choice of a simple program control without feedback
2. Providing restrictions on the tension force

   $T > 0$

3. Providing restrictions on the angular movement of satellites
4. Providing restrictions on the speed of the tether $V_{end} \approx 0 \ (V_l > 0)$

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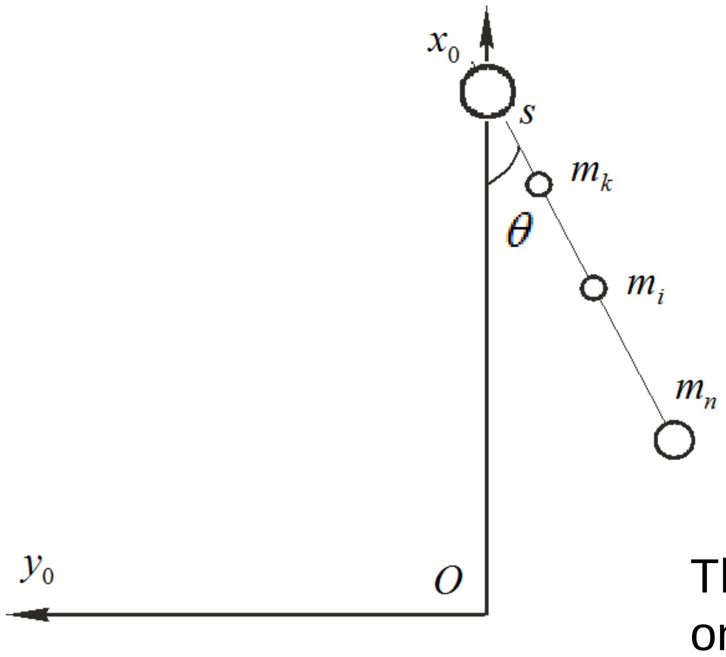

Fig. 1 Tether system

The system includes:
a base spacecraft and a few nanosatellites (NS)

After deployment,
the system is located near the vertical

The system is formed sequentially by separating one NS from the base spacecraft

Methods of formation: "slow" and "fast" deployment

Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial \dot{q}_j}\right)-\frac{\partial T_c}{\partial q_j}=-\frac{\partial P}{\partial q_j}+Q_j \qquad (1)$$

Generalized coordinate $q_1 = s$, $q_2 = \theta$

T_c and P - kinetic and potential energy.

$$T_c=\frac{1}{2}\sum_{i=k}^{n}\left(\dot{x}_i^2+\dot{y}_i^2\right),\; P=\sum_{i=k}^{n}P_i \qquad (2)$$

$$P_i=-Km_i/r_i,\; r_i=\sqrt{r_1^2+\left[s+(i-k)\Delta L\right]^2-2r_1\left[s+(i-k)\Delta L\right]\cos\theta}$$

$$x_i=x_{oi}\cos u-y_{oi}\sin u,\; y_i=x_{oi}\sin u+y_{oi}\cos u,$$

$$x_{oi}=r_1-\left[s+(i-k)\Delta L\right]\cos\theta,\; y_{oi}=-\left[s+(i-k)\Delta L\right]\sin\theta$$

$$\Delta L=L_{\text{end}}/(n-1),\; i=k,k+1,\ldots n$$

Model for building a nominal program:

$$\ddot{s} = \left[m_k(s)\left(\Omega + \dot{\theta}\right)^2 + m_k(s)\Omega^2\left(3\cos^2\theta - 1\right) - T \right] / M_k$$

$$\ddot{\theta} = -2m_s(s)\dot{s}\left(\Omega + \dot{\theta}\right) / J_k(s) - 1.5\Omega^2 \sin 2\theta \qquad (3)$$

Where $M_k = m(n-k) + m_n$

$$m_k(s) = m_n\left[s + (n-k)\Delta L\right] + m\sum_{i=k}^{n-1}\left[s + (i-k)\Delta L\right]$$

$$J_k(s) = m_n\left[s + (n-k)\Delta L\right]^2 + m\sum_{i=k}^{n-1}\left[s + (i-k)\Delta L\right]^2$$

Dynamic deployment program:

$$T = v_e\Omega^2\left[a(s - \Delta L) + b\dot{s}/\Omega + 3\Delta L\right] \qquad (4)$$

Where $v_e = m_k(\Delta L) = m_n(n+1-k) + m\sum_{i=k}^{n-1}(i+1-k)$

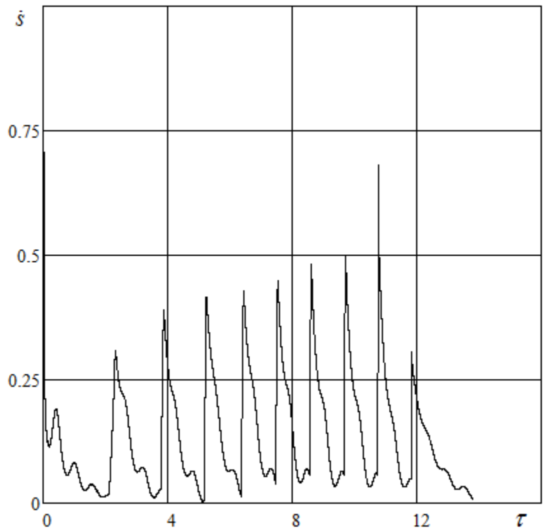

Fig.2 Tether speed

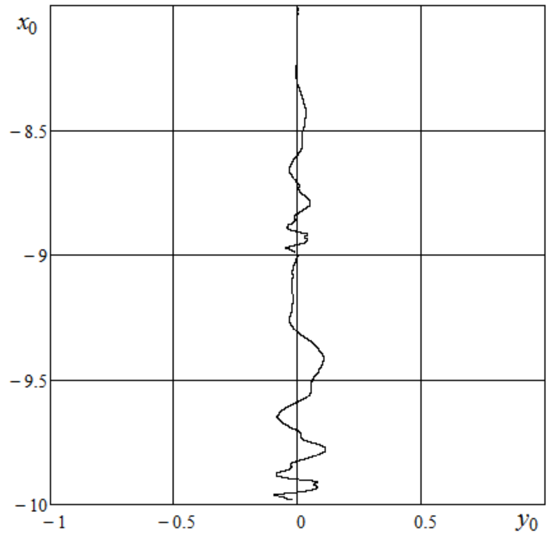

Fig.3 Trajectory of the two lower NS at the last stage of deployment

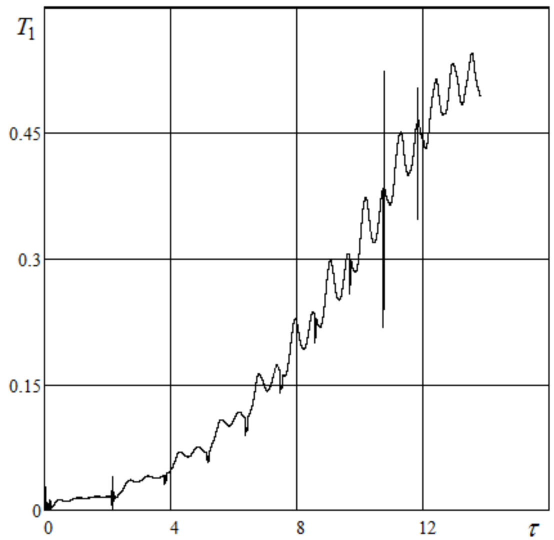

Fig. 4 Tension force

System Deployment Features

1. Deployment consists of n stages
2. Each stage consists of acceleration and deceleration sections
3. After completion of each stage, the system is located near the vertical
4. All satellites are located almost on the same line
5. After the last stage, the system is located near the vertical and has zero speed

Equivalent tether density: the mass of satellites is distributed evenly over the tether

$$\rho = m(n-2)/L_{end} \qquad (5)$$

Where n - number of satellites; m - mass of nanosatellites;
L_{end} - total tether length

Equations of motion of the tether system:

$$(m_n + \rho L)\ddot{L} = (m_n + \rho L/2)L F_{11} - T - \rho \dot{L}^2$$
$$(m_n + \rho L/3)L^2\ddot{\theta} = -2(m_n + \rho L/2)L\dot{L} F_{21} + (m_n + \rho L/3)L^2 F_{22} \qquad (6)$$

Where

$$F_{11} = \dot{\theta}^2 + 2\Omega\dot{\theta} + 3\Omega^2\cos^2\theta,\ F_{21} = \dot{\theta} + \Omega,\ F_{22} = -1.5\,\Omega^2\sin 2\theta$$

Nominal deployment program:

$$T = (m_n + \rho L/2)\Omega^2\left[a(L - L_{end}) + b\dot{L}/\Omega + 3L_{end}\right] \qquad (7)$$

Where m_n - mass of the lower nanosatellite, a, b - program parameters

You can test the features of the "fast" deployment program by using the following model:

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{V}_i, \quad m_i \frac{d\mathbf{V}_i}{dt} = \mathbf{G}_i + \mathbf{T}_i - \mathbf{T}_{i+1}, \; i = 1, 2, \ldots, n \qquad (8)$$

Where

$\mathbf{r}_i, \mathbf{V}_i, m_i$ - radiuses, velocities, masses of satellites,

$\mathbf{G}_i, \mathbf{T}_i$ - the gravitational forces and the tension forces of the tether

The tension forces of the tether

$$\mathbf{T}_i = \begin{cases} c(\gamma_i - 1)\Delta\mathbf{L}_i / \Delta L_{0i}, & if \;\; \gamma_i \geq 1 \\ 0, & if \;\; \gamma_i < 0 \end{cases}, \qquad \gamma_i = \frac{\Delta L_i}{\Delta L_{0i}} \qquad (9)$$

Where

c - the stiffness of the tether, γ_i - relative deformation of the tether section

$$\Delta\mathbf{L}_i = \mathbf{r}_{i+1} - \mathbf{r}_i$$

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Equations of operation of the control mechanism :

$$m_e \frac{dV}{dt} = T_1 - F_c, \quad \frac{dl}{dt} = V \tag{10}$$

where the parameter m_e takes into account the inertia of the control mechanism,

V , l - speed and undeformed length of the tether, F_c - control force,

T_1 - tension force on the first section of the tether, counting from the base spacecraft

Control force:

$$F_c = T + p_L \left(l - L\right) + p_V \left(V - \dot{L}\right) \tag{11}$$

Where

T - nominal tension force, p_l, p_V - feedback ratios,

L, $\dot{L}$ - program values of tether length and speed

NUMERICAL RESULTS USING THE PROGRAM "FAST" DEPLOYMENT

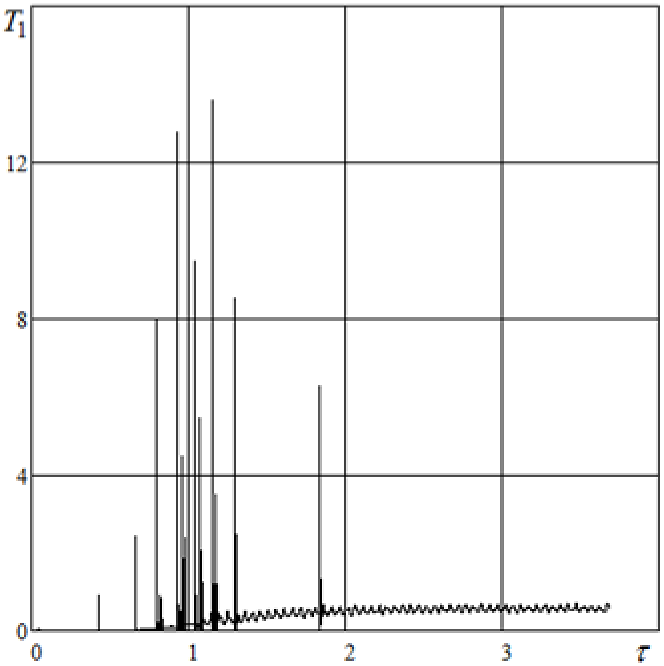

Fig. 5 The tension force of the tether

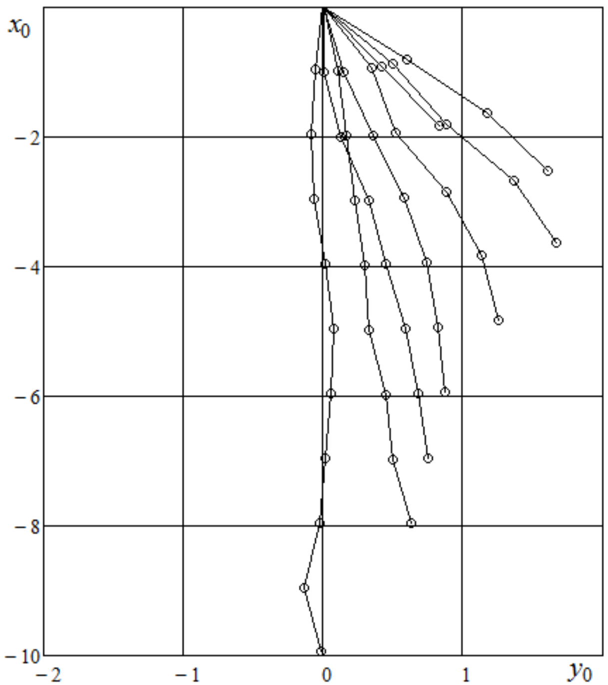

Fig.6 Positions of system in various point in time

Problems of "fast" deployment:

1. Need a rotary mechanism for the separation of the NS
2. It is necessary to have given the speed of separation of the satellite

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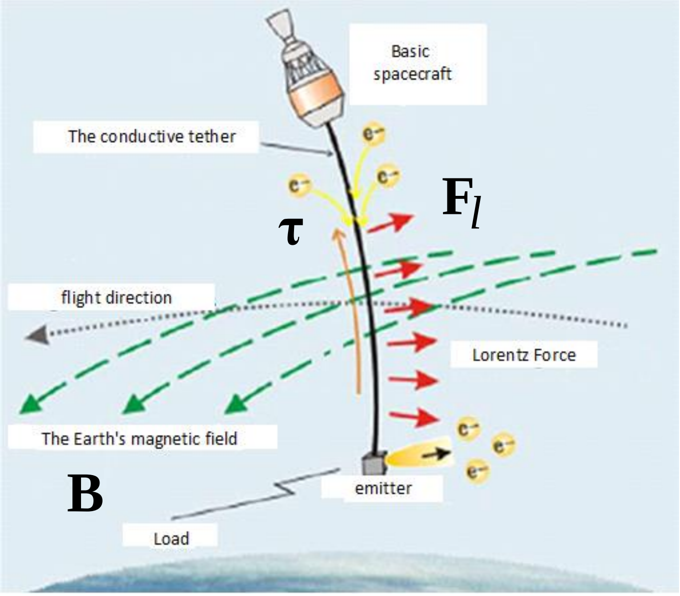


Recently, a very promising application of the electrodynamic tether systems. More than ten real tether experiments have already been conducted in different countries of the world.

In the interaction of the conducting tether with the magnetic field of the Earth, the Lorentz force arises, with which you can control the movement of the system

$$\mathbf{F}_l = I\,\boldsymbol{\tau} \times \mathbf{B}$$

Where $\mathbf{B}$ **- magnetic induction vector,** I **- current**

$\boldsymbol{\tau}$ **- the unit vector defines the direction of the current**

Use cases of the electrodynamic system:

1. **Using an insulated conductive tether.**
2. **Using a bare conductive tether.**

Other cases from the point of view of dynamics and control:

1. **Vertical (radial) EDTS.**
2. **Rotating EDTS.**

The use of rotating EDTS is due to the fact that vertical systems are not stable at constant current and require additional stabilization

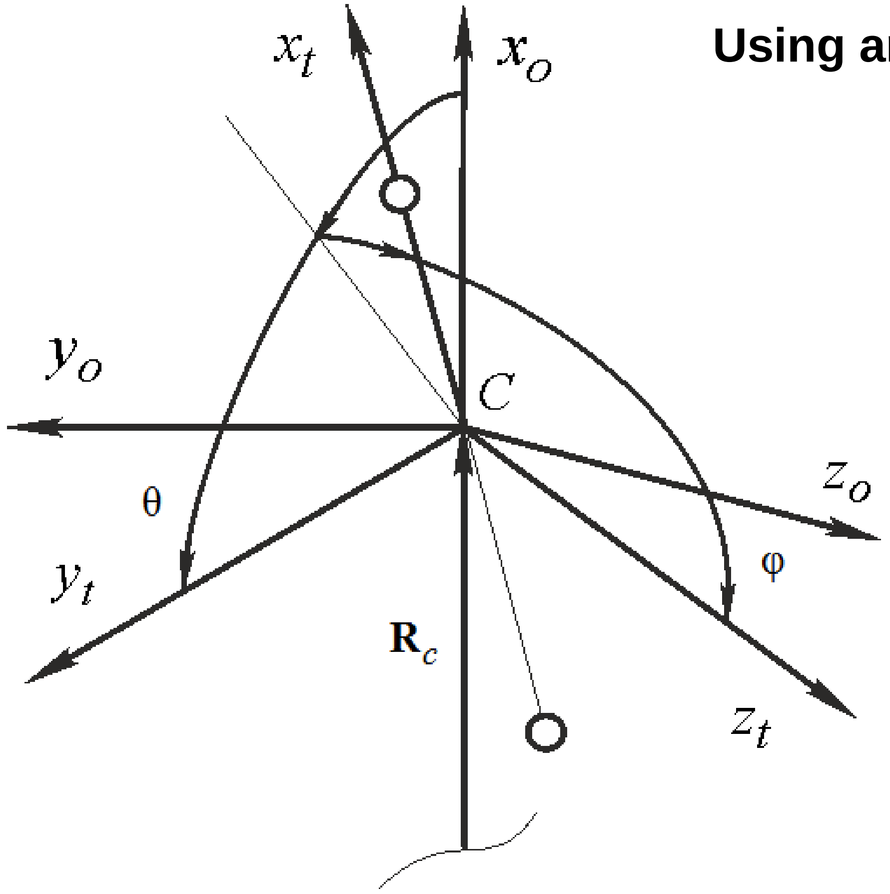


Fig. 1

Using an insulated conductive tether

Lorentz force

$$\mathbf{F} = L(\mathbf{I} \times \mathbf{B}) \quad (1)$$

The moment of the Lorentz force

$$\mathbf{M} = \mathbf{r}_D \times \mathbf{F} \quad (2)$$

Mathematical model of motion

$$\ddot{r} - r\left[\dot{\varphi}^2 + \left(\dot{\theta}+\omega\right)^2 \cos^2\varphi + \nu^{-1}\omega^2\left(3\cos^2\theta\cos^2\varphi - 1\right)\right] = Q_1 / m_e \quad (3)$$

$$\ddot{\theta} + \dot{\omega} + 2\left(\dot{\theta}+\omega\right)\left(\dot{r}/r - \dot{\varphi}\,\mathrm{tg}\varphi\right) + 1.5\nu^{-1}\omega^2 \sin 2\theta = Q_2 / m_e r^2 \cos^2\varphi \quad (4)$$

$$\ddot{\varphi} + 2\dot{\varphi}\dot{r}/r + \left[0.5\left(\dot{\theta}+\omega\right)^2 + 1.5\nu^{-1}\omega^2\cos^2\theta\right]\sin 2\varphi = Q_3 / m_e r^2 \quad (5)$$

where r, θ, φ - generalized coordinates, $m_e = m_1 m_2 / m,$ $m_{1,2}$ - masses of NS,

$m = m_1 + m_2,$ $\omega = \dot{\vartheta} = \left(K / p^3\right)^{0.5} \nu^2$ - orbital angular velocity,

$\dot{\omega} = \ddot{\vartheta} = -2Ke\sin\vartheta / p^3,$ $\nu = 1 + e\cos\vartheta,$ ϑ - the true anomaly,

p - orbit parameter, e - eccentricity of the orbit,

$Q_{1,2,3}$ - generalized forces.

Mathematical model of motion

The Earth's magnetic field is a direct dipole

$$\mathbf{B} = B_0\left[\mathbf{e}_z - 3(\mathbf{e}_z \cdot \mathbf{e}_R)\mathbf{e}_R\right] \quad (6)$$

where $\mathbf{e}_R = \mathbf{R}_c / R_c$, $\mathbf{e}_z$ - a unit vector directed along the axis of rotation of the Earth;

$B_0 = \mu_m / R_c^3$, $\mu_m = 8 \cdot 10^6 \,\text{Tl km}^3$

Generalized forces

$$Q_1 = -0.5 B_0 \cos i |I| r \left(\operatorname{ctg}\psi \cos^2\varphi + \psi^{-1}\sin^2\varphi\right)$$

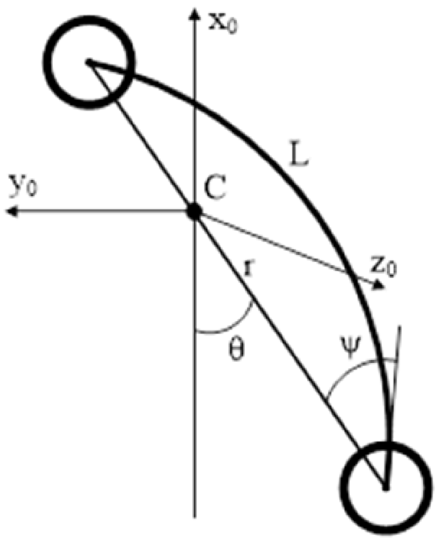
$$Q_2 = B_0\, I\, r\, \Delta\left[\cos\varphi\cos i - \sin\varphi \sin i \sin(\theta + u)\right] + \Delta Q_2 \quad (7)$$

$$Q_3 = Q_{3\psi} + B_0 I r \Delta \sin i \left[\cos(\theta + u) + 3\sin\theta\sin u\right]$$

where $\Delta = 0.5 r (m_2 - m_1)/m$, $Q_{3\psi} = 0.5 B_0 \cos i |I| r^2 \sin\varphi\cos\varphi\left(\operatorname{ctg}\psi - \psi^{-1}\right)$

$$\Delta Q_2 = 3 B_0 I r \Delta \sin i \cos\theta \sin\varphi \sin u$$

r - chord



The tether under the action of a distributed load is bends

Nonlinear equation for determining the angle ψ

$$r = L\gamma / \left[\sin^2\varphi + \cos^2\varphi \left(\psi / \sin\psi \right)^2 \right]^{0.5} \qquad (8)$$

where γ - tether elongation

L - undeformed tether length

VERTICAL EQUILIBRIUM POSITIONS OF EDTS $Q_{1,2,3} = 0$

$$\theta_1 = \frac{1}{2}\arcsin(\sigma), \qquad \theta_2 = \theta_1 + \pi \qquad (9)$$

$$\psi_k = \operatorname{arctg}\left[\frac{\mu |I|}{6Km_e \cos^2\theta_k} \right], \quad r_k = L\gamma \frac{\sin\psi_k}{\psi_k}, \quad \dot{\theta}_k = \dot{r}_k = \varphi = \dot{\varphi} = 0$$

where

$$\sigma = \frac{\mu I (m_2 - m_1)}{3K\, m_1 m_2}, \qquad k = 1, 2$$

Static stability condition $|\sigma| < 1$ (10)

Linearization of the equation of motion

$$dy / d\tau = A y \qquad (11)$$

where $\tau = \Omega t$ - dimensionless time, Ω - angular orbital velocity,

$y = \left(\Delta\theta, \Delta\dot{\theta}, \Delta r, \Delta\dot{r}, \varphi, \Delta\dot{\varphi}\right)$ - vector of deviations from equilibrium positions

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ A_{21} & 0 & 0 & A_{24} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ A_{41} & A_{42} & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & A_{65} & 0 \end{pmatrix} \qquad (12)$$

where

$A_{21} = -3\cos 2\theta_{1,2}$, $A_{26} = -2 / r_{1,2}$, $A_{41} = -3 r_{1,2} \sin 2\theta_{1,2}$, $A_{42} = 2 r_{1,2}$

$$A_{43} = 3\cos^2\theta_{1,2} - |b| \operatorname{ctg}\psi_{1,2} - \frac{r_{1,2} |b| \psi_{1,2}^2}{L\gamma \sin^2\psi_{1,2} \left(\sin\psi_{1,2} - \psi_{1,2}\cos\psi_{1,2}\right)}$$

Instability of the linearized system

Characteristic equation

$$\left|A-\lambda E\right|=0 \tag{13}$$

or

$$\left(\lambda^4+a\lambda^2+b\lambda+c\right)\left(\lambda^2+d\right)=0$$

where $d>0, \quad b=-6\sin 2\theta_1=\dfrac{2B_o I\left(m_2-m_1\right)}{m_1 m_2\omega^2}$

Eigenvalues

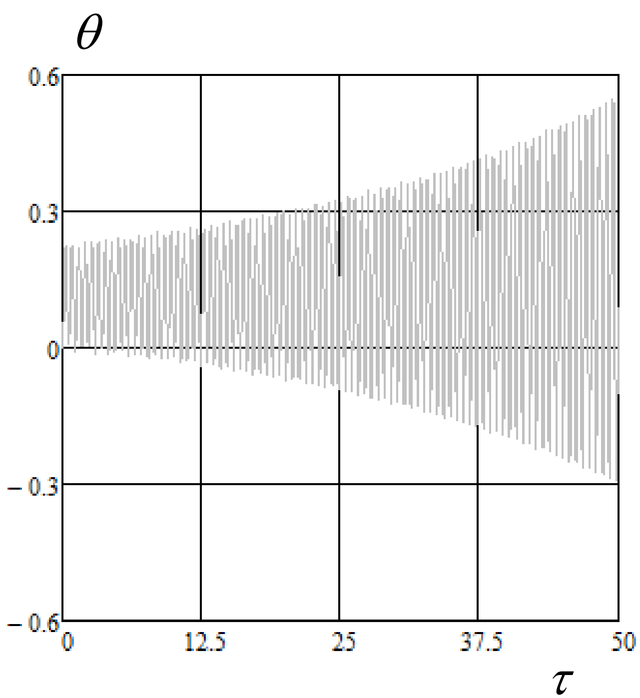
$$\lambda_{1,2}=x_1\pm ix_2, \qquad \lambda_{3,4}=x_3\pm ix_4, \qquad \lambda_{5,6}=\pm ix_5,$$

Always one real part is positive (Instability)

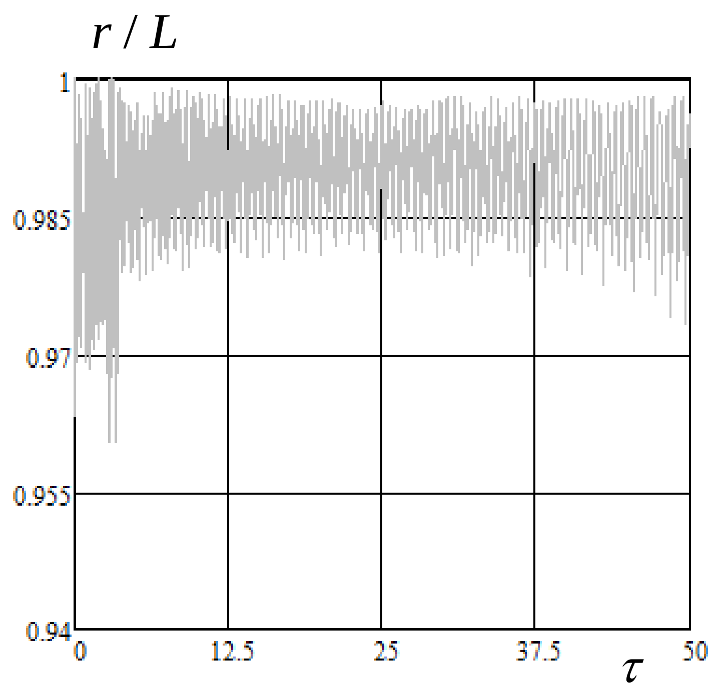
$\left(x_1>0, x_3<0\right)$ or $\left(x_1<0, x_3>0\right)$

When $b=0$ all roots are imaginary.

Numerical results



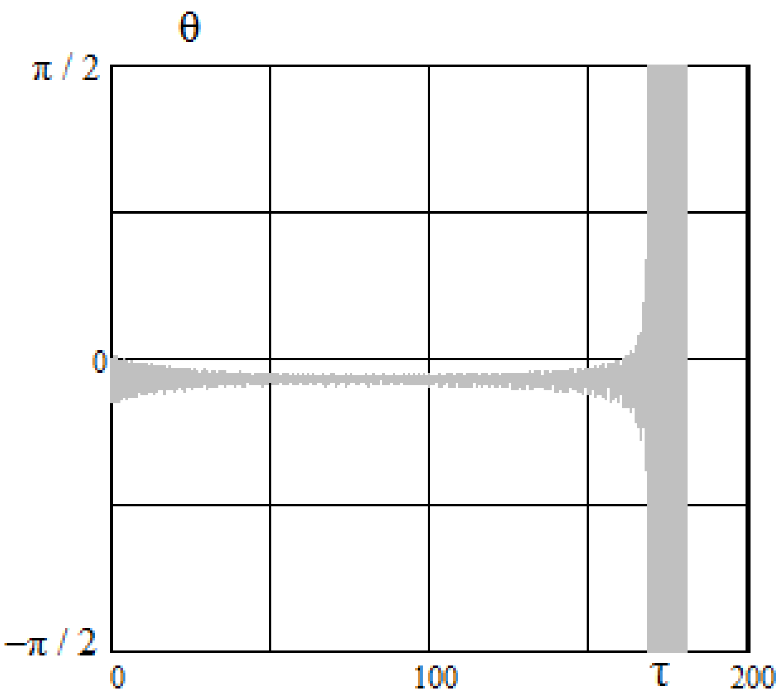
Instability in the orbital plane



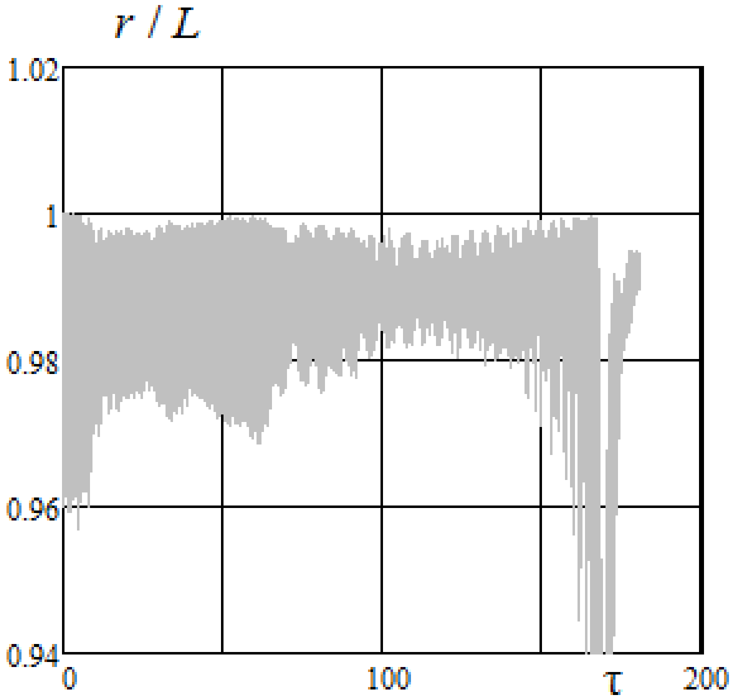
Bending the tether

r - chord, L - Length of tether

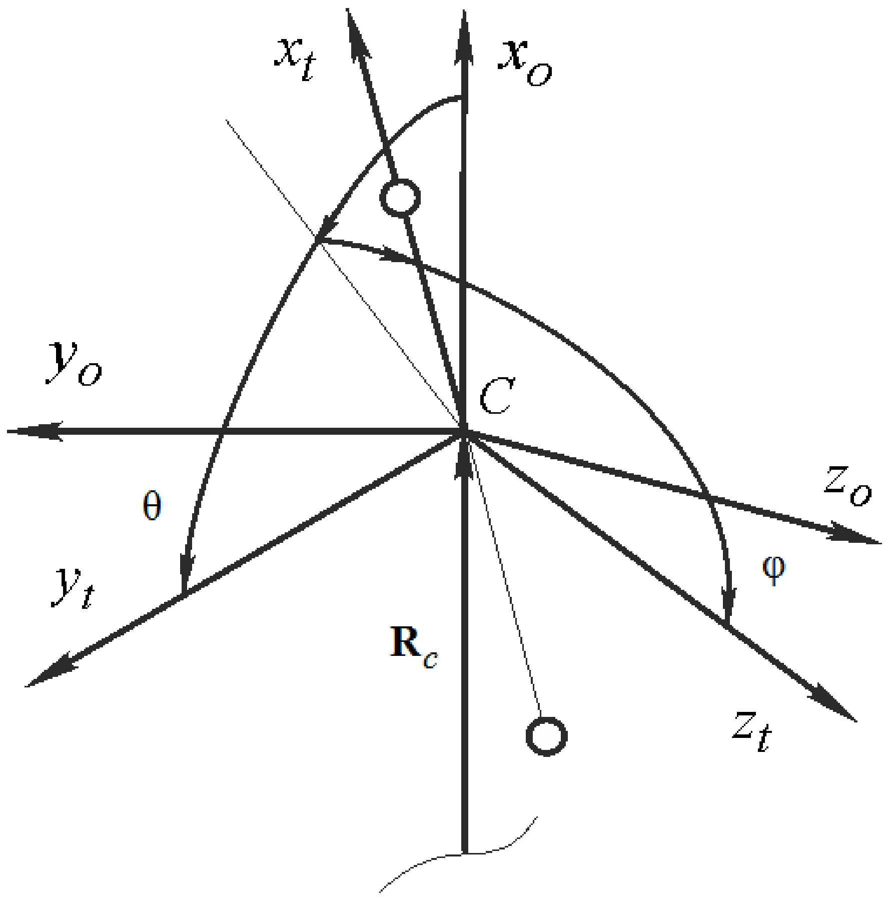
Numerical results



Fluctuations in the plane of the orbit



Instability of bending vibrations of the tether



Lorentz force

$$\mathbf{F} = L(\mathbf{I} \times \mathbf{B}) \qquad (1)$$

The moment of the Lorentz force

$$\mathbf{M} = \mathbf{r}_D \times \mathbf{F} \qquad (2)$$

To stabilize the movement of the EDTS near the vertical , the following are used corrections to the nominal current value

$$I = I_n + \Delta I \qquad (3)$$

Moreover, the current does not change direction

Linearized equations with control

$$dy / d\tau = Ay + mu \qquad (4)$$

where

$$u = \Delta I, \qquad m = \left(0, \frac{B_o}{2m_b\Omega^2}, 0, \frac{B_o}{2m_e\Omega^2}, 0, 0\right)^T$$

Controller synthesis

$$\Delta I = K_{\dot\theta}\,\dot\theta + K_{\dot r}\,\dot r + K_\theta\,\Delta\theta + K_r\,\Delta r \qquad (5)$$

where $K_{\dot\theta}, K_{\dot r}, K_\theta, K_r$ - feedback coefficients

Quadratic optimality criterion

$$J = \int_0^{\tau_k} \left(y^T a y + c u^2\right) d\tau \qquad (6)$$

Weight coefficients of the optimality criterion (7)

$$a = D$$ - diagonal matrix

Relationship between coefficients

$$c + \sum a_{ii} = 1, \qquad a_{ii} = a, \qquad a / c = \nu \tag{8}$$

The Bellman dynamic programming principle is used

Bellman 's condition

$$\min_{u}\left[\frac{\partial V}{\partial y}\left(Ay + mu\right) + y^{T} ay + cu^{2}\right] = 0 \tag{9}$$

Optimal control

$$u^{o} = -\frac{1}{2c}\frac{\partial V}{\partial y} m \tag{10}$$

The Bellman equation

$$\frac{\partial V}{\partial y}Ay + y^T ay - \frac{1}{2c}\left(\frac{\partial V}{\partial y}m\right)^2 = 0 \qquad (11)$$

Solving the Bellman equation

$$V = y^T By \qquad (12)$$

where B - positive definite symmetric matrix

The matrix B from the solution of a system of nonlinear equations is determined by

$$BA + A^T B + a - \frac{1}{c}Bmm^T B = 0 \qquad (13)$$

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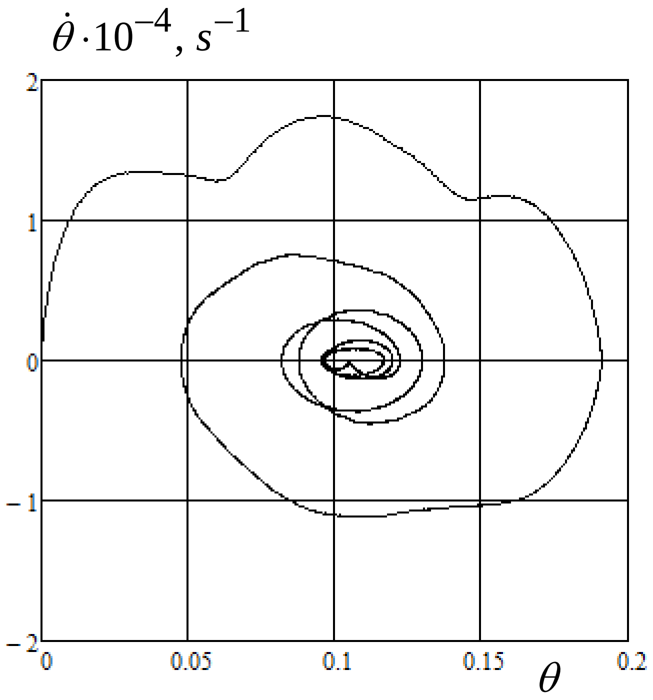
Numerical results

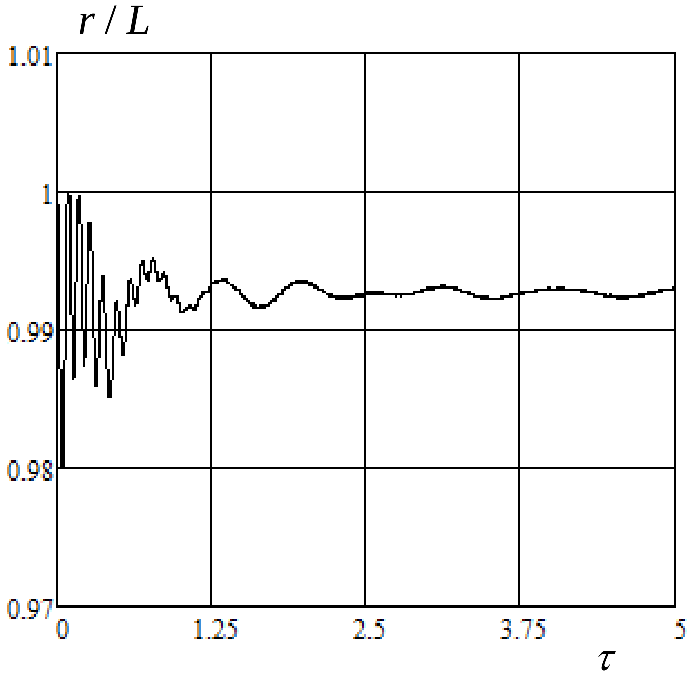
Relations for weight coefficients $a / c = \nu = 0.01$

Feedback coefficients

$$K_{\dot{\theta}} = -0.1168,\ K_{\dot{r}} = -0.1002,\ K_{\theta} = -0.0007,\ K_{r} = -0.1782$$



Motion of system on the phase plane



Flexural vibrations of the tether

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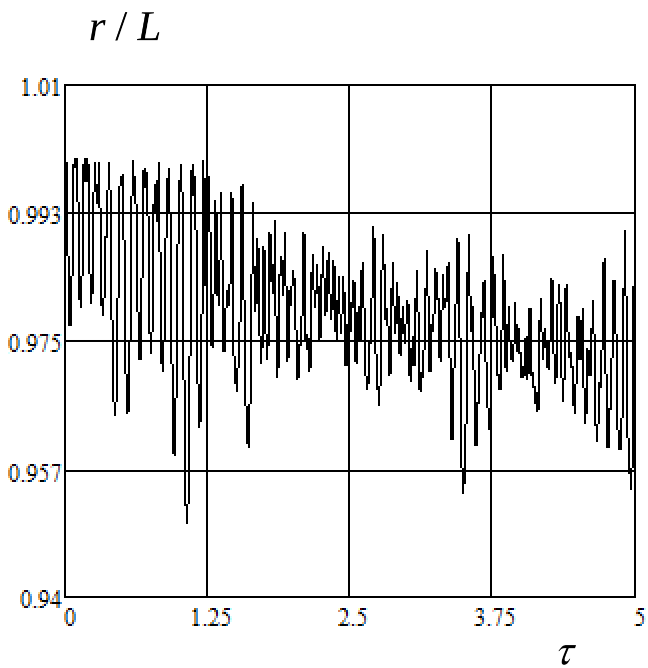
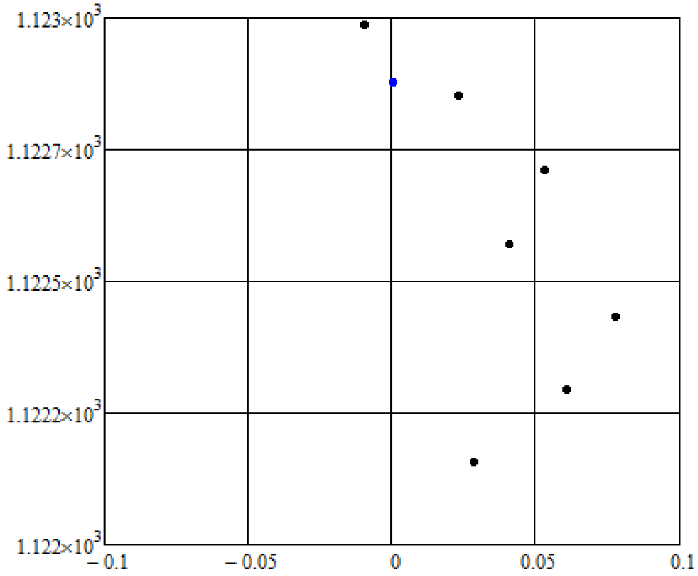
Numerical results

An example of motion instability when $K_{\dot{r}} = K_r = 0$



Flexural vibrations of the tether



Large deformation of the tether

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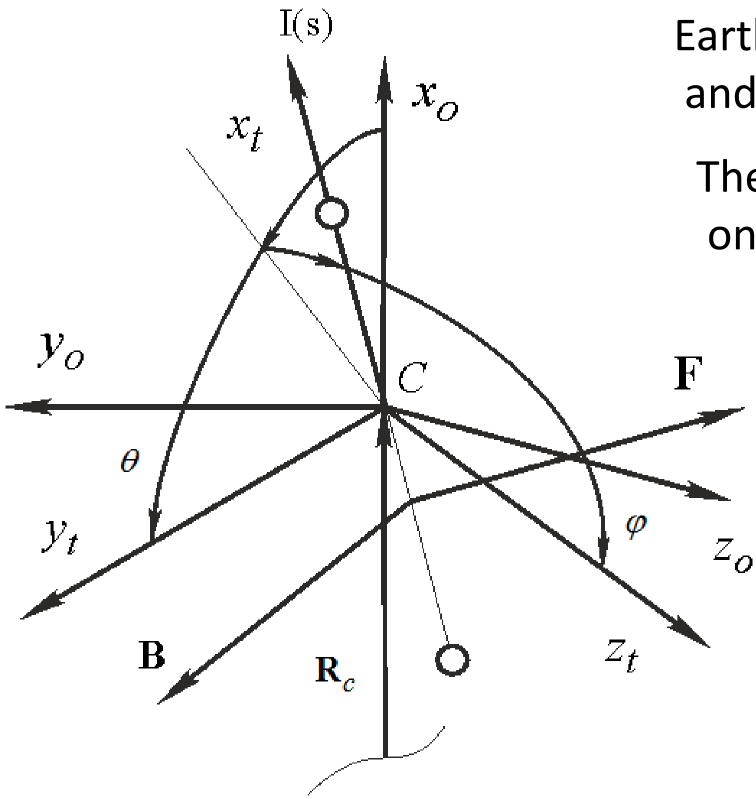
Using a bare conductive tether



When the conductive tether moves in the Earth's magnetic field, an electric field is induced and a current arises

The intensity of the electric field in the projection on the direction of the tether

$$E_m = (\mathbf{V_r} \times \mathbf{B}) \cdot \boldsymbol{\tau} \qquad (1)$$

The current on an conductive tether is distributed unevenly along the length of the tether $I(s)$

Lorentz force acting on the tether

$$\mathbf{F} = \int_L I(s)(\boldsymbol{\tau} \times \mathbf{B}) ds \qquad (2)$$

Equations determining the current distribution over the tether:

$$\frac{dI}{ds} = q_e n_\infty D\, f(\Delta V), \qquad \frac{d\Delta V}{ds} = -\frac{I}{\sigma A} - E_m \qquad (3)$$

where

$$f(\Delta V) = \begin{cases} 0 & if\ \Delta V \le 0 \\ -\left(2 q_e \Delta V / m_e\right)^{0.5} & if\ \Delta V > 0 \end{cases}$$

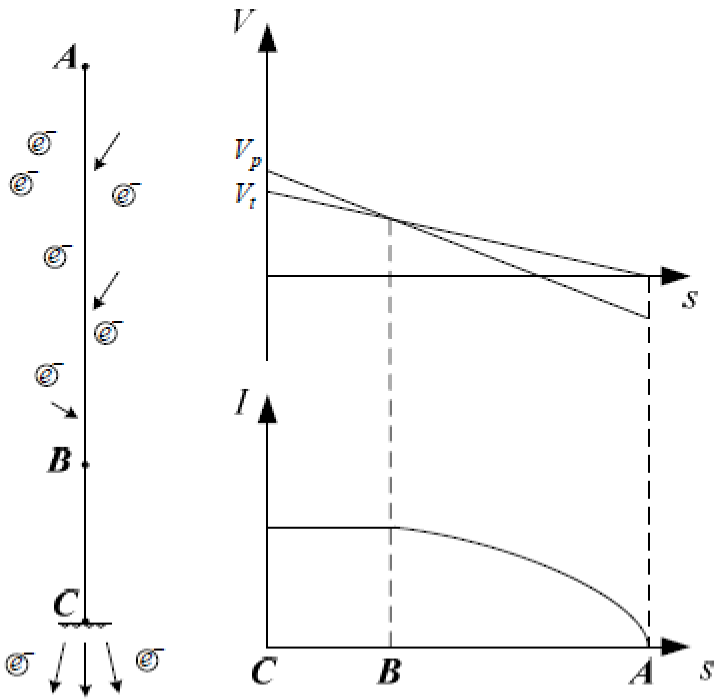
ΔV - potential difference, q_e - electron charge, D - tether diameter,

n_∞ - plasma concentration, σ - electrical conductivity of the tether material.

This source is used:

Xin Chen and J. R. Sanmartín Bare-tether cathodic contact through thermionic emission by low-work-function materials // Physics of Plasmas. American Institute of Physics. 19, 073508 (2012), pp. 1-9.

Equations (3) in dimensionless form



Current distribution

$$\frac{d\gamma}{d\varepsilon} = \frac{3}{4}\sqrt{\lambda} \qquad (4)$$

$$\frac{d\lambda}{d\varepsilon} = \gamma - 1 \qquad (5)$$

Where $\varepsilon = s / L_0$, $\gamma = I / I_0$,

$\lambda = \Delta V / V_0$,

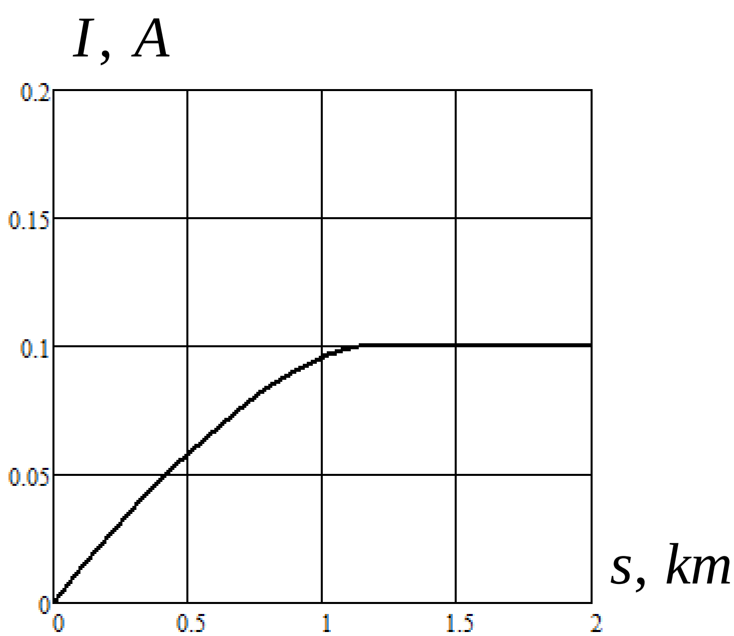
L_0 , I_0 , V_0 - characteristic values.

A – Anode, C – Cathode, B - the point of zero potential

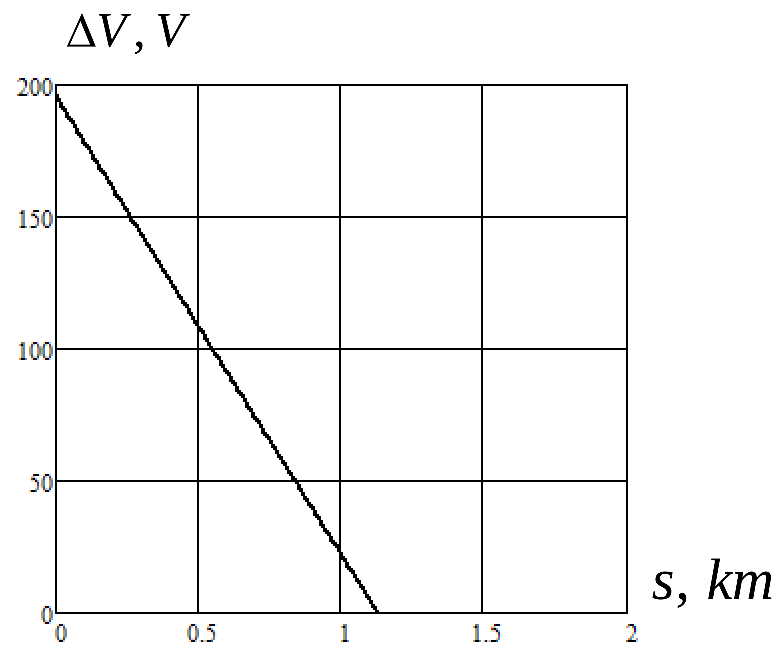
Example of calculating the current distribution over a tether:

$$m_2 = m_A = 2\,kg\,,\quad m_1 = m_c = 6\,kg\,,\quad \rho = 0.25\,kg\,/\,km,\quad L = 2\,km\,,$$

$$I_c = 0.1\;A,\quad D = 1mm\,,\quad \sigma = 3.4\cdot 10^7\;Om^{-1}m^{-1}\;(Al)$$



Current distribution



Potential difference

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Equations of spatial angular motion of the EDTS

$$\ddot{\theta}+\ddot{\vartheta}-2\left(\dot{\theta}+\dot{\vartheta}\right)\dot{\varphi}\operatorname{tg}\varphi+1.5\nu^{-1}\dot{\vartheta}^{2}\sin 2\theta=Q_{\theta}/m_{e}L^{2}\cos^{2}\varphi \qquad (8)$$

$$\ddot{\varphi}+\left[0.5\left(\dot{\theta}+\dot{\vartheta}\right)^{2}+1.5\nu^{-1}\dot{\vartheta}^{2}\cos^{2}\theta\right]\sin 2\varphi=Q_{\varphi}/m_{e}L^{2} \qquad (9)$$

where $m_{e}=\frac{1}{M}\left(m_{1}m_{2}+\frac{1}{3}m_{1}m_{t}+\frac{1}{3}m_{2}m_{t}+\frac{1}{12}m_{t}^{2}\right),\quad \nu=1+e\cos\vartheta,$

e - eccentricity, ϑ - the true anomaly,

$$Q_{\theta}=B_{0}\,J(L)\,L\,\Delta\left[\cos\varphi\cos i-\sin\varphi\sin i\sin\left(\theta+u\right)\right]+\Delta Q_{\theta},$$

$$Q_{3}=B_{0}J\left(L\right)L\Delta\sin i\left[\cos\left(\theta+u\right)+3\sin\theta\sin u\right],$$

$$\Delta Q_{\theta}=3B_{0}J(L)r\Delta\sin i\cos\theta\sin\varphi\sin u, \qquad J(L)=\int_{0}^{L}I(s)ds,$$

Δ - arm of the resultant the Lorentz force,

u - latitude argument, m_{t} - tether mass

Equations of motion of the center of mass in the orbital elements of the EDTS

$$\frac{dA}{dt} = 2\nu\sqrt{\frac{A^3}{K\left(1-q^2-k^2\right)}}\left[a_s\frac{q\sin u - k\cos u}{\nu} + a_t\right]$$

$$\frac{dq}{dt} = \sqrt{\frac{p}{K}}\left[a_s\sin u + a_t\left(1+\frac{1}{\nu}\right)\cos u + \frac{1}{\nu}\left(qa_t + ka_w ctgi\sin u\right)\right]$$

$$\frac{dk}{dt} = \sqrt{\frac{p}{K}}\left[-a_s\cos u + a_t\left(1+\frac{1}{\nu}\right)\sin u + \frac{1}{\nu}\left(ka_t - qa_w ctgi\sin u\right)\right] \qquad (10)$$

$$\frac{di}{dt} = \frac{a_w}{\nu}\sqrt{\frac{p}{K}}\cos u, \qquad \frac{d\Omega}{dt} = \frac{a_w}{\nu}\sqrt{\frac{p}{K}}\frac{\sin u}{\sin i}, \qquad \frac{du}{dt} = \frac{1}{\nu}\sqrt{\frac{p}{K}}\left[\nu^3\frac{K}{p^2} - a_w ctgi\sin u\right]$$

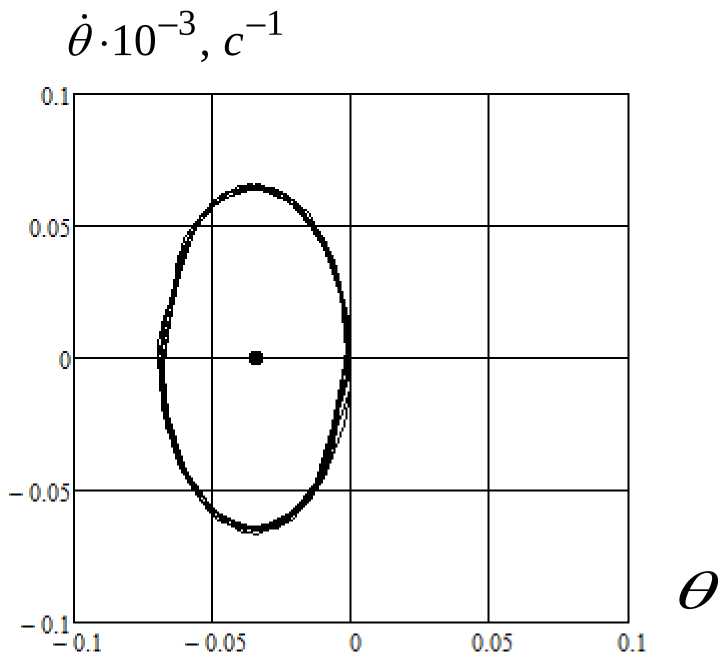
where A - large semi - axis, $q = e\cos\omega_\pi$, $k = e\sin\omega_\pi$,

ω_π - the pericenter argument, Ω - longitude of the ascending node,

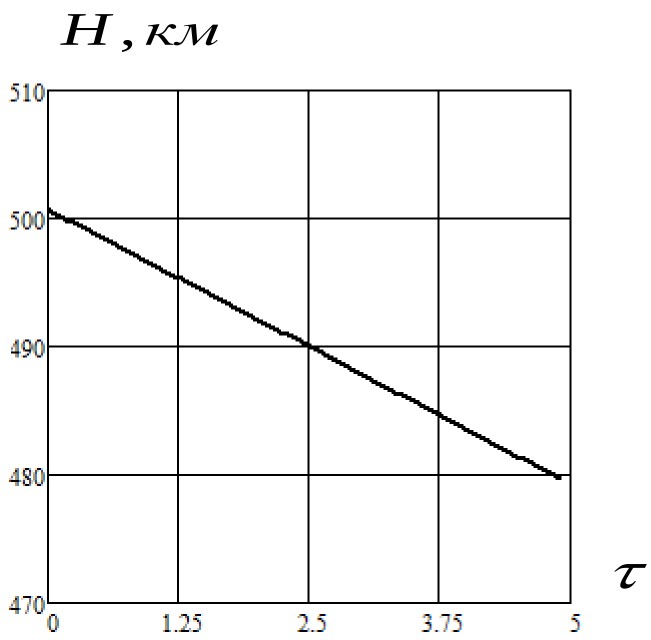
a_s, a_t, a_w - acceleration components from the Lorentz force along the axes of the orbital coordinate system $Cx_oy_oz_o$

The movement of the EDTS in an almost circular orbit with an inclination $i = 0$

$$m_1 = 6\,kg,\; m_2 = 2\,kg$$



Oscillations on the phase plane

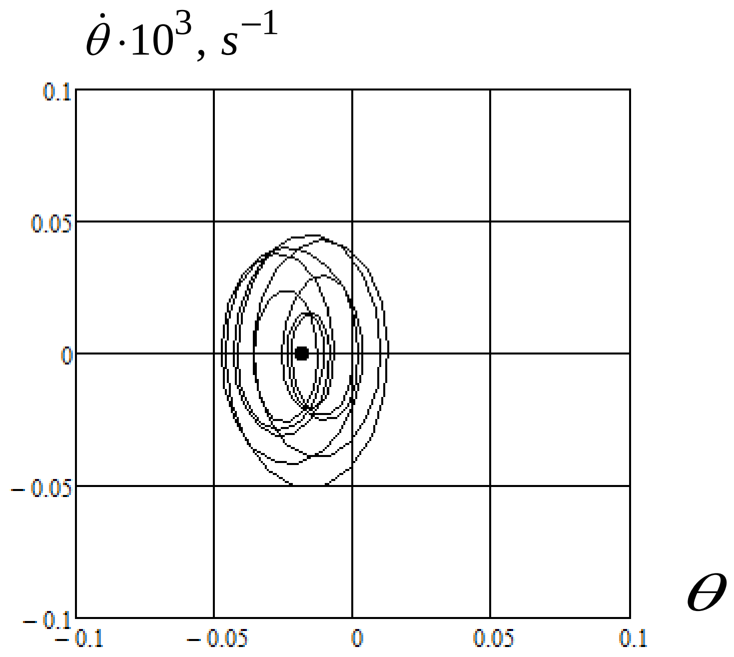


Changing the altitude of the orbit.

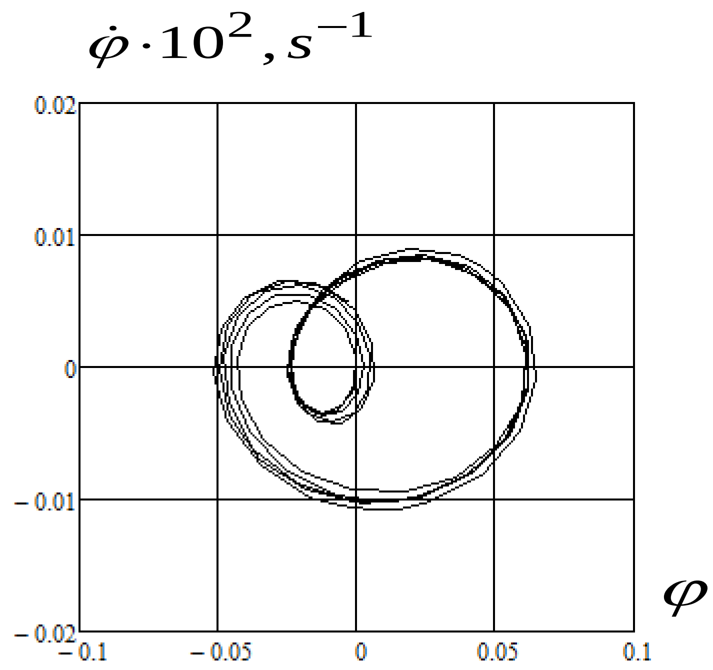
The TS for lowering the orbit is used

Fluctuations of the EDTS in an elliptical orbit with inclination

$$e = 0.01 \qquad i = \pi / 3$$



Oscillations in the plane of the orbit



Oscillations outside the plane of the orbit

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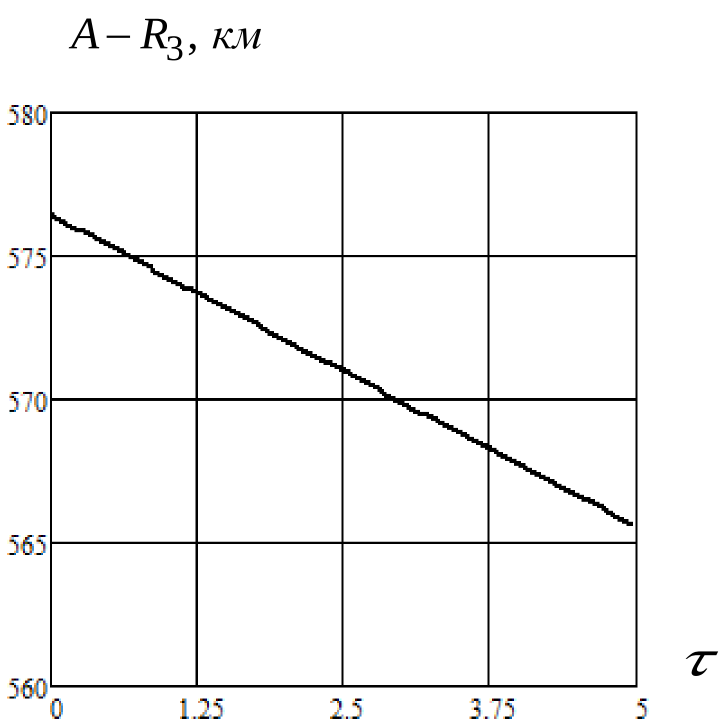
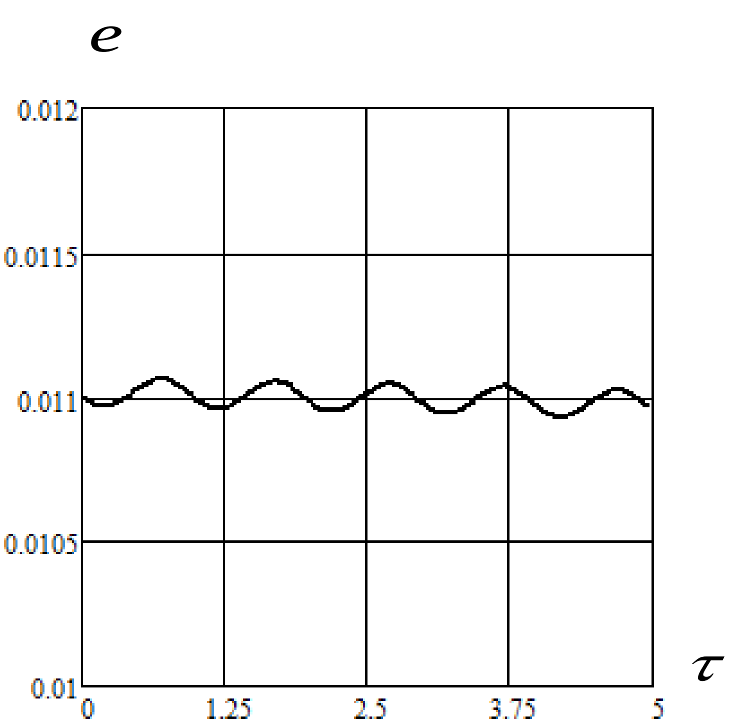
Changing the parameters of the elliptical orbit

$e = 0.011 \qquad i = \pi / 3$



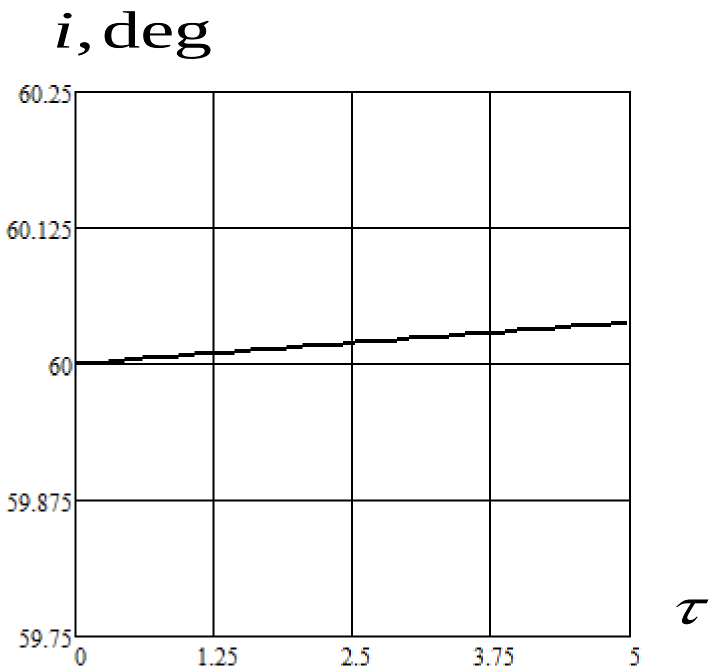
The large semi-axis minus the average radius of the Earth



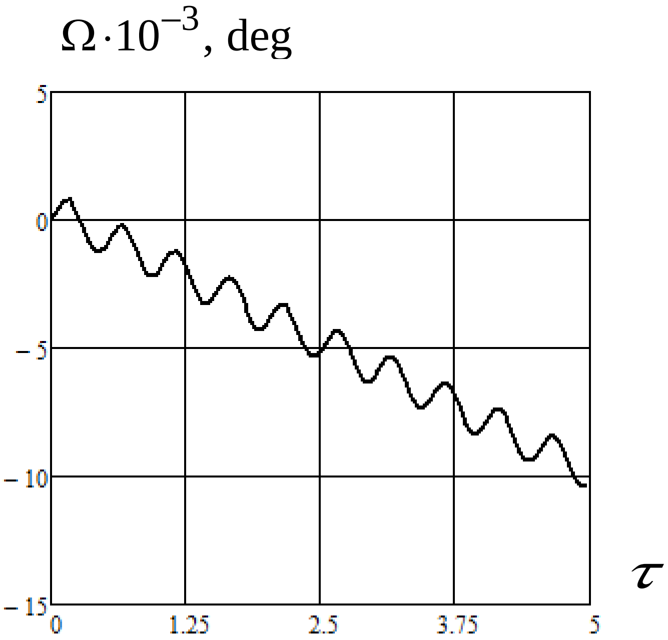
Eccentricity

Changing the parameters of the elliptical orbit

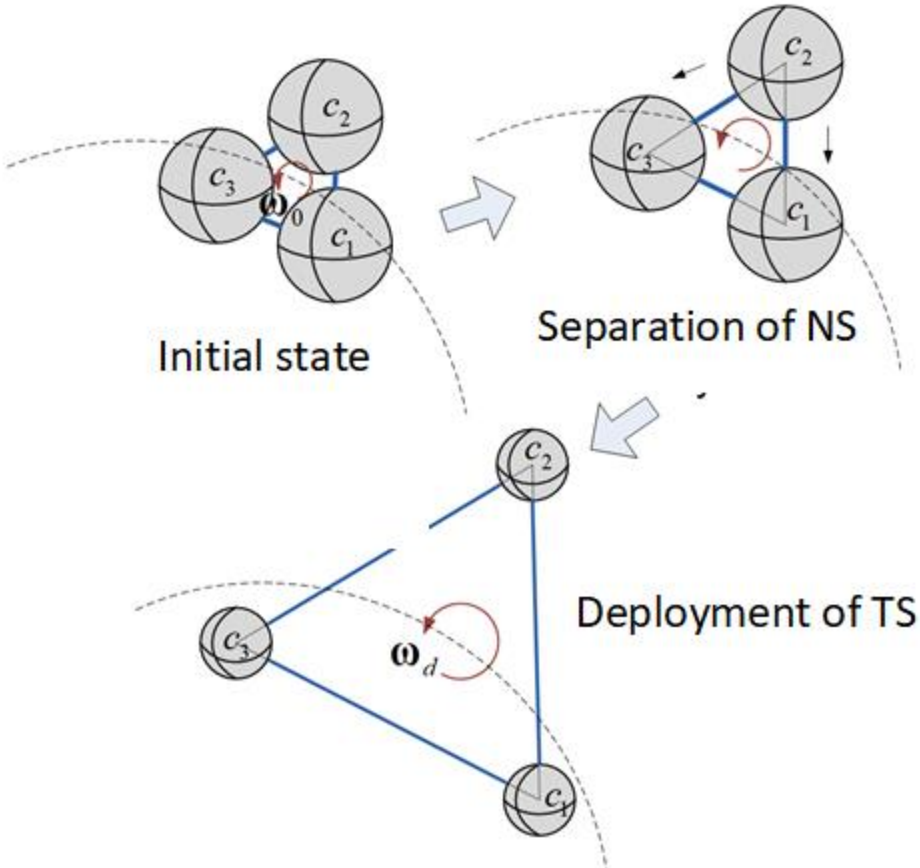
$$e = 0.011 \qquad i = \pi / 3$$



Inclination of the orbit



Longitude of the ascending node of the orbit



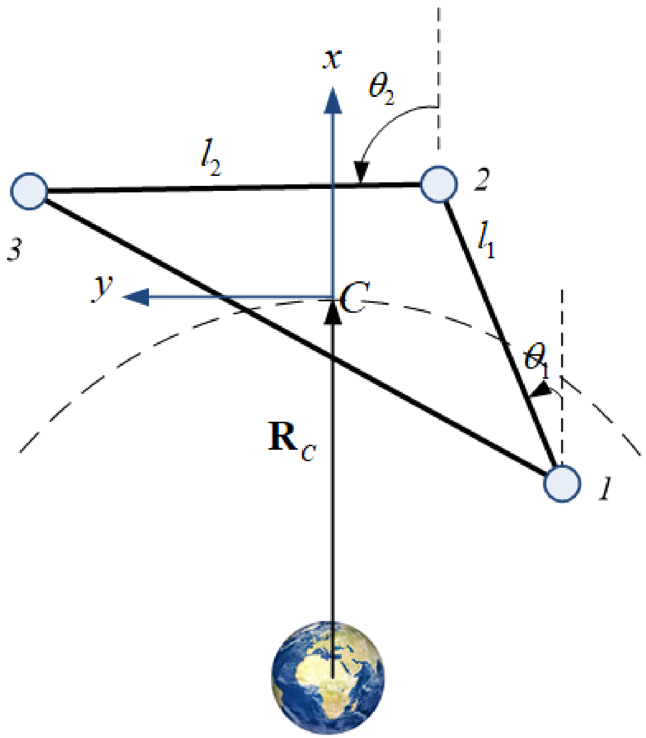
1. In the initial state, until the moment of separation, the system is a regular triangle and the solid body.

2. In the final state, the system is a the right triangle that rotates at a given angular velocity.

Composition of the control system:

1. Low-thrust jet engines located on satellites.
2. Control mechanisms for the release of tethers for regulating tension forces, working only on their braking

Mathematical model of motion



Lagrange`s equations

$$\frac{d}{dt}\left(\frac{\partial T_C}{\partial \dot{q}_j}\right)-\frac{\partial T_C}{\partial q_j}=-\frac{\partial \Pi}{\partial q_j}+Q_{q_j} \qquad (1)$$

where

$$q_1=l_1,\ q_2=l_2,\ q_3=\theta_1,\ q_4=\theta_2$$

– generalized coordinates,

T_C, Π – kinetic and potential energies of the system,

Q_{q_j} – generalized non-potential forces.

Mathematical model of motion

$$
\begin{aligned}
&m\mu_1(\mu_2+\mu_3)\left\{\ddot{l}_1 - l_1\left[(\dot\theta_1+\omega)^2+\omega^2\left(3\cos^2\theta_1-1\right)\right]\right\}+\\
&+m\mu_1\mu_3\left\{\left[\ddot{l}_2 - l_2\dot\theta_2(\dot\theta_2+2\omega)\right]\cos\Delta\theta+\left[l_2\ddot\theta_2+2\dot{l}_2(\dot\theta_2+\omega)\right]\sin\Delta\theta-3\omega^2 l_2\cos\theta_1\cos\theta_2\right\}=Q_{l_1},
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
&m\mu_3(\mu_1+\mu_2)\left\{\ddot{l}_2 - l_2\left[(\dot\theta_2+\omega)^2+\omega^2\left(3\cos^2\theta_2-1\right)\right]\right\}+\\
&+m\mu_1\mu_3\left\{\left[\ddot{l}_1 - l_1\dot\theta_1(\dot\theta_1+2\omega)\right]\cos\Delta\theta-\left[l_1\ddot\theta_1+2\dot{l}_1(\dot\theta_1+\omega)\right]\sin\Delta\theta-3\omega^2 l_1\cos\theta_1\cos\theta_2\right\}=Q_{l_2},
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
&m\mu_1(\mu_2+\mu_3)l_1^2\left[\ddot\theta_1+2\left(\dot{l}_1/l_1\right)(\dot\theta_1+\omega)+3\omega^2\sin\theta_1\cos\theta_1\right]+m\mu_1\mu_3 l_1 l_2\left\{3\omega^2\sin\theta_1\cos\theta_2+\right.\\
&\left.+\left[\ddot\theta_2+2\left(\dot{l}_2/l_2\right)(\dot\theta_2+\omega)\right]\cos\Delta\theta+\left[(\dot\theta_2+\omega)^2-\omega^2-\left(\ddot{l}_2/l_2\right)\right]\sin\Delta\theta\right\}=Q_{\theta_1},
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&m\mu_3(\mu_1+\mu_2)l_2^2\left[\ddot\theta_2+2\left(\dot{l}_2/l_2\right)(\dot\theta_2+\omega)+3\omega^2\sin\theta_2\cos\theta_2\right]+m\mu_1\mu_3 l_1 l_2\left\{3\omega^2\sin\theta_2\cos\theta_1+\right.\\
&\left.+\left[\ddot\theta_1+2\left(\dot{l}_1/l_1\right)(\dot\theta_1+\omega)\right]\cos\Delta\theta+\left[\omega^2-(\dot\theta_1+\omega)^2+\left(\ddot{l}_1/l_1\right)\right]\sin\Delta\theta\right\}=Q_{\theta_2}.
\end{aligned}
\tag{5}
$$

where $m=\sum_{k=1}^{3} m_k,\ \mu_k=m_k/m,\quad \Delta\theta=\theta_1-\theta_2.$

- The law of change of tension forces

$$T_k = T_k^0 + p_k\left(l_k - L_{d_k}\right) + w_k \dot{l}_k,\ k = \overline{1,3}\,, \quad (9)$$

where p_k, w_k – feedback coefficients, L_{d_k} – final tether lengths,

T_k^0 – the magnitude of the tension forces in the ideal case when the configuration of the system is a "symmetrical triangle". When $F_k = 0,\ \ddot{l}_k = \dot{l}_k = 0,\ \ddot{\theta}_k = 0$

$$T_k^0 = m\,l_k\left(\dot{\theta}_k + \omega\right)^2 / 9,\ k = 1,2,$$

$$T_3^0 = m\,l_3\left(\dot{\theta}_2 + \omega\right)^2 / 9.$$

- The law for reactive Forces

$$F_k = \begin{cases} F, & \text{при } t < t_e \\ 0, & \text{при } t \geq t_e \end{cases},\ k = \overline{1,3}\,. \quad (10)$$

where t_e – the moment of switching off the low-thrust engines.

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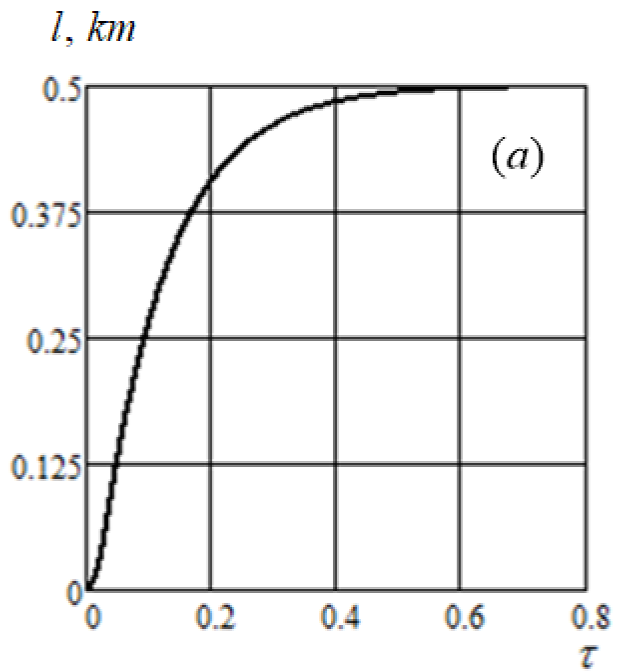
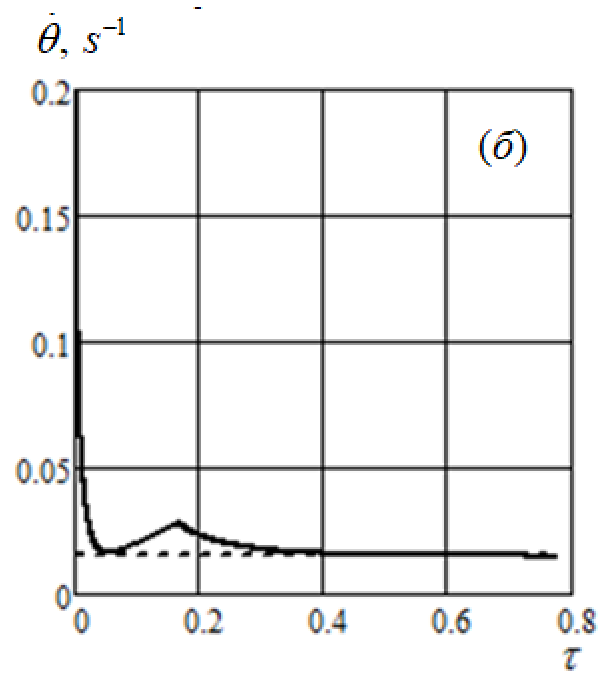
- Example of a nominal program

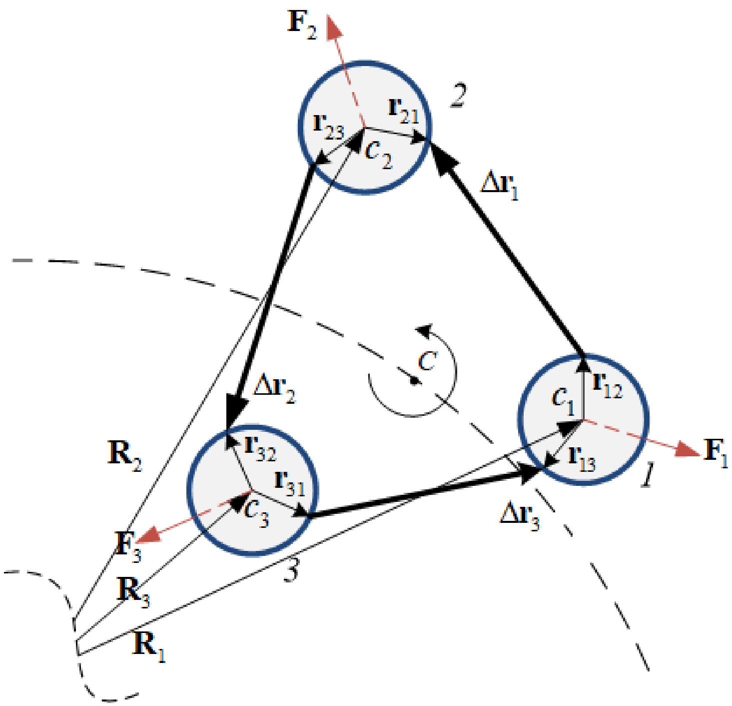


Changing the length of the tethers



Changing the rotation speed of the tethers

Mathematical model of the movement of the TS "triangle" taking into account the movements of the NS relative to its centers of mass



Equations of motion of the centers of mass NS

$$m_1\ddot{\mathbf{R}}_1 = \mathbf{G}_1 + \mathbf{T}_1 - \mathbf{T}_3 + \mathbf{F}_1 \ , \qquad (14)$$

$$m_k\ddot{\mathbf{R}}_k = \mathbf{G}_k + \mathbf{T}_k - \mathbf{T}_{k-1} + \mathbf{F}_k, \ \ k = 2,3,$$

where

$\mathbf{G}_k = -\mu_e m_k \mathbf{R}_k / R_k^3$ – gravitational forces.

$\mathbf{T}_k, \mathbf{F}_k$ – tension forces and reactive forces.

$$\mathbf{T}_k = T_k \frac{\Delta\mathbf{r}_k}{\Delta r_k}, \qquad (15)$$

$$T_k = \begin{cases} ES\dfrac{\Delta r_k - L_k}{L_k}, & \Delta r_k - L_k \geq 0, \\ 0 \qquad , & \Delta r_k - L_k < 0, \end{cases}$$

where ES – stiffness of the tether material.

Mathematical model of the movement of the TS "triangle" taking into account the movements of the NS relative to its centers of mass

Equations that take into account the dynamics of the release mechanism of tethers

$$m_e \ddot{L}_k = T_k - U_k,\ k = \overline{1,3}, \qquad (16)$$

where m_e – coefficient of inertia of control mechanisms.

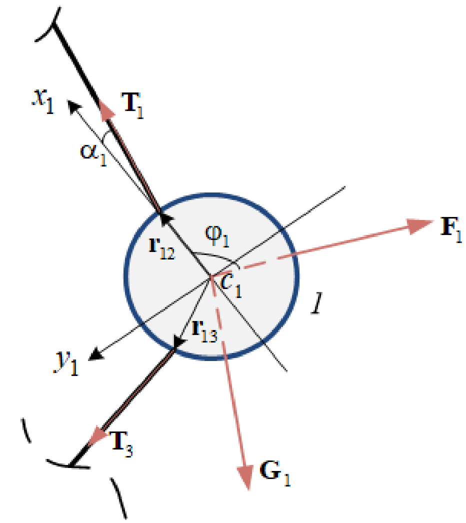
L_k - undeformed tether length

Controlling forces U_k

$$U_k = p_g\left(L_k - l_k\right) + w_g\left(\dot{L}_k - \dot{l}_k\right), k = \overline{1,3}\,, \qquad (17)$$

feedback coefficients.

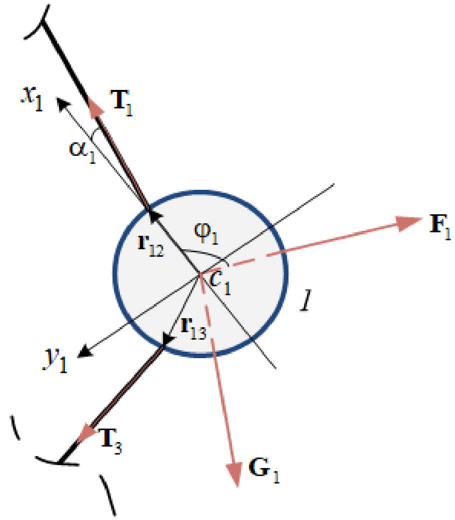
nominal values.



Equations describing angular motions of satellites

$$\dot{\boldsymbol{\omega}}_k = J_k^{-1}\left(\mathbf{M}_k - \boldsymbol{\omega}_k \times J_k \boldsymbol{\omega}_k\right), k = \overline{1,3}, \qquad (18)$$

$$\dot{\mathbf{e}}_{xk} = \boldsymbol{\omega}_k \times \mathbf{e}_{xk},\ \dot{\mathbf{e}}_{yk} = \boldsymbol{\omega}_k \times \mathbf{e}_{yk},\ \dot{\mathbf{e}}_{zk} = \boldsymbol{\omega}_k \times \mathbf{e}_{zk}, \qquad (19)$$



where

$\boldsymbol{\omega}_k$ – the angular velocity of the satellite,

J_k – satellite inertia tensor,

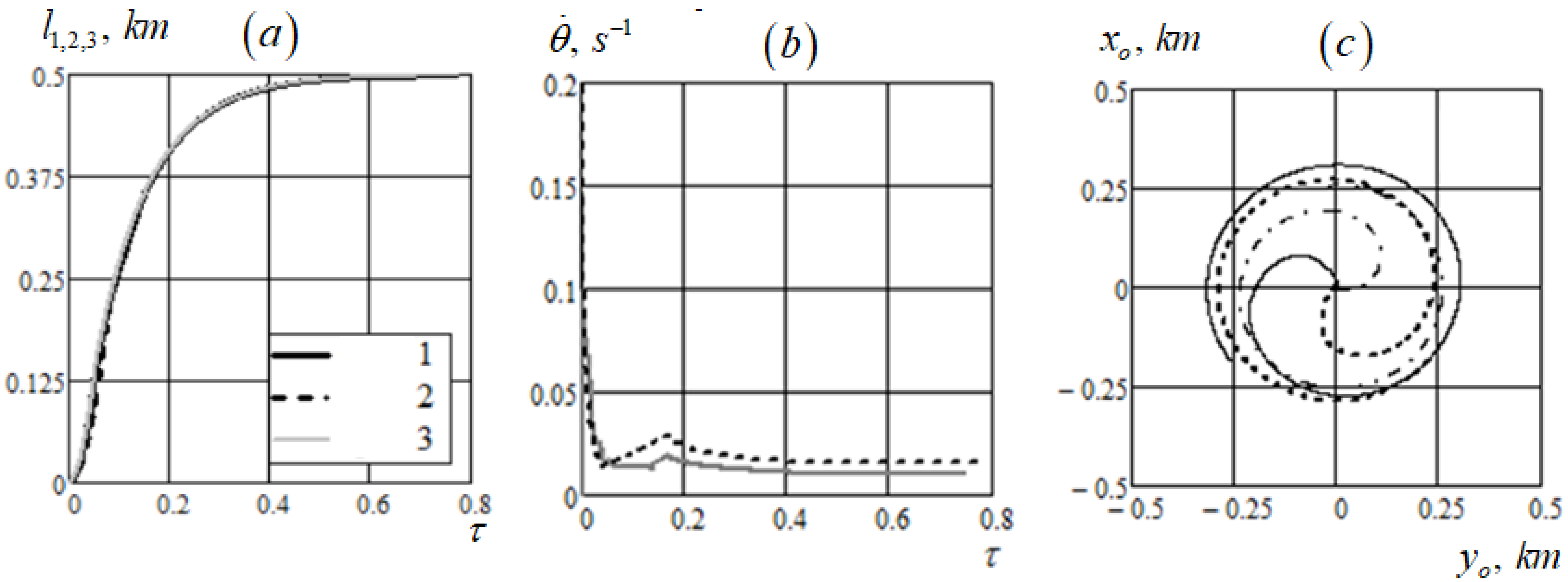
$\mathbf{e}_{xk}, \mathbf{e}_{yk}, \mathbf{e}_{zk}$ – orts of related coordinate systems $c_k x_k y_k z_k,\ k = \overline{1,3}.$

Angle between the directions of the tethers and the characteristic axes of the satellites

$$\alpha_k = \arccos\left(\frac{\Delta\mathbf{r}_k \cdot \mathbf{e}_{xk}}{\Delta r_k}\right),\ k = \overline{1,3};\ \alpha_1^{(3)} = \arccos\left(\frac{\Delta\mathbf{r}'_3 \cdot \mathbf{e}_{x1}}{\Delta r_3}\right); \qquad (20)$$

$$\alpha_k^{(k-1)} = \arccos\left(\frac{\Delta\mathbf{r}'_{k-1} \cdot \mathbf{e}_{xk}}{\Delta r_{k-1}}\right),\ k = 2,3.$$

Simulation results based on a more complete model



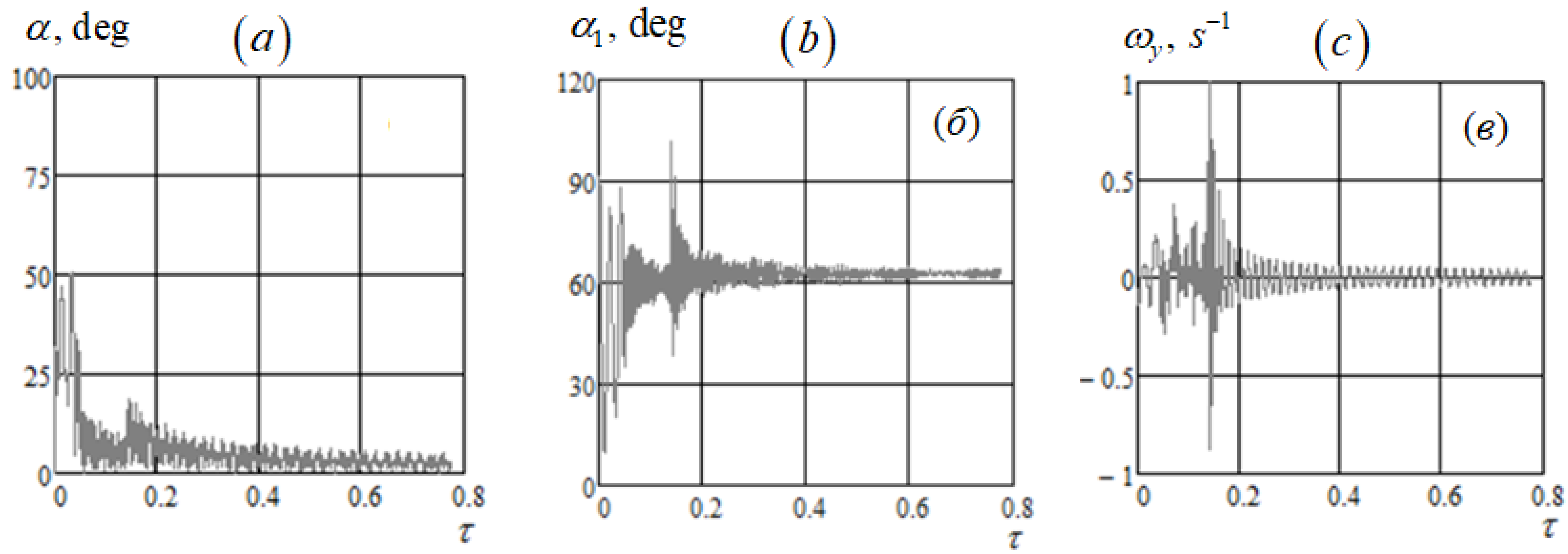
(*a*) – Changing tethers lengths from time;

(*b*) – Changing the speed of rotation of tethers from time;
(dashed line – nominal value) ;

(*c*) – Trajectories of NS relative to the center of mass of the TS (0,0).

Simulation results based on a more complete model

In the figures, the angles between the characteristic axes of the NS and the tethers are given



(a) – The angle between the axis (c_3x_3) NS 3 and the direction of the tether 3 (in the nominal case: $\alpha_3 = 0$);

(b) – The angle between the axis (c_1x_1) NS 1 and the direction of the tether 3 (in the nominal case : $\alpha_1^{(3)} = \pi/3$);

(c) – The change in the component of the angular velocity of NS 2 from time.

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Thank you for attention

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