



**САМАРСКИЙ УНИВЕРСИТЕТ**  
SAMARA UNIVERSITY

# **XVII International Summer Space School: Future Space Technologies and Experiments in Space**

## **Solar sail**

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04 Sep 2023

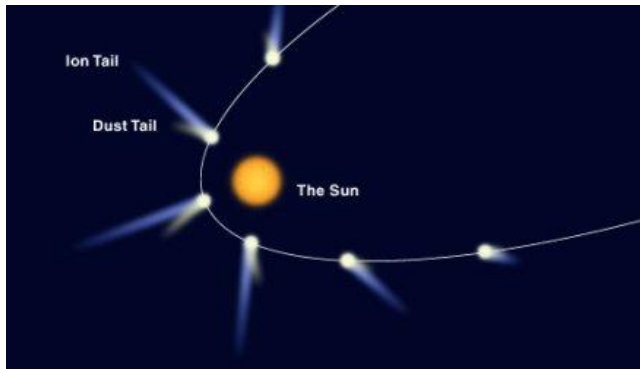


# Solar sailing

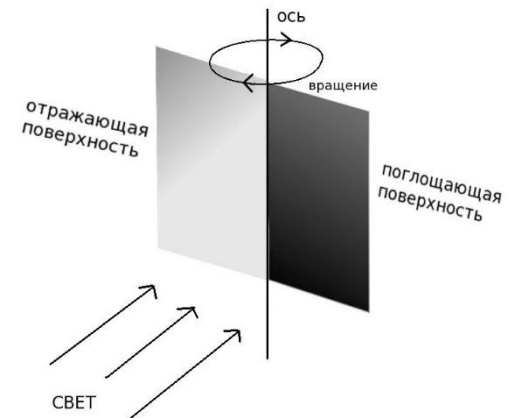




# Solar power pressure

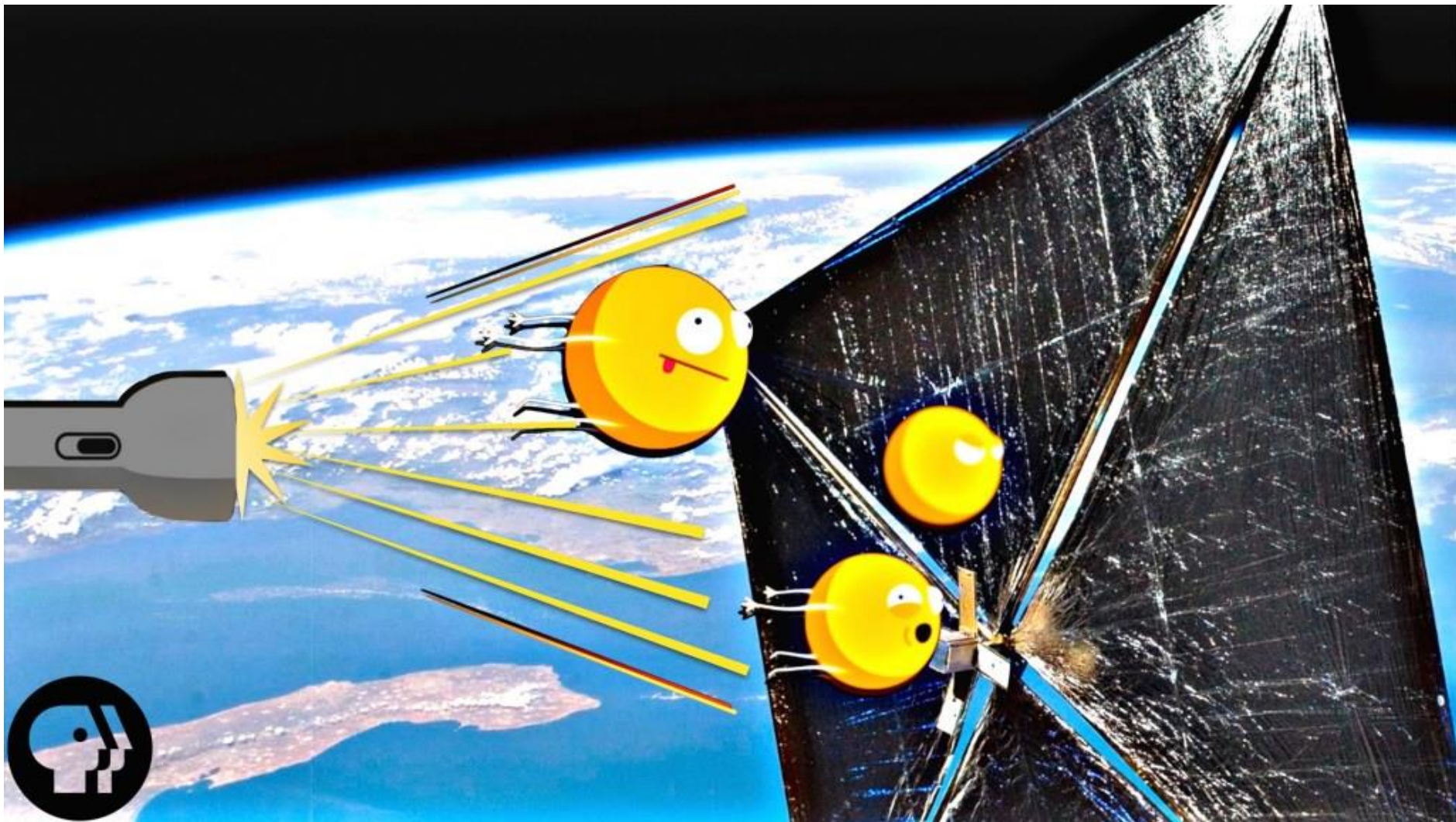


$$P_r = \frac{(1 + \varepsilon) S_r}{c}$$





# Solar power pressure





# Solar power pressure

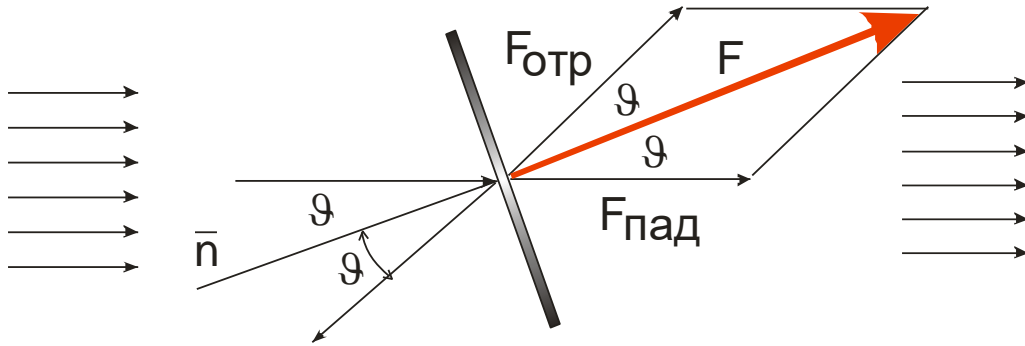


Figure 1 - Thrust magnitude and direction for a perfectly reflective sail

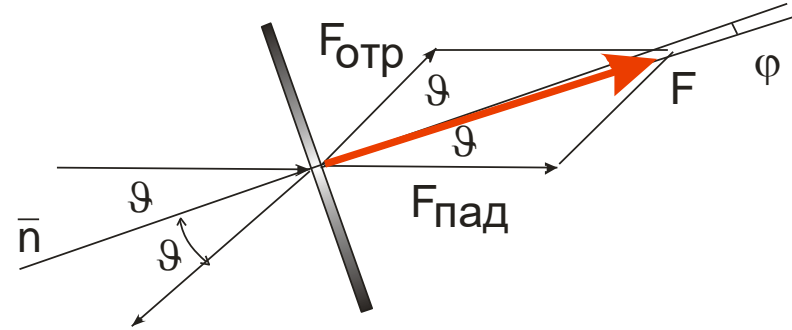


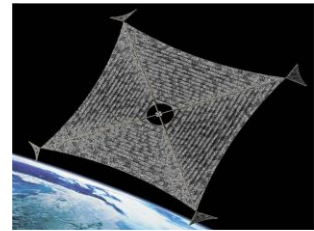
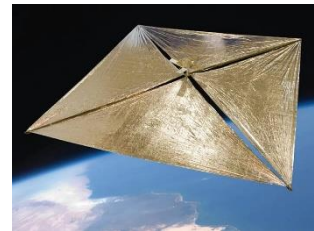
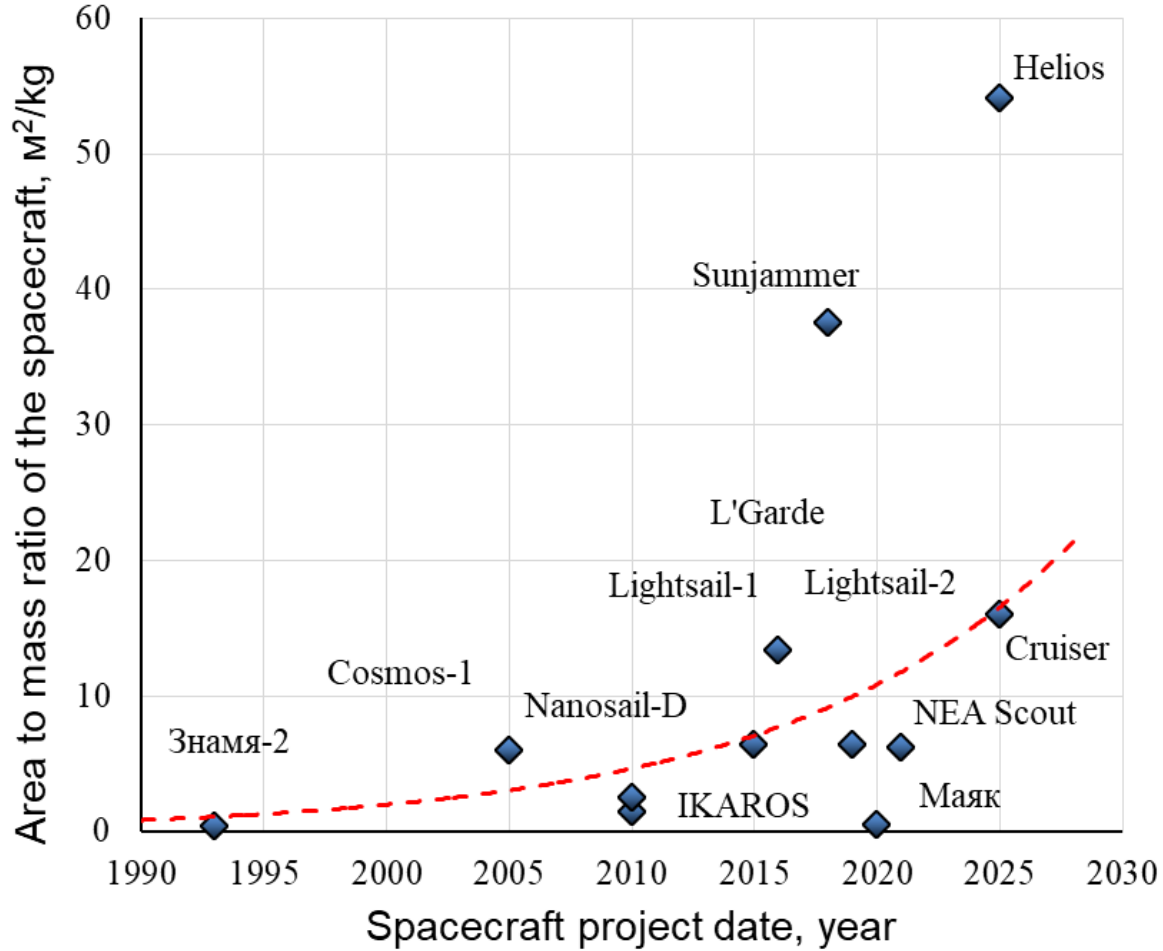
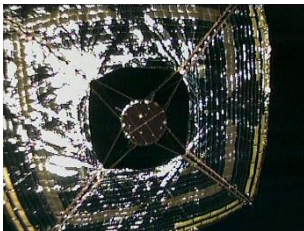
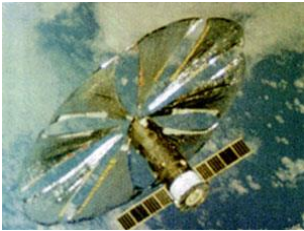
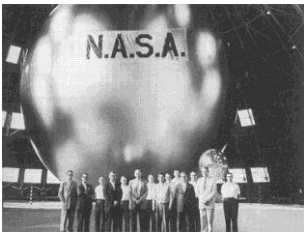
Figure 2 - Thrust magnitude and direction for a non-perfectly reflective sail

## The value of light pressure on the orbits of the planets of the Solar system

Planet	Absorbing sail	Perfectly reflective sail
Mercury	$3,1 \cdot 10^{-5}$	$6,2 \cdot 10^{-5}$
Venus	$8,9 \cdot 10^{-6}$	$1,78 \cdot 10^{-5}$
Earth	$4,64 \cdot 10^{-6}$	$9,28 \cdot 10^{-6}$
Mars	$2,0 \cdot 10^{-6}$	$4,0 \cdot 10^{-6}$
Jupiter	$1,7 \cdot 10^{-7}$	$3,4 \cdot 10^{-7}$

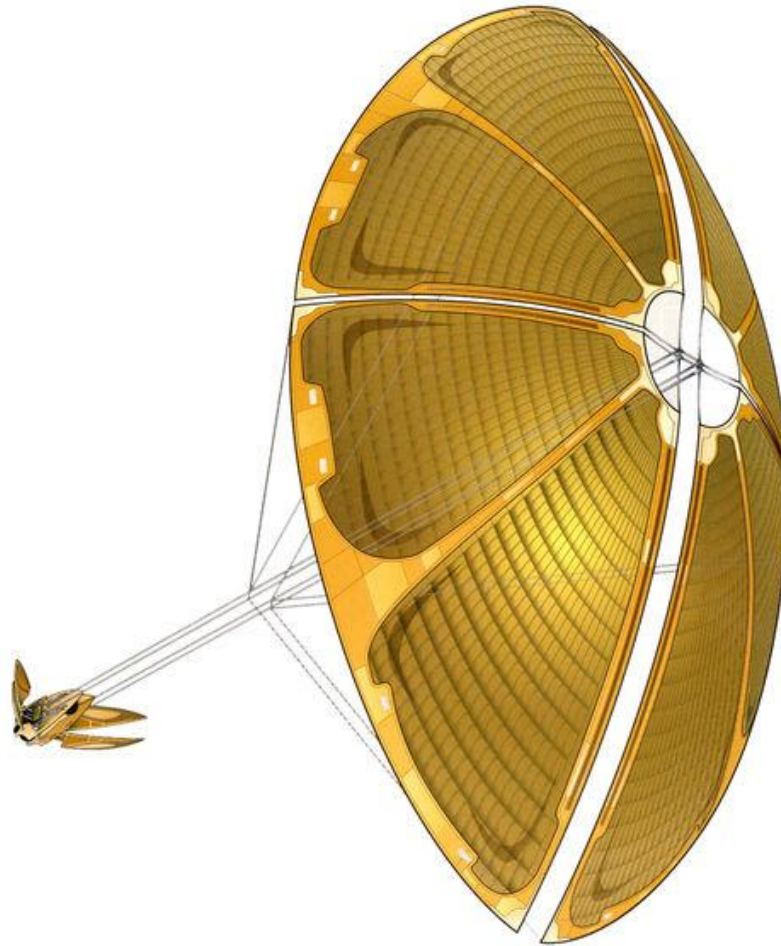
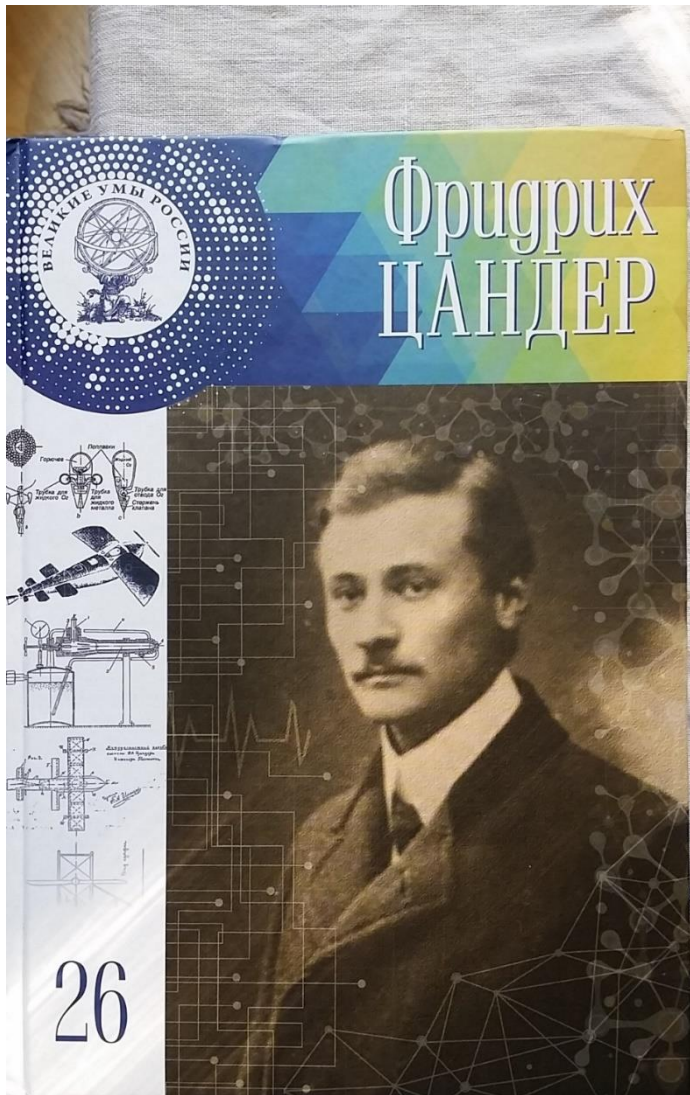


# Design parameters of solar sails





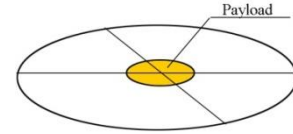
# Solar power pressure



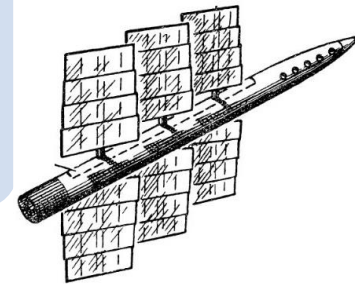
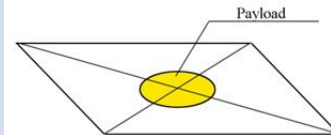


# Design types of solar sails

Disc Sail



Square Sail (Yankee Clipper)



## Frame

Single-layer

Box layered

## Rotating

Rotating the canvas

Petal

Heliogyro

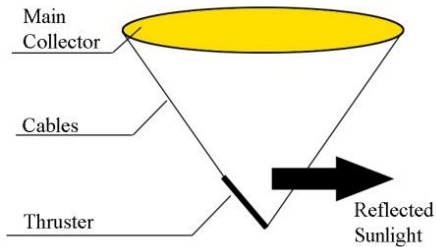
## Inflatable

Hollow body

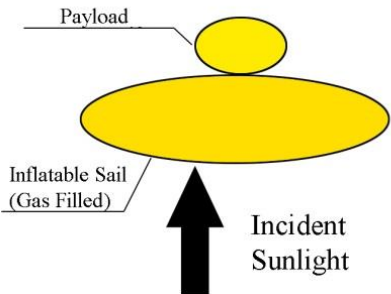
Inflatable reflectors

Inflatable frame

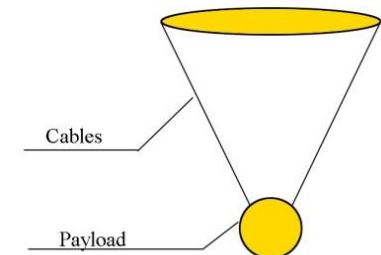
Parabolic Sail (Solar Photon Truster)



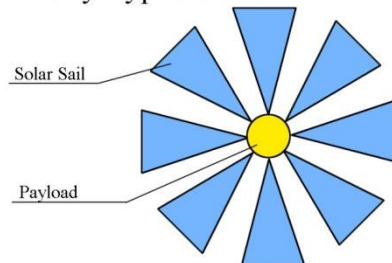
Hollow Body (Pillow Sail)



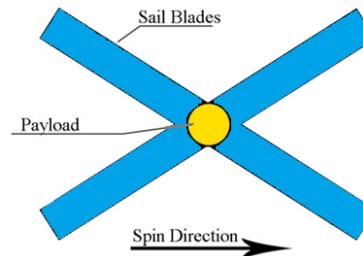
Parachute Sail



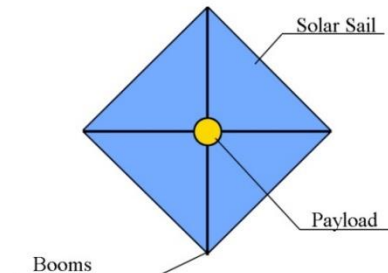
Rotary-Type Solar Sail



Heliogyro



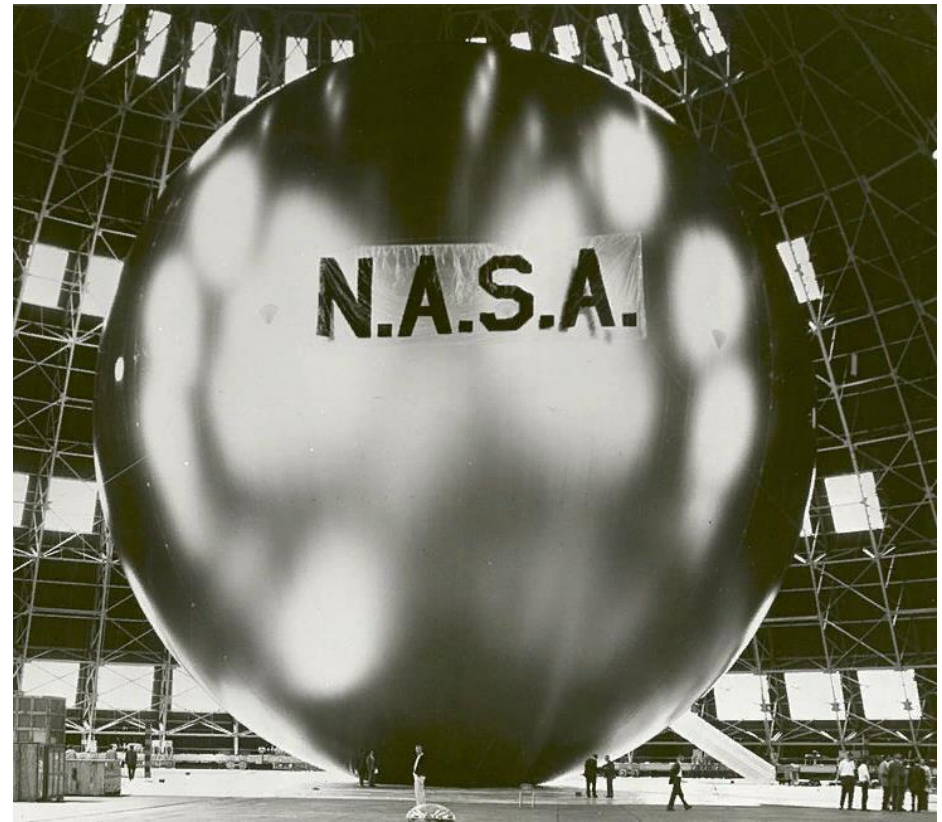
Frame-Type Solar Sail







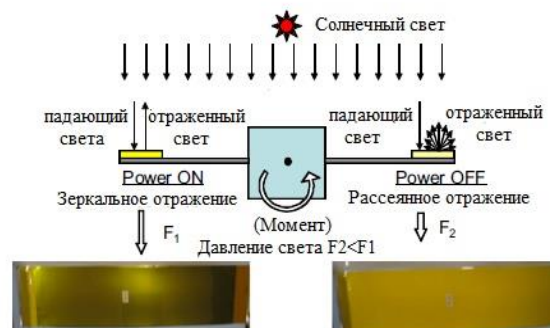
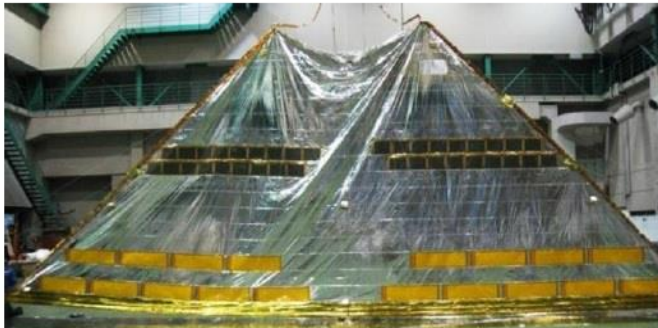
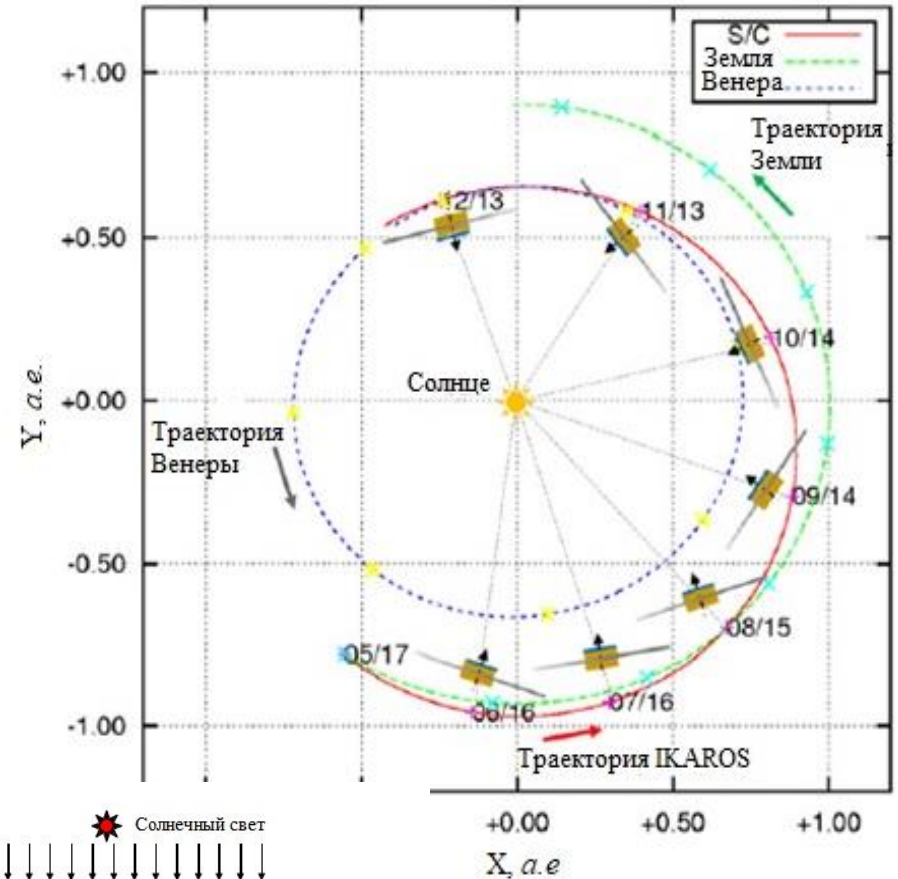
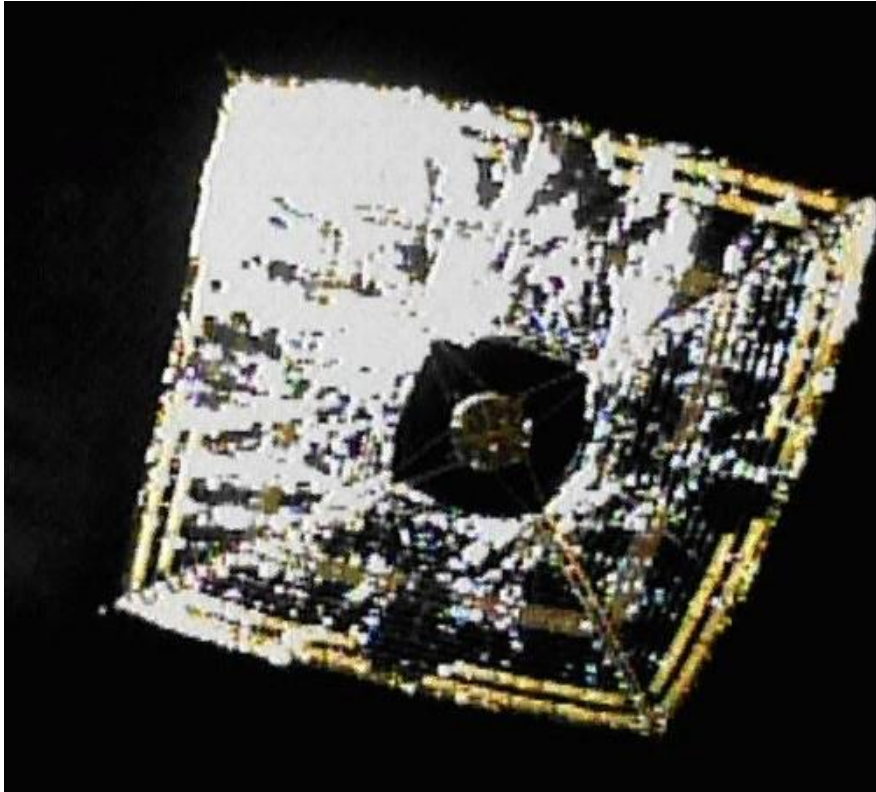
## Satellites repeaters Echo 1 and Echo 2



Satellites repeaters Echo-1 (12.08.1960) и Echo-2 (25.01.1964)

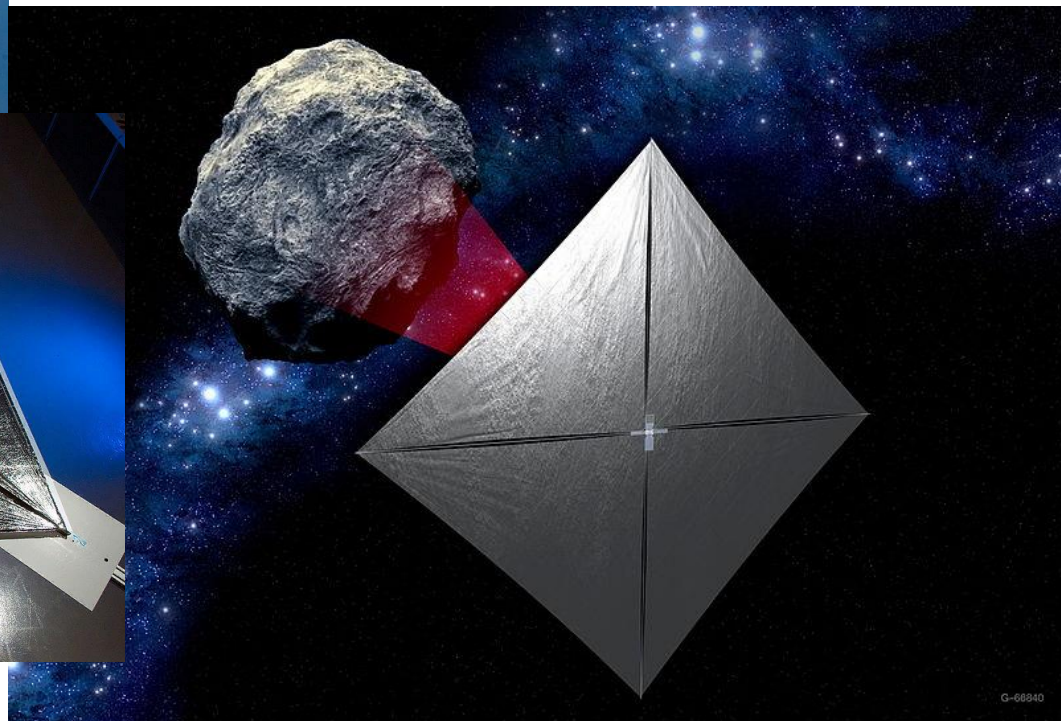
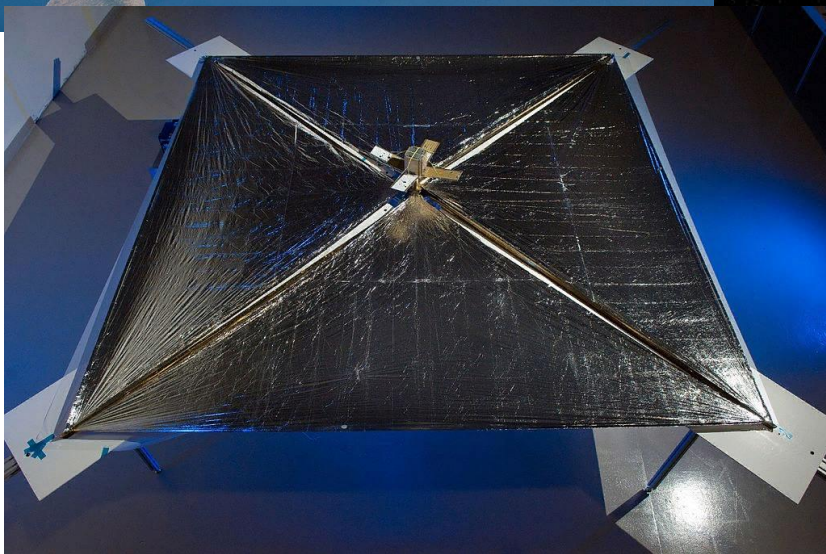
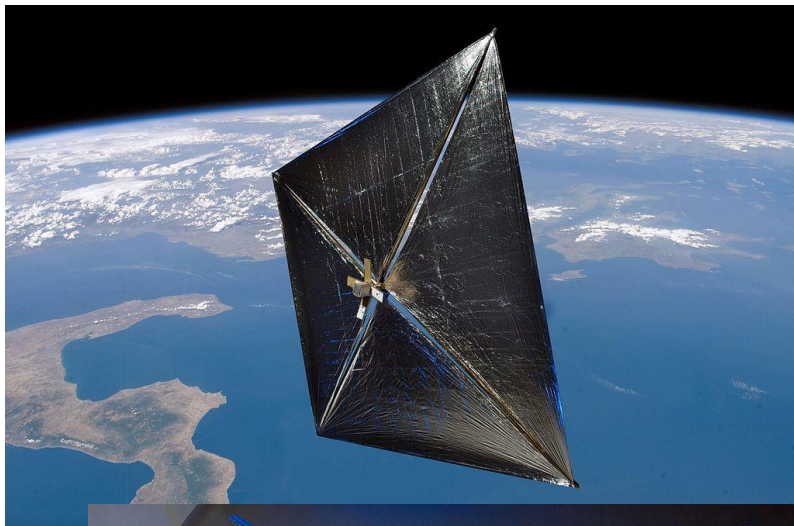


# IKAROS – the first interplanetary solar sail spacecraft



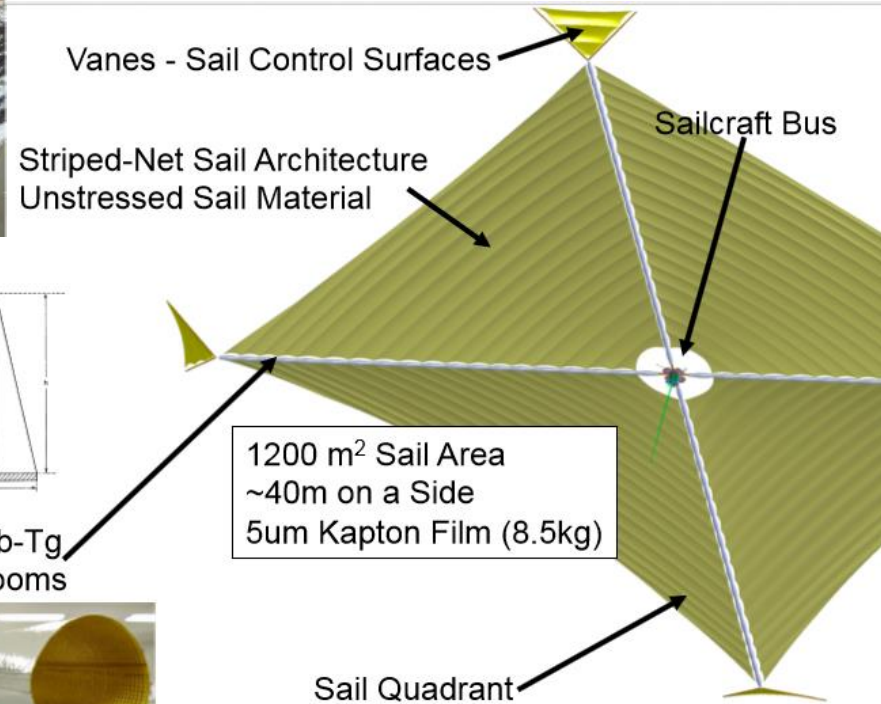


# Nano-sails on nanosatellites from Nanosail D2 to NEA Scout

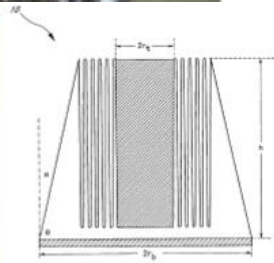




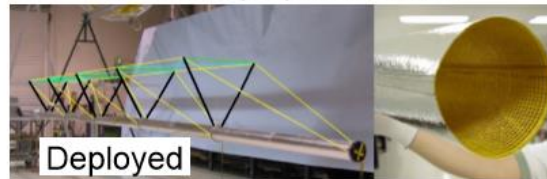
# Sunjumper or L'Garde



Stowed



L'Garde Patented Sub-Tg  
Conical Deployable Booms

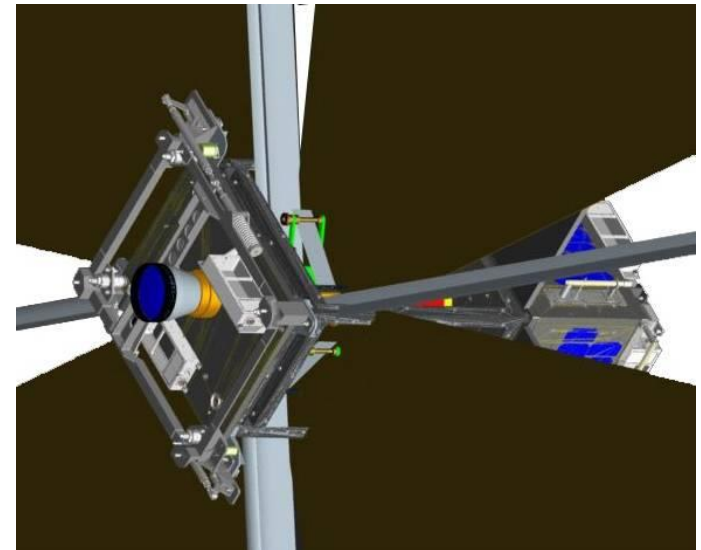


Deployed



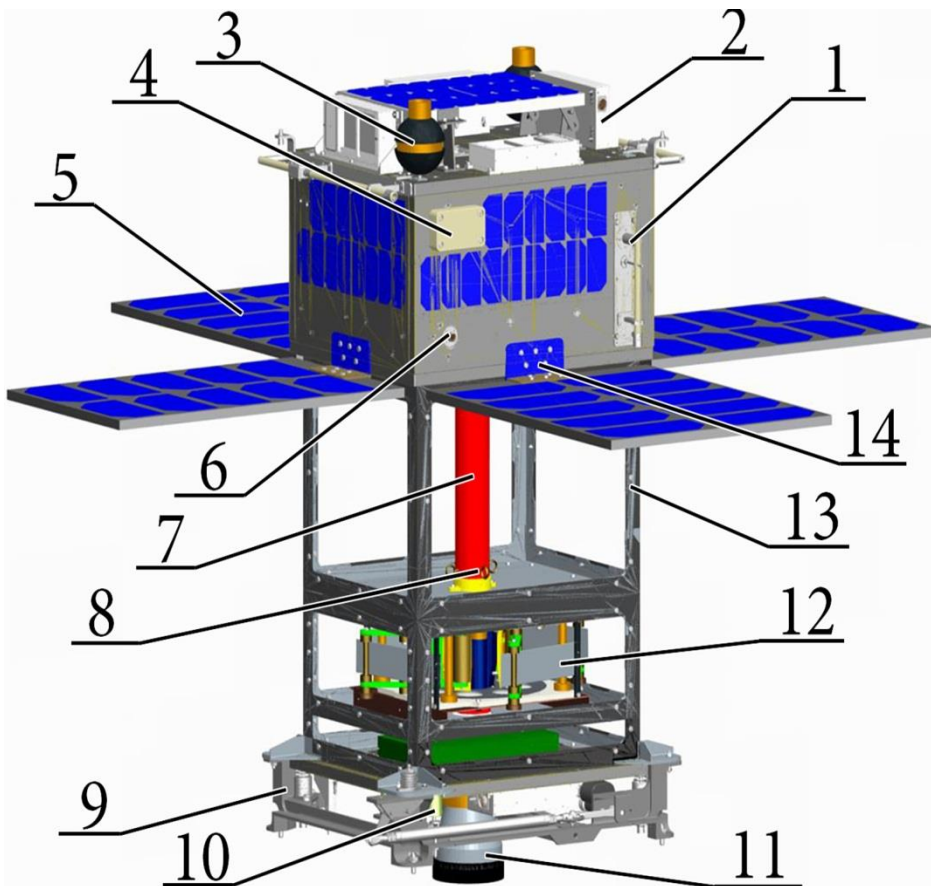
## Our Solar sail spacecraft Helios has the following main design parameters

- the mass of a spacecraft is 100 kg and the total area of a solar sail is 10,000 m<sup>2</sup>;
- the film thickness is 3.5 micrometer;
- the area to the mass ratio of the solar sail spacecraft is 100 m<sup>2</sup>/kg;
- the optical parameters  $\rho=0.98$ ,  $\zeta=0.94$ ,  $\varepsilon_f=0.05$ ,  $\varepsilon_b=0.55$ ;
- the degraded parameters  $d_\rho=0.1$ ,  $\delta_\rho=0.231$ ,  $d_\zeta=0.1$ ,  $\delta_\zeta=0.139$ ,  $d_{\varepsilon_f}=0.1$ ,  $\delta_{\varepsilon_f}=0.231$ ;
- $T_{\max} = 1000$  K.





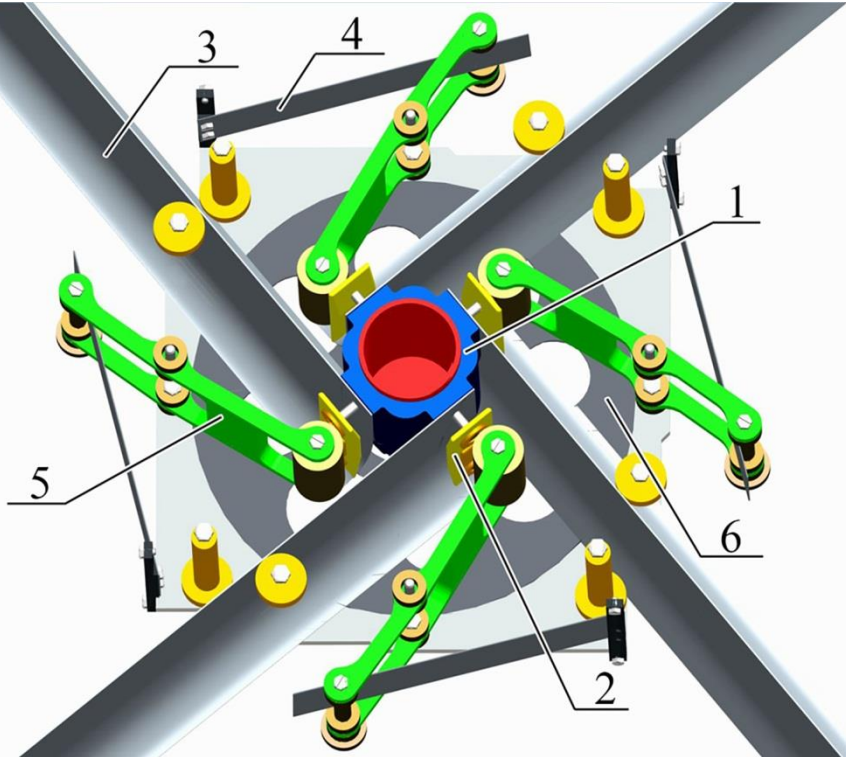
## Our Solar sail spacecraft Helios



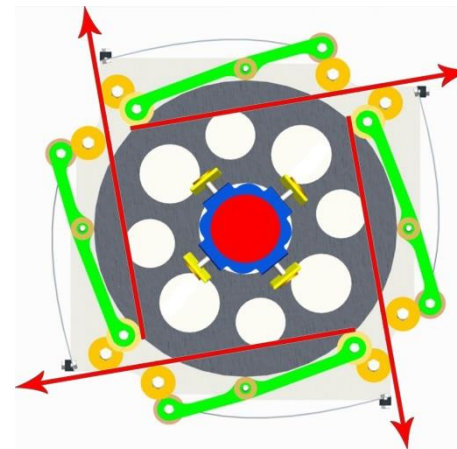
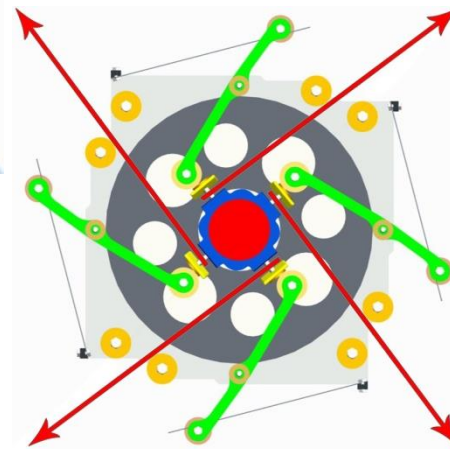
- 1 – Antenna-feeder device
- 2 – Orientation system devices
- 3 – Photo equipment
- 4 – Automatic control unit
- 5 – Solar panel
- 6 – Process connector
- 7 – Fixing pipe
- 8 – Attachment of solar sail
- 9 – Separation device
- 10 – Pin sensor
- 11 – Electron-optical equipment
- 12 – Deploying mechanism
- 13 – Frame Space craft
- 14 – Attachment of the solar panel



# Our Solar sail spacecraft Helios



- 1 – Swivel axle;
- 2 – Locking clamping device;
- 3 – Deploying beam;
- 4 – Bending plates;
- 5 – Tension unit;
- 6 – Pressure plate.



**Red arrows** – the direction of the beam exit from the deployment mechanism



A six dimensional state  $\mathbf{x}(t) = (r \ u \ V_r \ V_\varphi \ \Omega \ i)^T \in \mathbf{X}$  describes a solar sail motion in heliocentric frame. Criterion of optimality – minimum flight time for a given mission's aim

$$\mathbf{u}_{opt}(t) = \arg \min_{\mathbf{u}(t) \in U} \left\{ t_m(\mathbf{u}(t), \mathbf{x}(t)) \mid \mathbf{x}(t_0) = \mathbf{x}_0, \mathbf{x}(t_k) = \mathbf{x}_k, T(t) \leq T_{max} \right\}$$

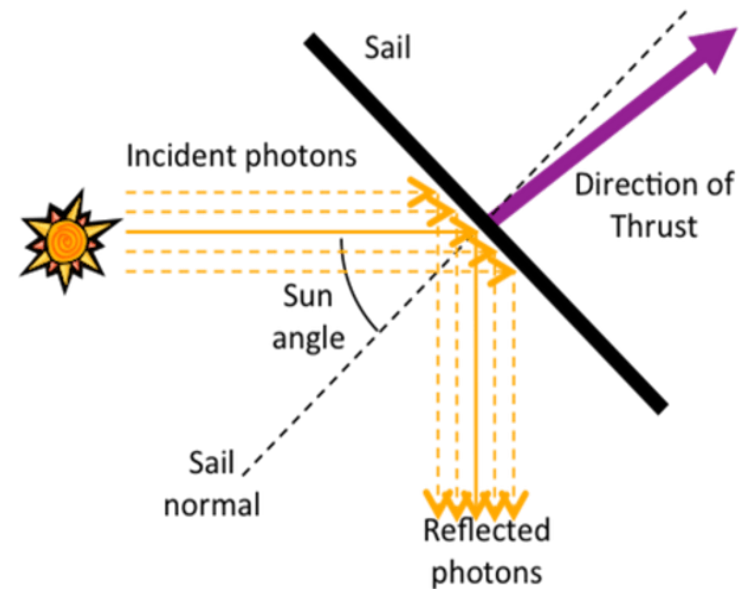
Thrust vector of the solar sail:

$$F_{\perp} = 2 \frac{S_r}{c} S \cdot \cos \vartheta \cdot (a_1 \cos \vartheta + a_2)$$

$$F_{\parallel} = 2 \frac{S_r}{c} S \cdot \cos \vartheta \cdot a_3 \sin \vartheta$$

$$a_1 = \frac{1}{2} (1 + \zeta \rho)$$

$$a_2 = \frac{1}{2} \left( B_f (1 - \zeta) \rho + (1 - \rho) \frac{\varepsilon_f B_f - \varepsilon_b B_b}{\varepsilon_f + \varepsilon_b} \right) \quad a_3 = \frac{1}{2} (1 - \zeta \rho)$$







$$\dot{r} = V_r,$$

$$\dot{V}_r = \frac{V_\theta^2}{r} + \frac{V_\varphi^2 \sin^2 \theta}{r \sin^2 \theta} - \frac{GM}{r^2} f + \frac{GM}{c^2 r^2} f^{-1} V_r^2 + \frac{f V_\varphi^2}{r} + \frac{2GJ}{c^2 r^3} f V_\varphi + a_r + f_r,$$

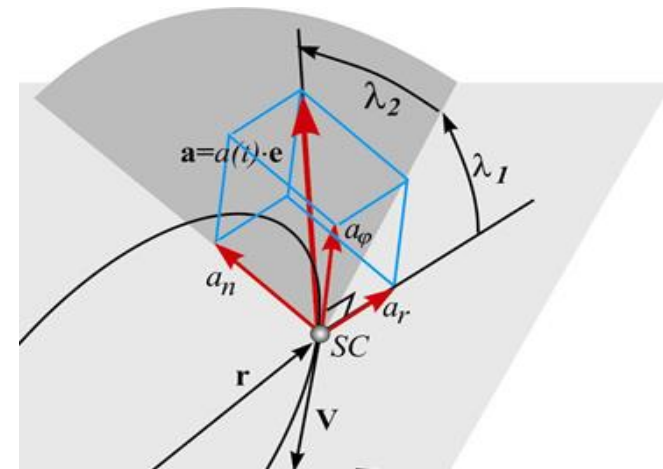
$$\dot{\varphi} = \frac{V_\varphi}{r \sin \theta},$$

$$\dot{V}_\varphi = -\frac{V_r V_\varphi}{r} - \frac{V_\varphi V_\theta \cos \theta}{r \sin \theta} + \frac{2GJ}{c^2 r^3} V_r + a_\varphi + f_\varphi,$$

$$\dot{\theta} = \frac{V_\theta}{r},$$

$$\dot{V}_\theta = \frac{V_\varphi^2 \cos \theta}{r \sin \theta} - \frac{V_r V_\theta}{r} - \frac{4GJ}{c^2 r^2} \frac{V_\varphi}{r} \cos \theta + a_\theta + f_\theta.$$

(1)





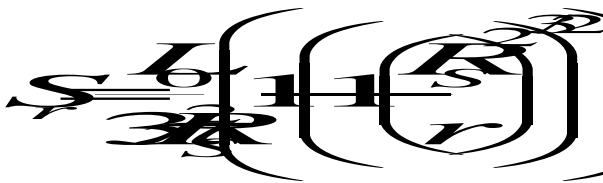
## Temperature of sail surface

If we do not take into account the dependence of the optical coefficients of the sail surface on the temperature and surface degradation due to the effects of space factors, the equilibrium temperature can be calculated by formula

$$T = \left( \frac{S_r}{\sigma_{SB}} \frac{1 - \rho}{\varepsilon_f + \varepsilon_b} \left( \frac{r_0}{r} \right)^2 \cos \vartheta \right)^{1/4}$$

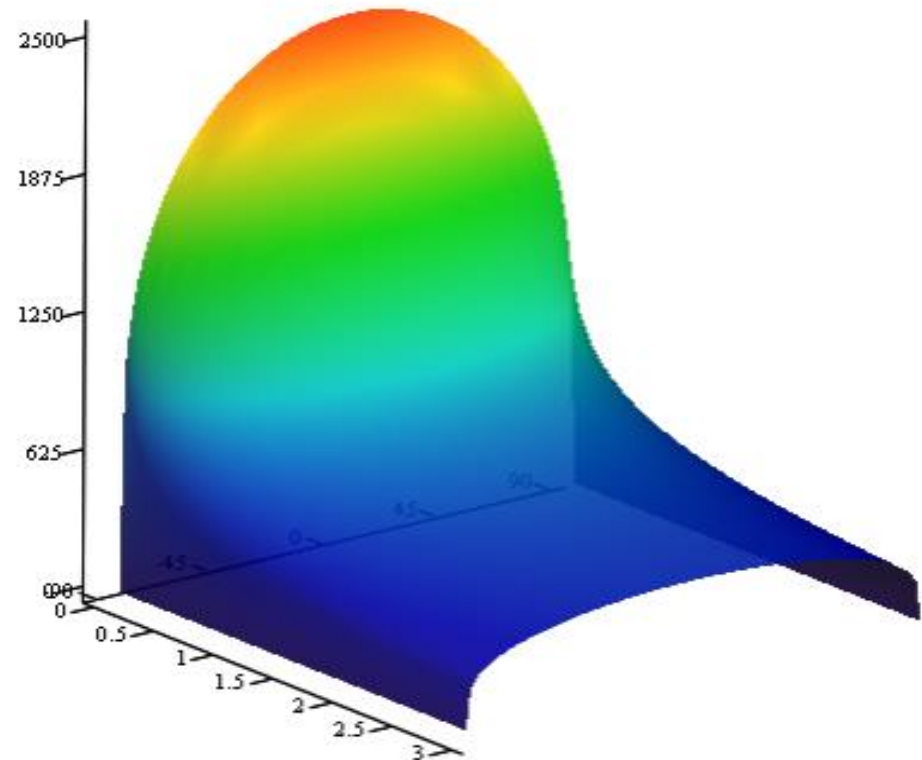
Here  $\sigma_{SB} = 5,67 \cdot 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$  is the Stefan-Boltzmann constant.

Sun as an extended source of radiation pressure



$m$  is the Sun radius.

Our design model of sail has a following optical parameters:  
 $\rho=0.88$ ,  $\varepsilon_f=0.05$ ,  $\varepsilon_b=0.55$



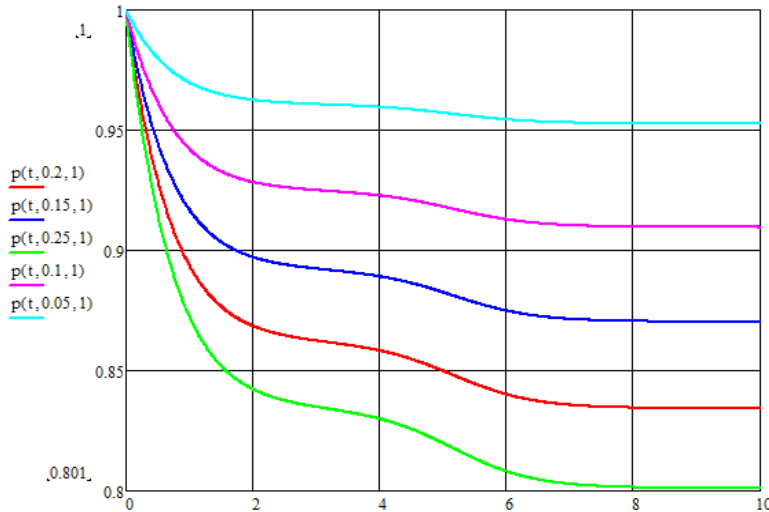
Dependence the surface equilibrium temperature to the heliocentric distance and the installation angle



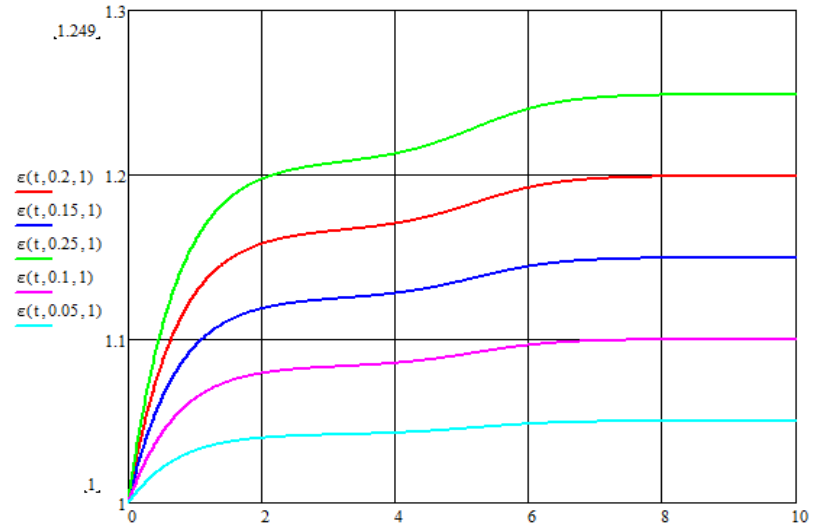
# Degradation of sail surface

The change in the optical parameter  $p$  in time depends on the total dose of solar radiation obtained the sail

$$\frac{p(t)}{p_0} = \begin{cases} \frac{1 + de^{-\lambda\Sigma(t)}}{1 + d} & \text{if } p \in \{\rho, \zeta\} \\ 1 + d(1 - e^{-\lambda\Sigma(t)}) & \text{if } p = \varepsilon_f \\ 1 & \text{if } p \in \{\varepsilon_b, B_f, B_b\} \end{cases} \quad \tilde{\Sigma}_0 = 15,768 \cdot 10^{12}$$



Dependence the reflection coefficient of the sail surface to the lifetime



Dependence the emission coefficient of the sail surface to the lifetime

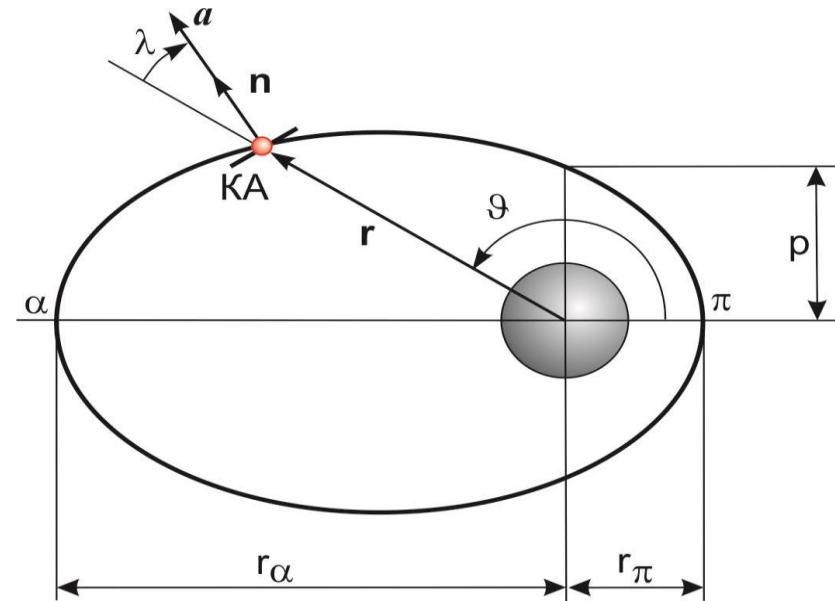


We use the planar Keplerian osculated elements to choosing control laws. All equations of Keplerian elements have form

$$\frac{dK}{dt} = f_1(p, e, \vartheta) \cos^3 \lambda_1 + f_2(p, e, \vartheta) \cos^2 \lambda_1 \sin \lambda_1$$

If we need keeping the element K constant, then steered angle has to be

$$\left[ \begin{array}{l} \operatorname{tg} \lambda_1 = -\frac{f_1(p, e, \vartheta)}{f_2(p, e, \vartheta)}, \\ \lambda_1 = \pm \frac{\pi}{2}. \end{array} \right. \quad (2)$$



For the most rapid change of the Keplerian element steering angle has to be

$$\lambda_1 = \frac{1}{2} \arcsin \frac{f_2(p, e, \vartheta) \left( f_1(p, e, \vartheta) - \sqrt{9f_1(p, e, \vartheta)^2 + 8f_2(p, e, \vartheta)^2} \right)}{3 \left( f_1(p, e, \vartheta)^2 + f_2(p, e, \vartheta)^2 \right)} \quad (3)$$



## Method of the heliocentric movement calculation

$$x(t_0) = \begin{pmatrix} r_E \\ \varphi_E \\ V_{rE} \\ V_{\phi E} \\ i_E \\ \Omega_T \end{pmatrix}$$



$$x(t_k) = \begin{pmatrix} r_{Pl} \\ \varphi_{Pl} \\ V_{rPl} \\ V_{\phi Pl} \\ i_{Pl} \\ \Omega_{Pl} \end{pmatrix}$$

Initial moment of the system  
(Earth heliocentric parameters)



Final moment of the system  
(Planet heliocentric parameters)





# Method of the heliocentric movement calculation

Step of numerical solution (1)

Eccentricity, parameter and true anomaly calculation

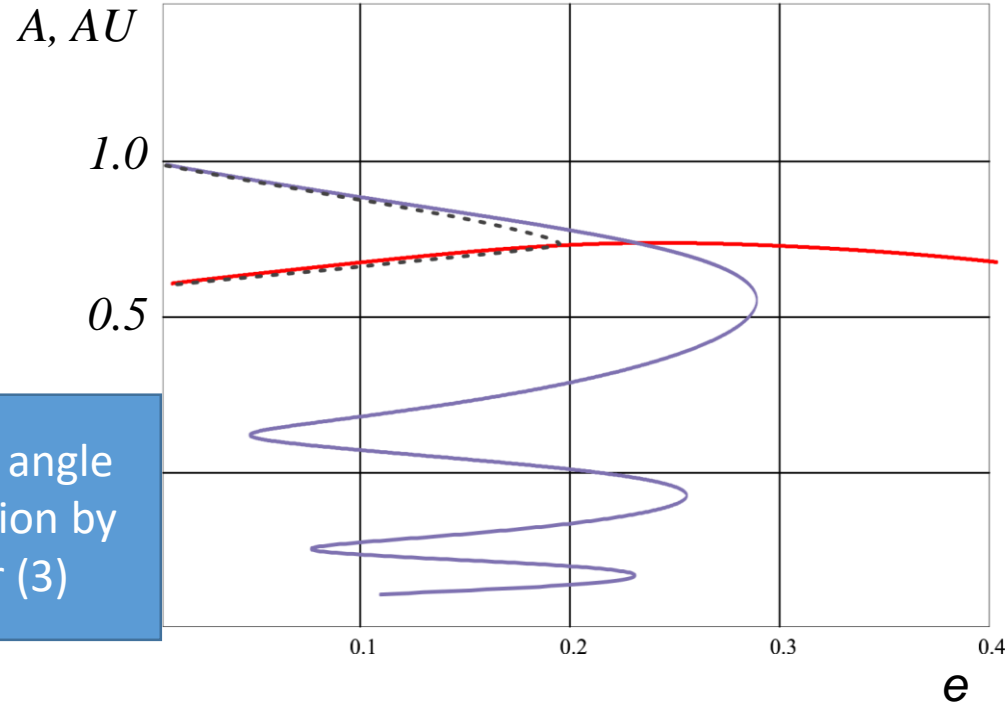
Control angle calculation by (2) or (3)

$$\sin \vartheta = \frac{rV_\varphi V_r}{\sqrt{(rV_r^2 - 1)^2 + (rV_\varphi V_r)^2}}$$

$$\cos \vartheta = \frac{rV_r^2 - 1}{\sqrt{(rV_r^2 - 1)^2 + (rV_\varphi V_r)^2}}$$

$$e = \sqrt{(rV_r^2 - 1)^2 + (rV_\varphi V_r)^2}$$

$$p = r(1 + e \cos \vartheta)$$



- decrease semi-major axes direct integration Earth
- decrease eccentricity contrary integration Venus
- ⋯ exact optimal solution



## Modeling of SSSP to Venus flight

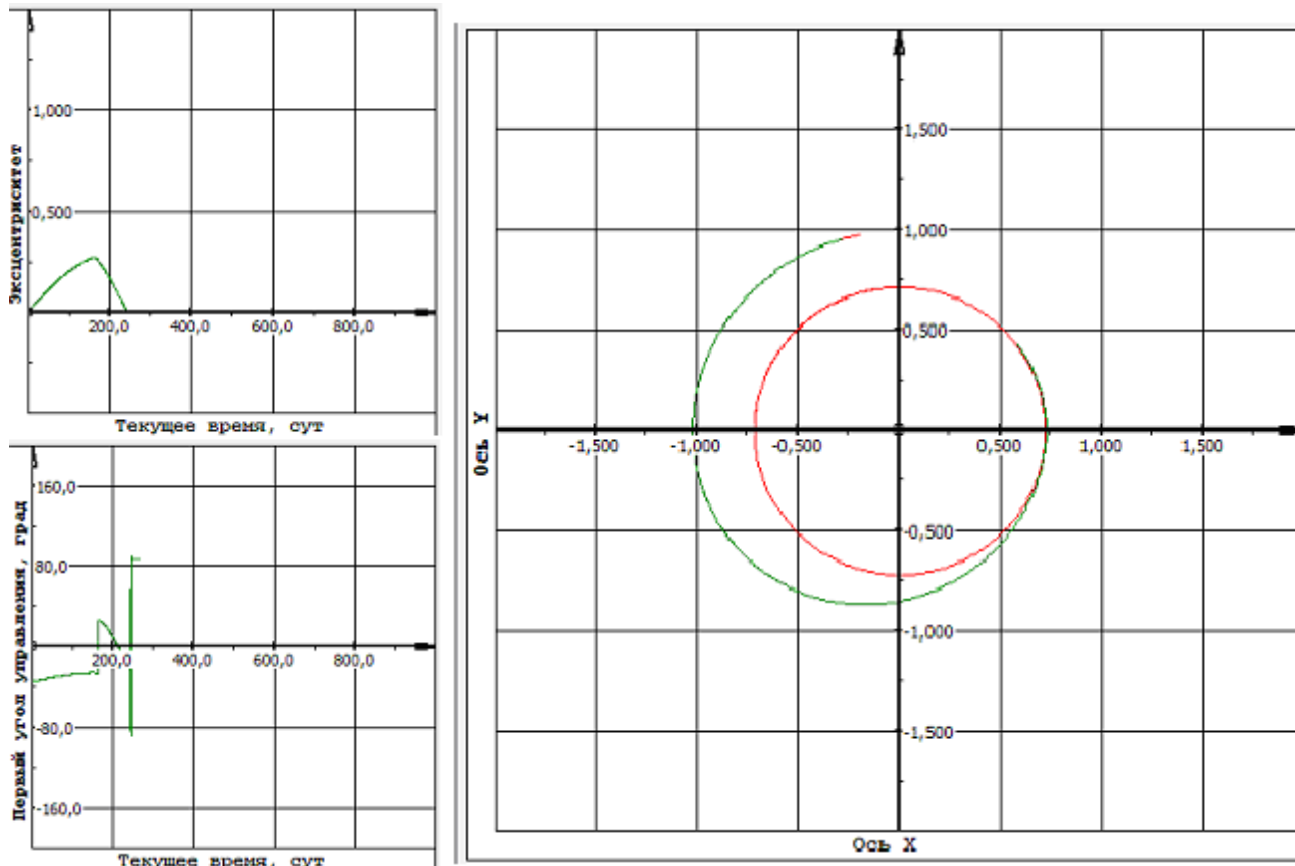
The orbit parameters were selected in two stages:

Stage 1 - the fastest reduction of the semi-axis (165 days).

Stage 2 - the fastest reduction of eccentricity (78 days).

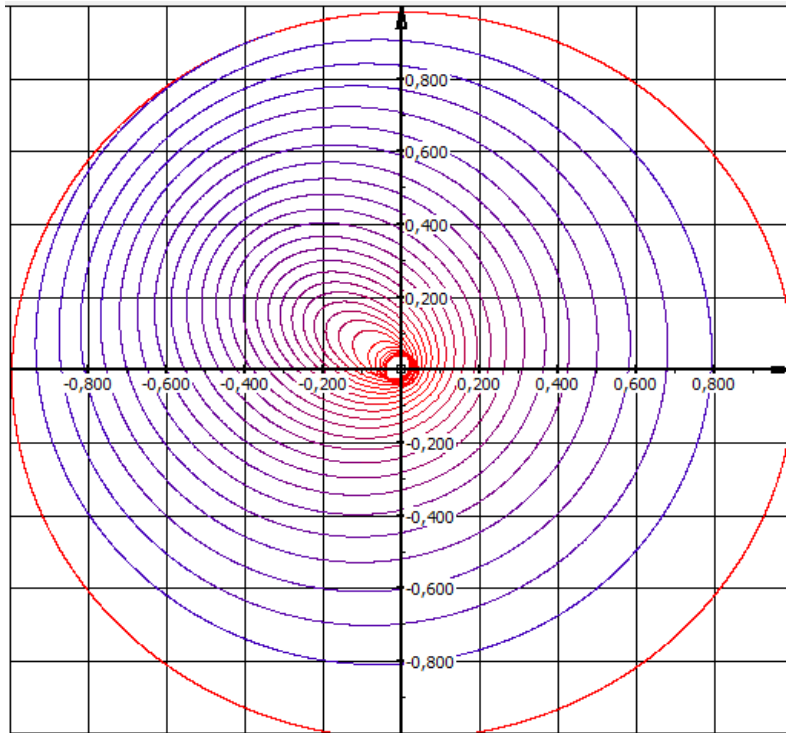
The heliocentric movement of SSSP to Venus lasts 243.0 days.

Including maneuver set parabolic speed, the full flight time is 593 days.





## Results for flight to near sun vicinity

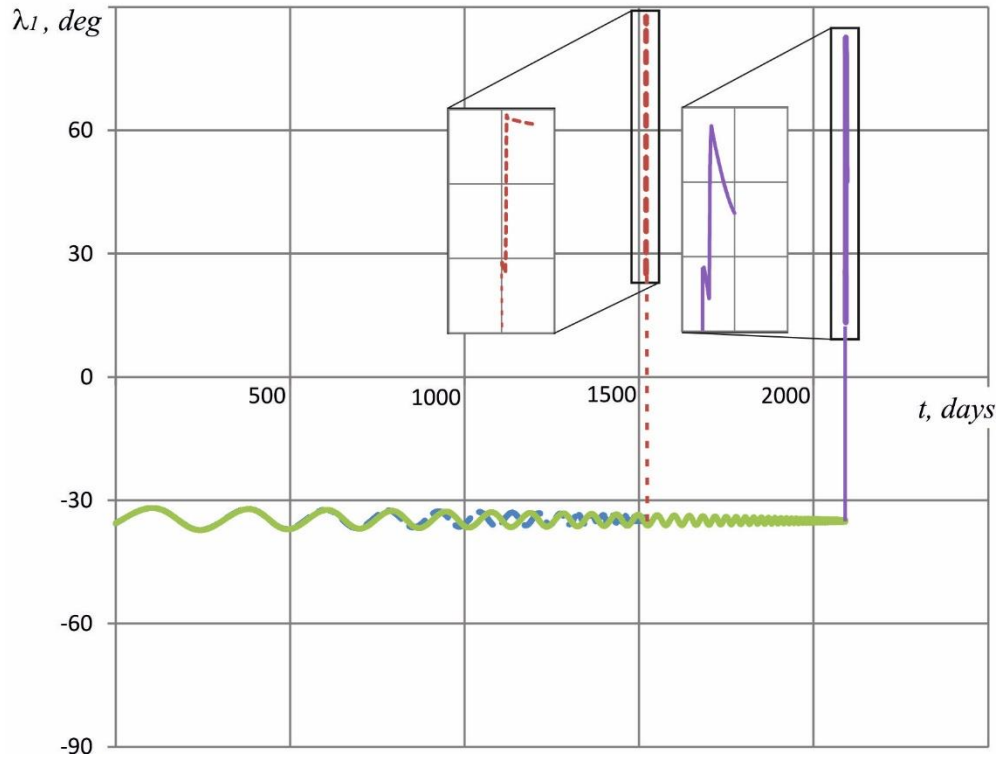


Start mass of spacecraft, kg	100
Area of sail, m <sup>2</sup>	2000
Mirror reflection coefficient of the front surface of the sail (Be)	0,98
Secondary emission coefficient of the front surface of the sail (Be)	0,01
Secondary emission coefficient of the rear surface of the sail (Cr)	0,75
Date of withdrawal from the Earth	10.01.2022
Duration of the phase of reduction of the radius of the pericenter, days	2102,7
The duration of the plot reduce the eccentricity of the orbit, days	37,5
Date of formation of the working orbit	14.10.2027
The maximum steady-state temperature of the sail surface Be/Cr*, K	1139

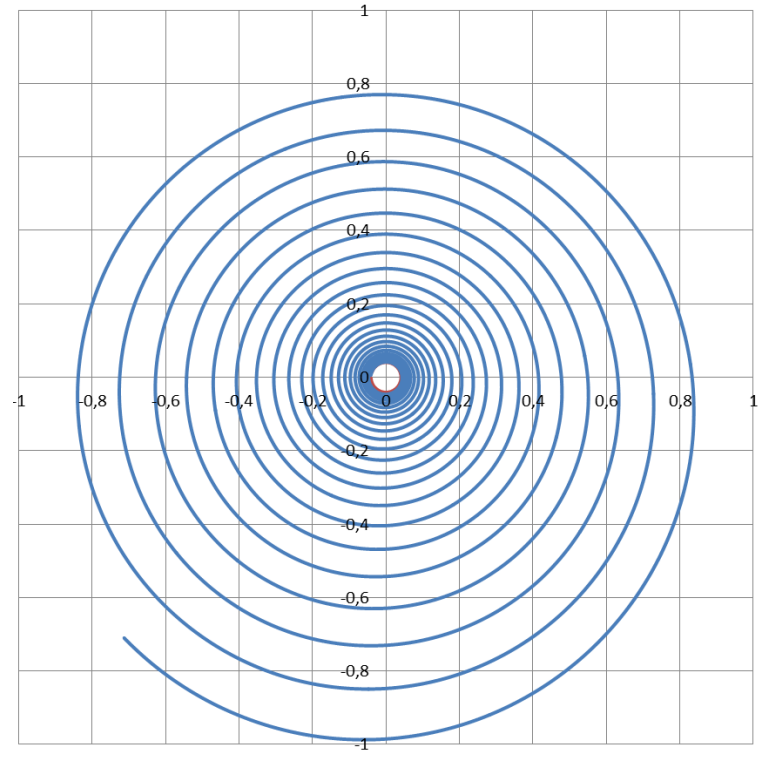




# Simulation results for flight to near sun vicinity without the temperature restriction



a)

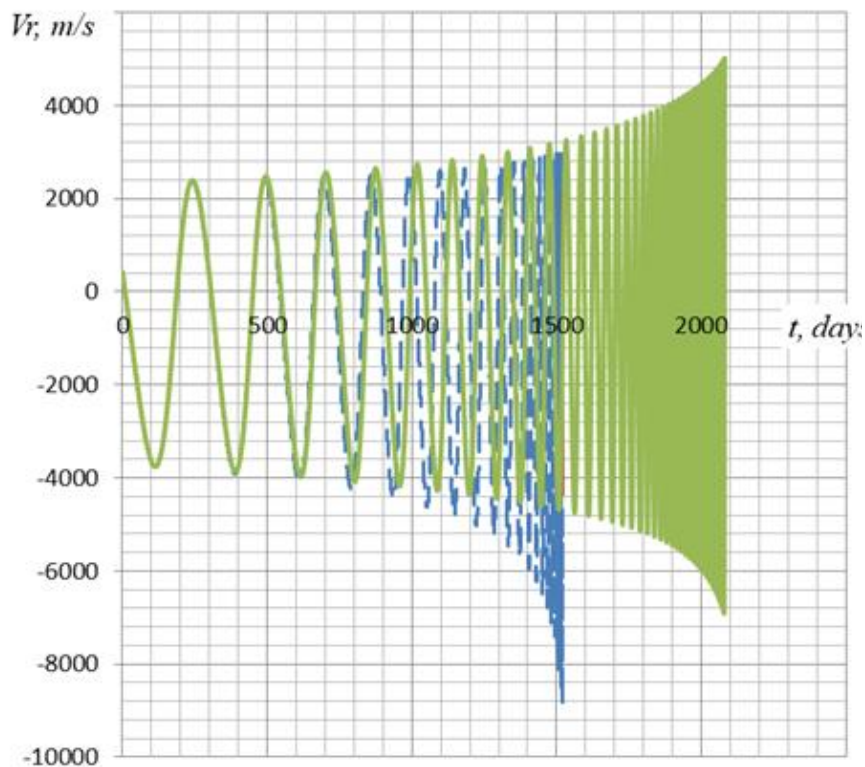


b)

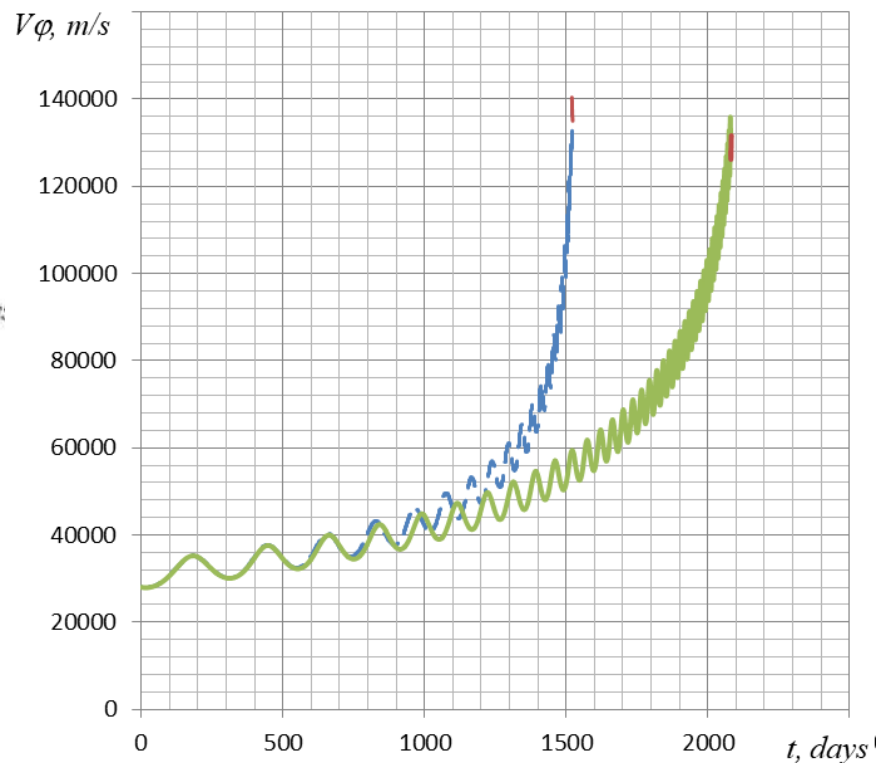
The local-optimal control law (a) and the corresponding flight trajectory (b)  
without the temperature restriction



# Simulation results for flight to near sun vicinity without the temperature restriction



a)

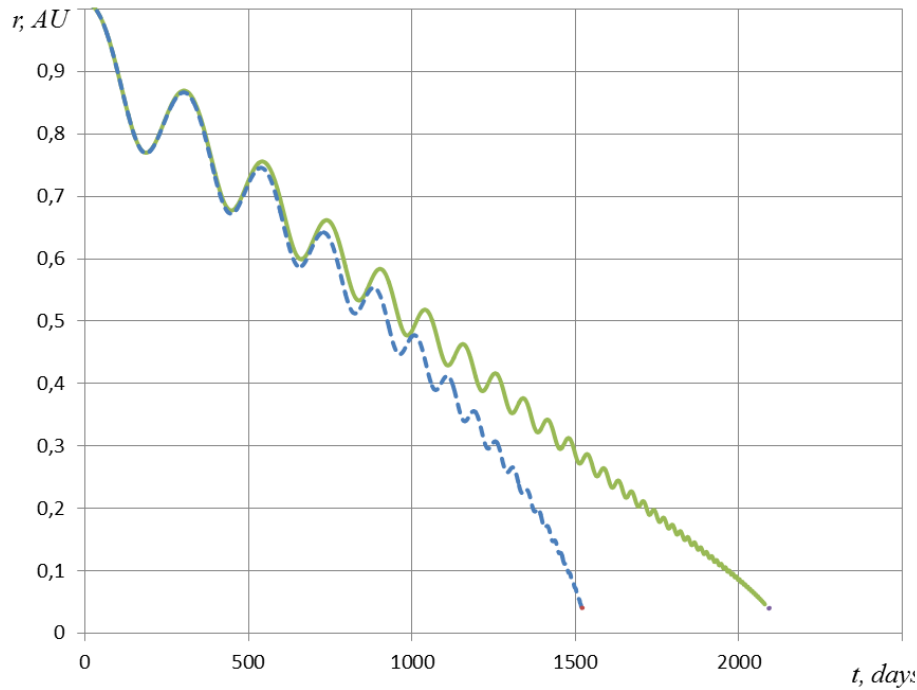


b)

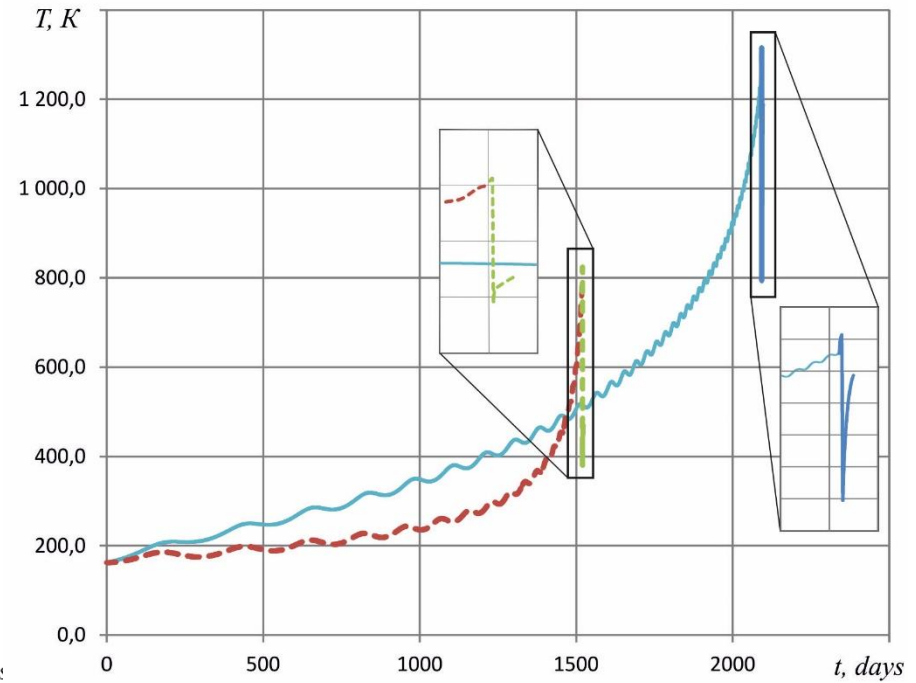
The dependence of the radial (a) and transversal components (b) of heliocentric velocity on flight duration without the temperature restriction



# Simulation results for flight to near sun vicinity without the temperature restriction



a)

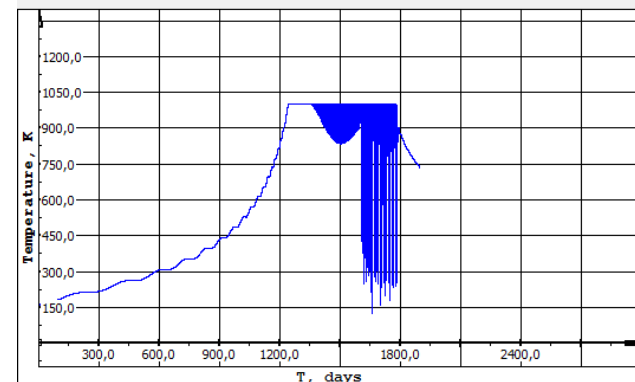
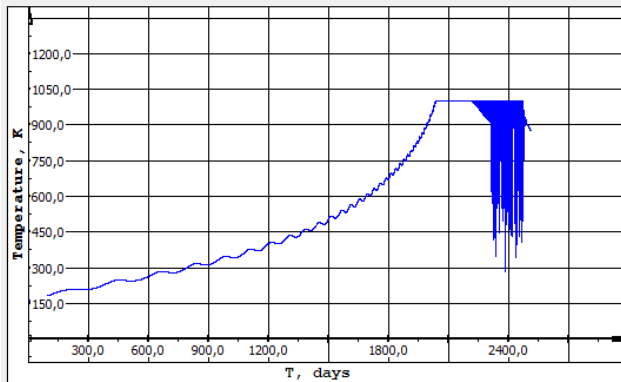
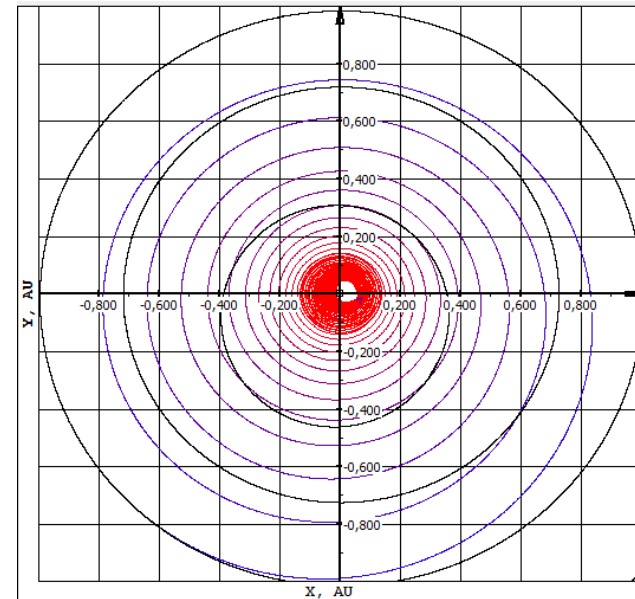
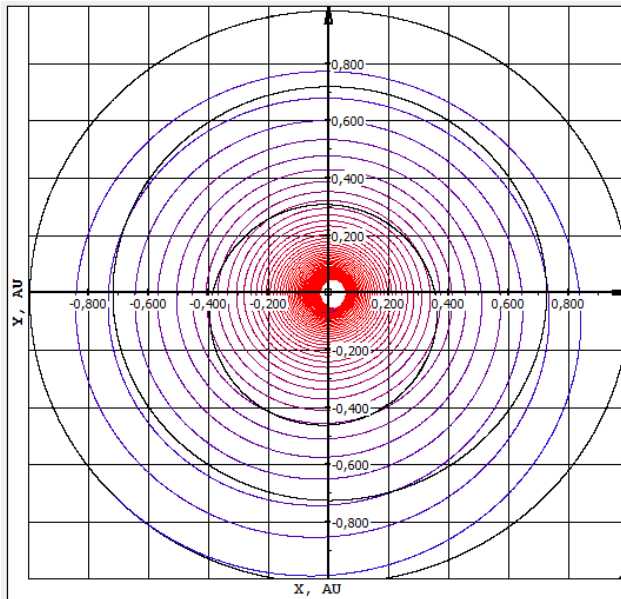


b)

The dependence of the heliocentric radius-vector (a) and the equilibrium temperature (b) on flight duration without the temperature restriction



# Simulation results for flight to near sun vicinity with the temperature restriction



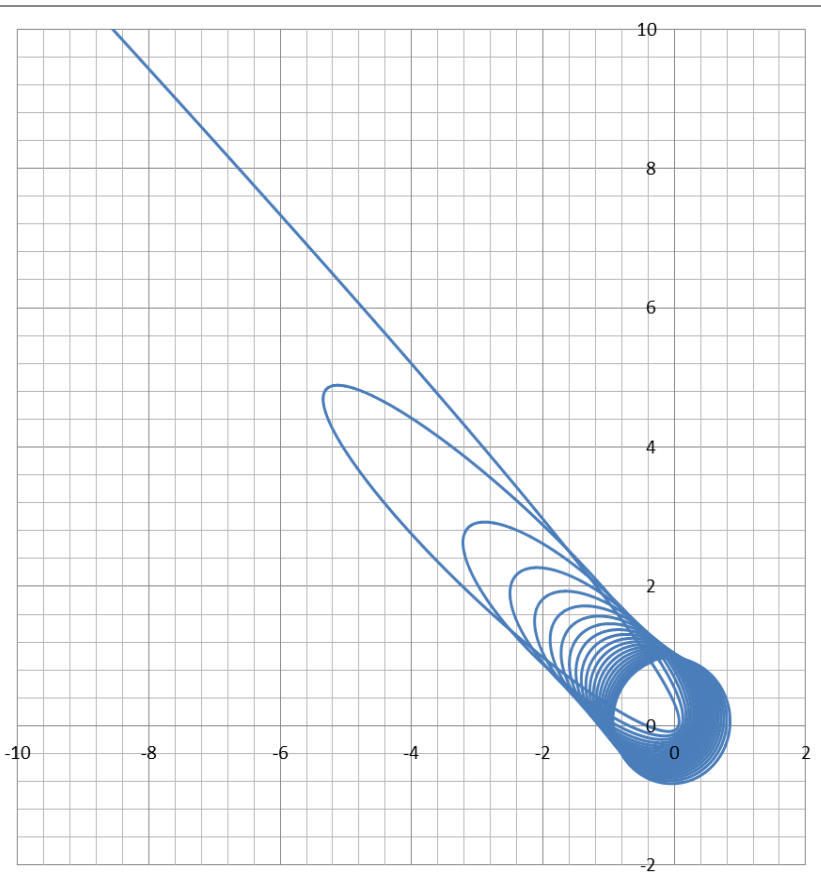
a)

b)

The flight trajectories to SSSC with  $\sigma=20$  kg/m<sup>2</sup> (a) and  $\sigma=30$  kg/m<sup>2</sup> (b) obtained with the help of our software



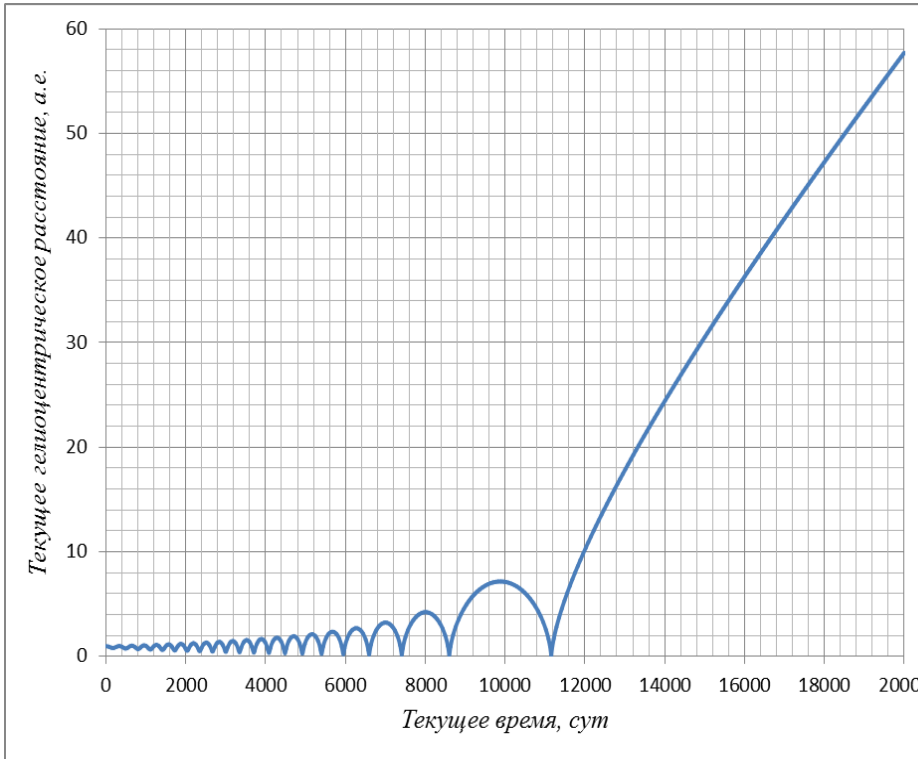
## Results for flight to inter stars space



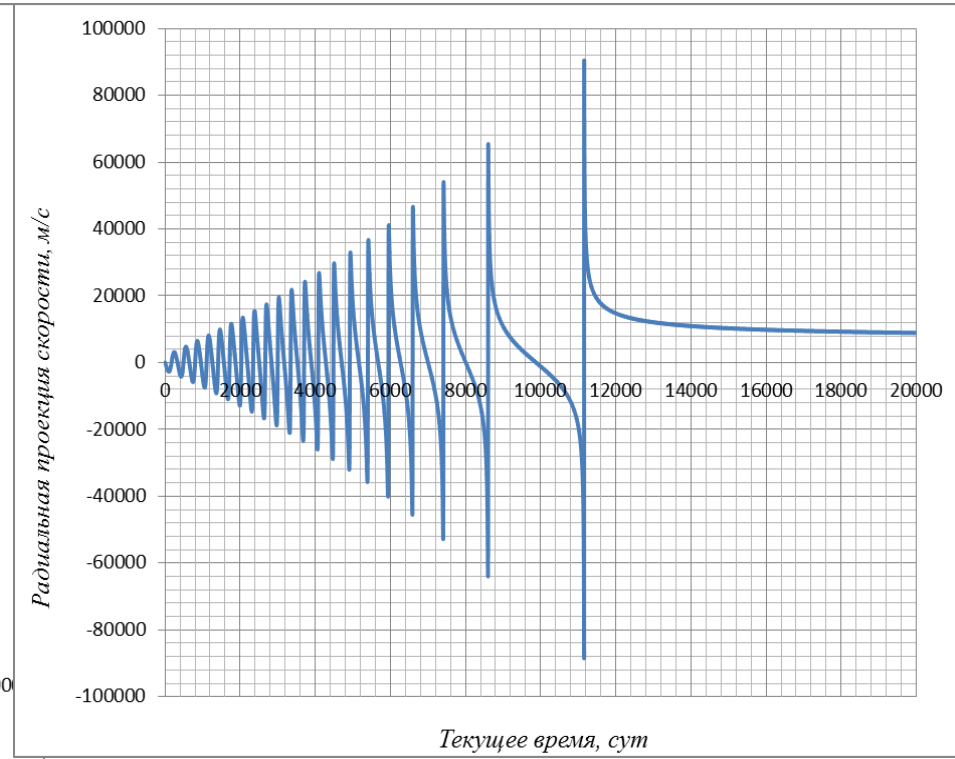
Start mass, kg	100
Area of sail, m <sup>2</sup>	2500
Reflection coefficient of front sail's surface (Be)	0,98
Emission coefficient of front sail's surface (Be)	0,01
Emission coefficient of back sail's surface (Cr)	0,75
Date of withdrawal from the Earth	10.01.2022
The duration of plot of increasing the eccentricity of the orbit, days	15000
Date of leaving the Sun	12.10.2076
he minimum heliocentric distance, A.U.	0,056
The maximum equilibrium temperature of the sail's surface Be/Cr*, K	480,26



# Results for flight to inter stars space



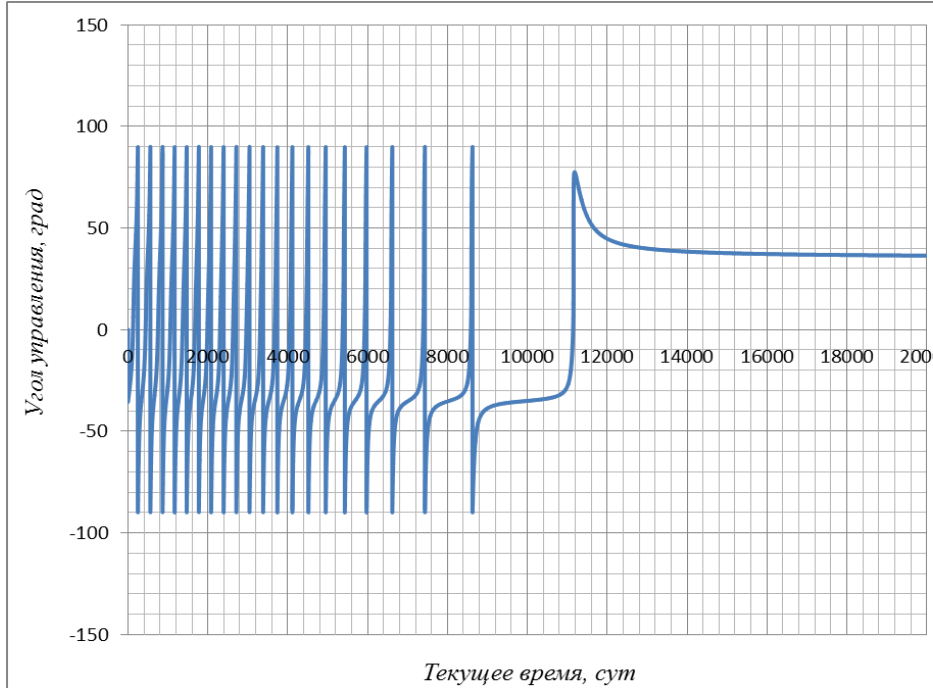
Heliocentric radius-vector of SSSP



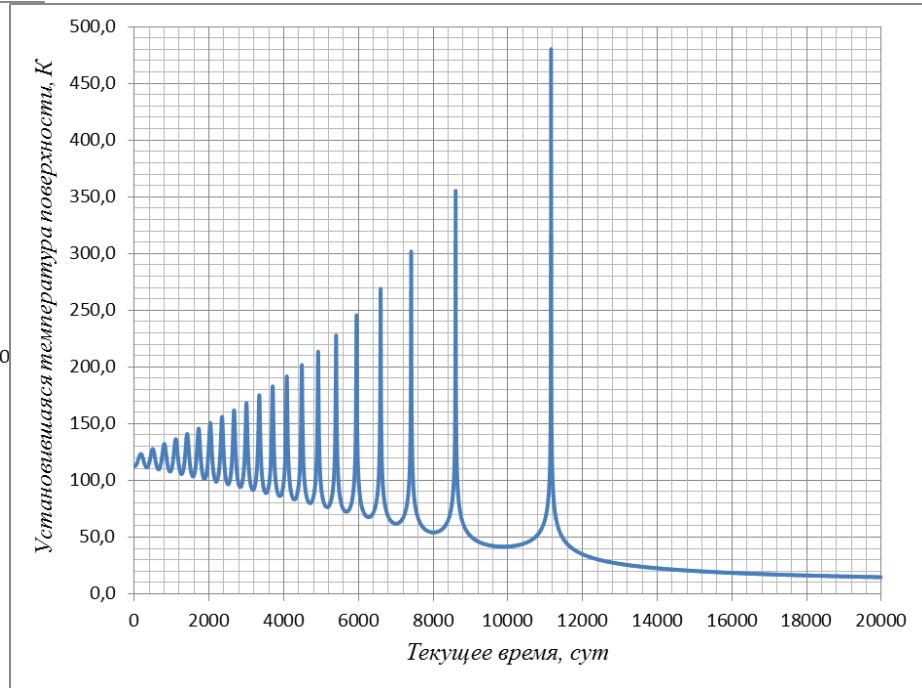
Radial projection of the SSSP velocity



# Results for flight to inter stars space



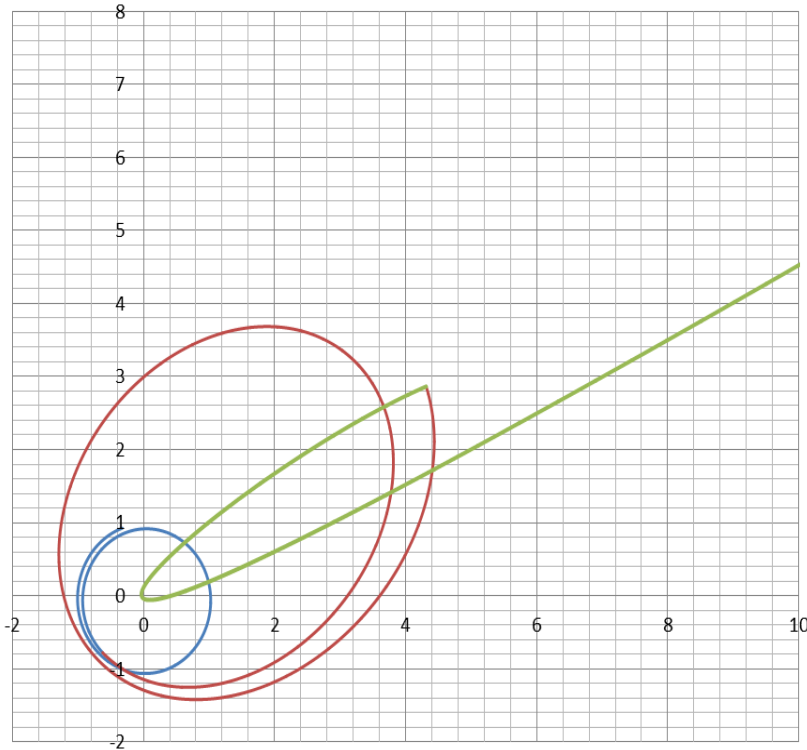
Installation angle of SSSC



Equilibrium temperature of SSSC surface



## Results for flight to inter stars space

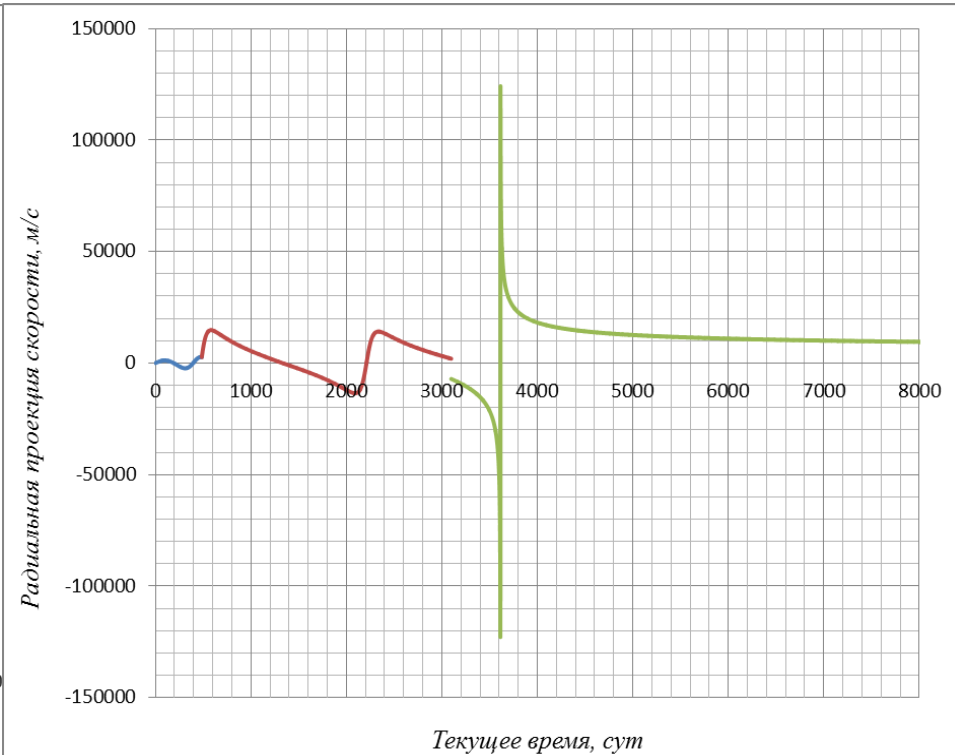
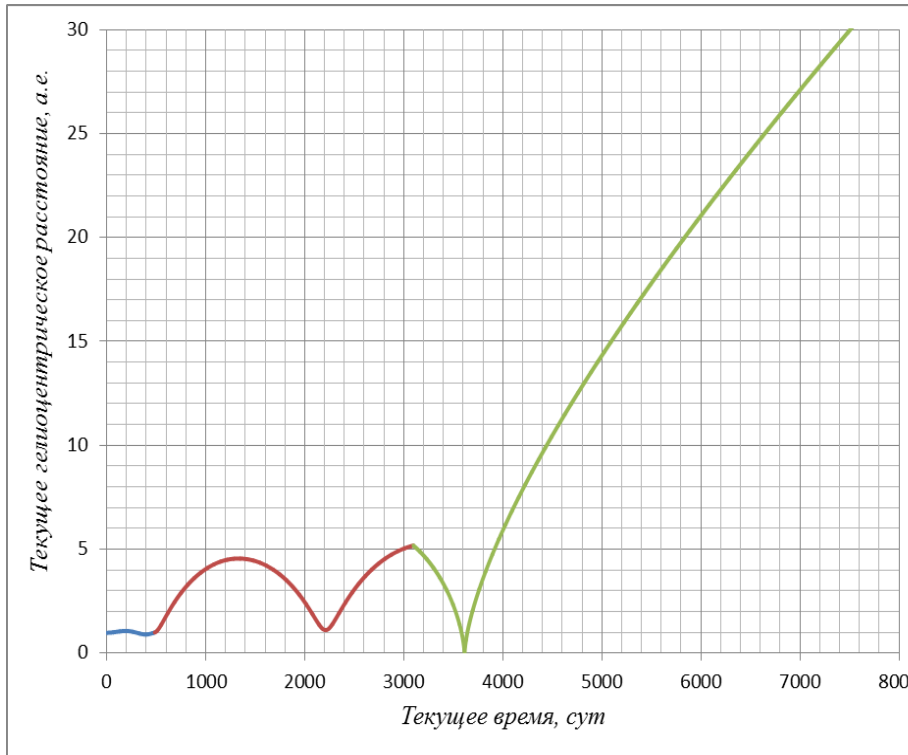


Start mass, kg	100
Area of surface, m <sup>2</sup>	2500
Reflection coefficient of sail's front surface (Be)	0,98
Emission coefficient of sail's front surface (Be)	0,01
Emission coefficient of sail's back surface (Cr)	0,75
Date of the exit in the Earth action sphere	10.01.2022
The length of the phase of increasing the orbit eccentricity, days	485,6
Date of the gravity assist in the Earth action sphere	10.05.2023
The duration of the phase of increase of the radius of apoapsis, days	2611
Date of the gravity assist in the Jupiter action sphere	3.07.2030
Date of leaving the Sun's action sphere	12.03.2044
The minimum heliocentric distance, A.U.	0,056
The maximum equilibrium temperature of the sail's surface Be/Cr*, K	480,26





# Results for flight to inter stars space

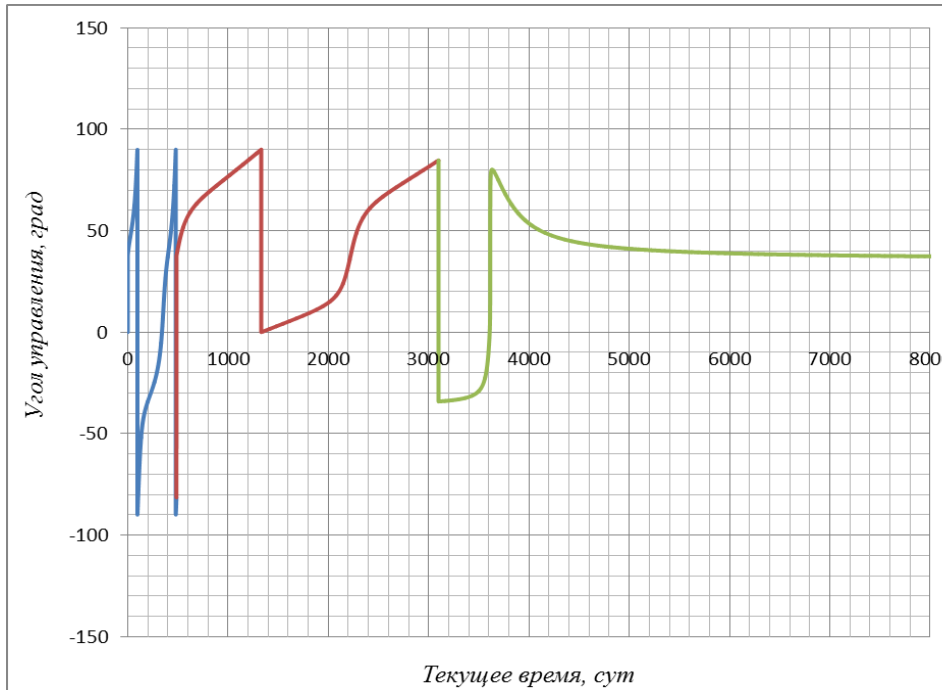


Heliocentric radius-vector of SSSP

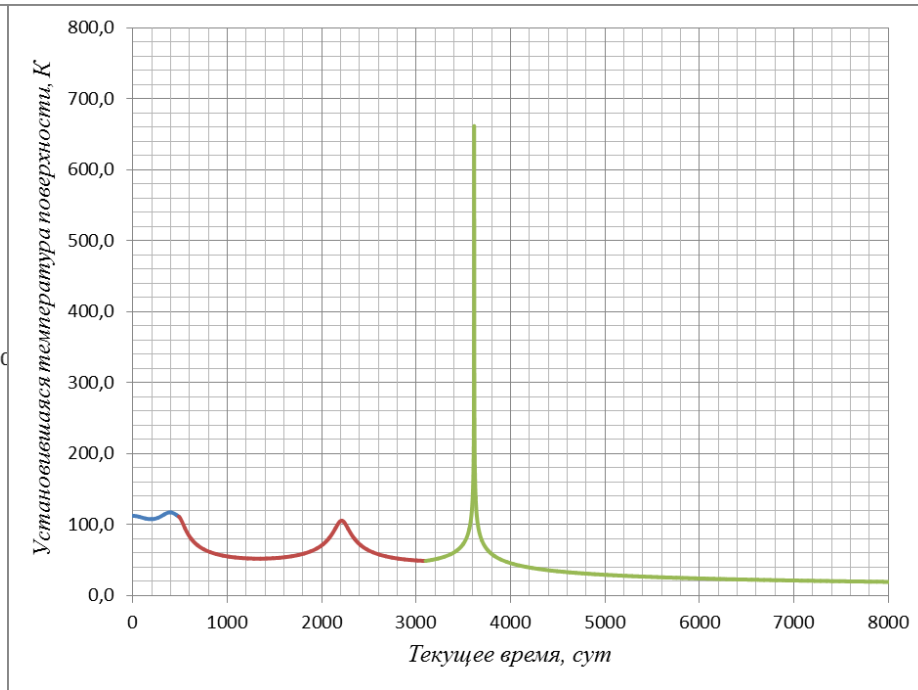
Radial projection of the SSSP velocity



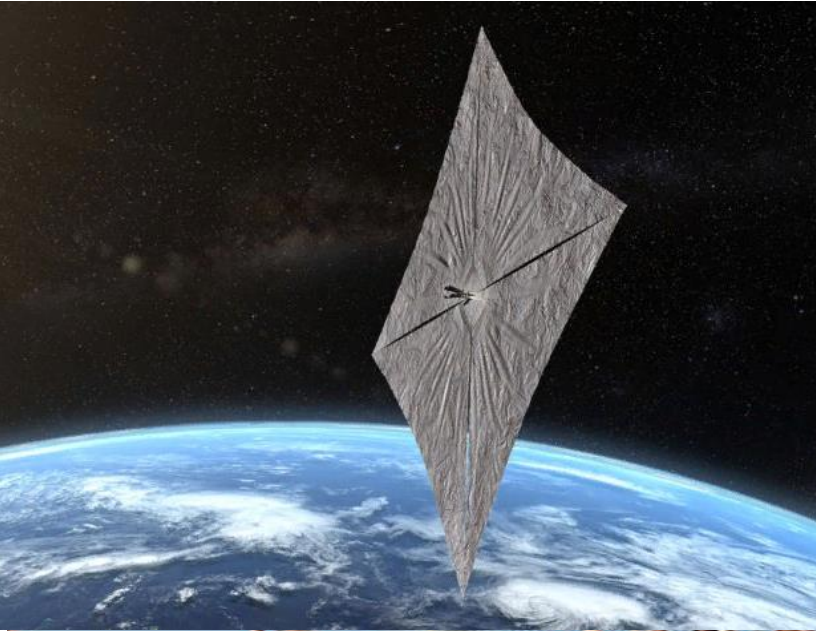
# Results for flight to inter stars space



Installation angle of SSSC



Equilibrium temperature of SSSC surface



THE PLANETARY SOCIETY



**САМАРСКИЙ** УНИВЕРСИТЕТ  
SAMARA UNIVERSITY

**THANK YOU FOR YOUR  
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