САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

Orbital Mechanics

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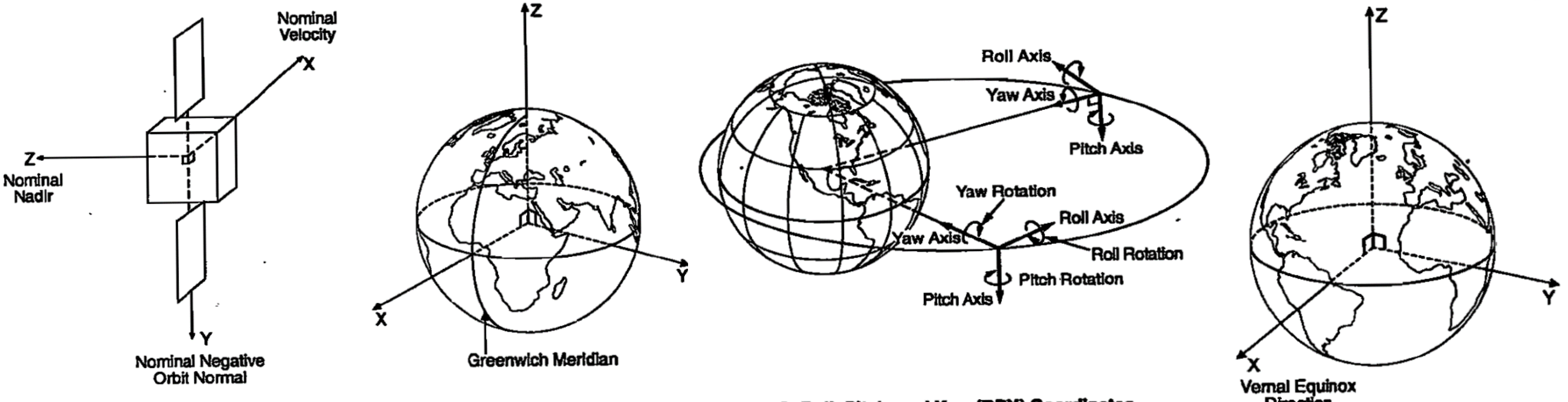
Samara 2023



A. Spacecraft-fixed Coordinates B. Earth-fixed Coordinates C. Roll, Pitch, and Yaw (RPY) Coordinates D. Celestial Coordinates

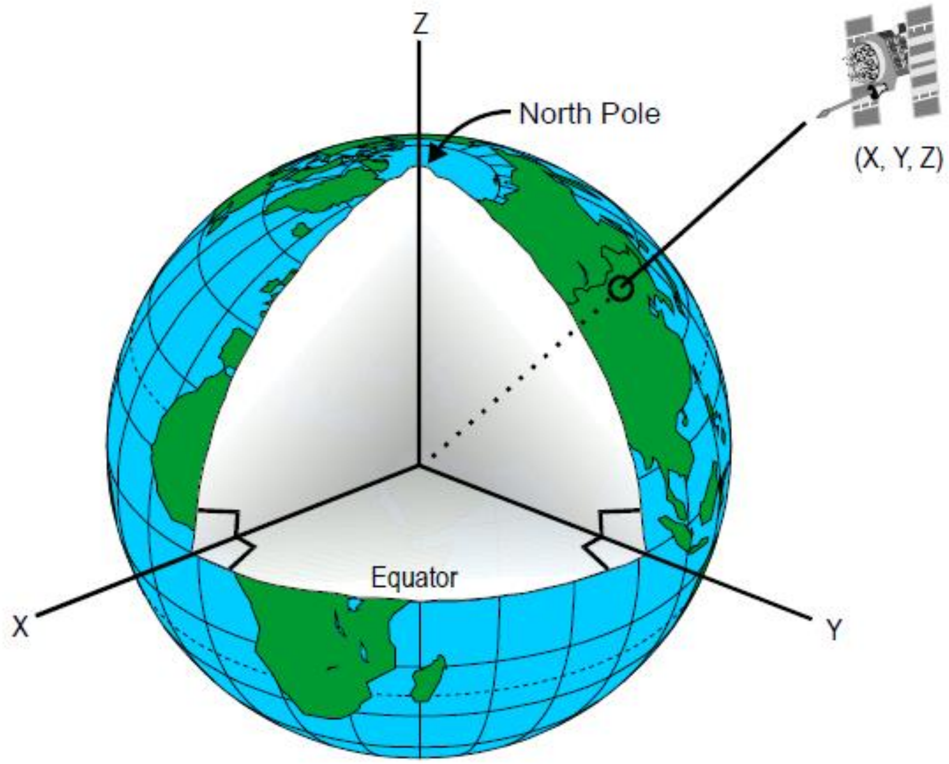
Coordinate Name	Fixed with Respect to	Center	Z-axis or Pole	X-axis or Ref. Point	Applications
Celestial (inertial)	Inertial space*	Earth† or spacecraft	Celestial pole	Vernal equinox	Orbit analysis, astronomy, inertial motion
Earth-fixed	Earth	Earth	Earth pole = celestial pole	Greenwich meridian	Geolocation, apparent satellite motion
Spacecraft-fixed	Spacecraft	Defined by engineering drawings	Spacecraft axis toward nadir	Spacecraft axis in direction of velocity vector	Position and orientation of spacecraft instruments
Local Horizontal‡	Orbit	Spacecraft	Nadir	Perpendicular to nadir toward velocity vector	Earth observations, attitude maneuvers
Ecliptic	Inertial space	Sun	Ecliptic pole	Vernal equinox	Solar system orbits, lunar/solar ephemerides

* Actually rotating slowly with respect to inertial space.
† Earth-centered inertial coordinates are frequently called *GCI (Geocentric Inertial)*.
‡ Also called *LVLH (Local Vertical/Local Horizontal), RPY (Roll, Pitch, Yaw)*, or *Local Tangent Coordinates*.

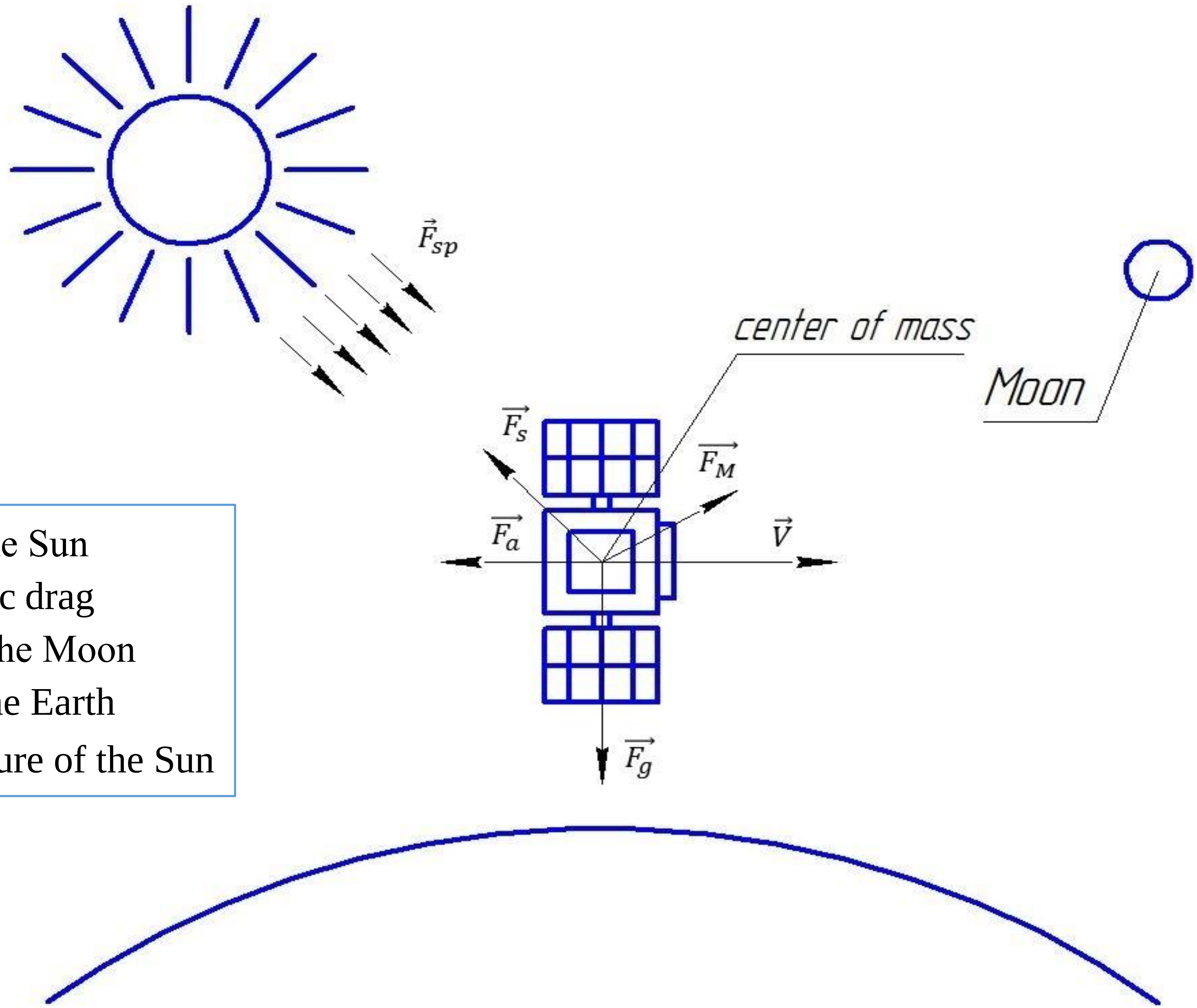
Some recommendations on choosing of a coordinate system:

- Earth-centered inertial for orbit problems
- Spacecraft-centered local horizontal for the Earth remote sensing missions
- Spacecraft-centered inertial for remote sensing missions of any other objects

Axis directions:
the X axis is directed to the point of the vernal equinox;
the Z axis is directed along the direction of the angular velocity vector of the Earth's rotation (to the north);
the Y axis completes the axis system to the right-handed coordinate system.

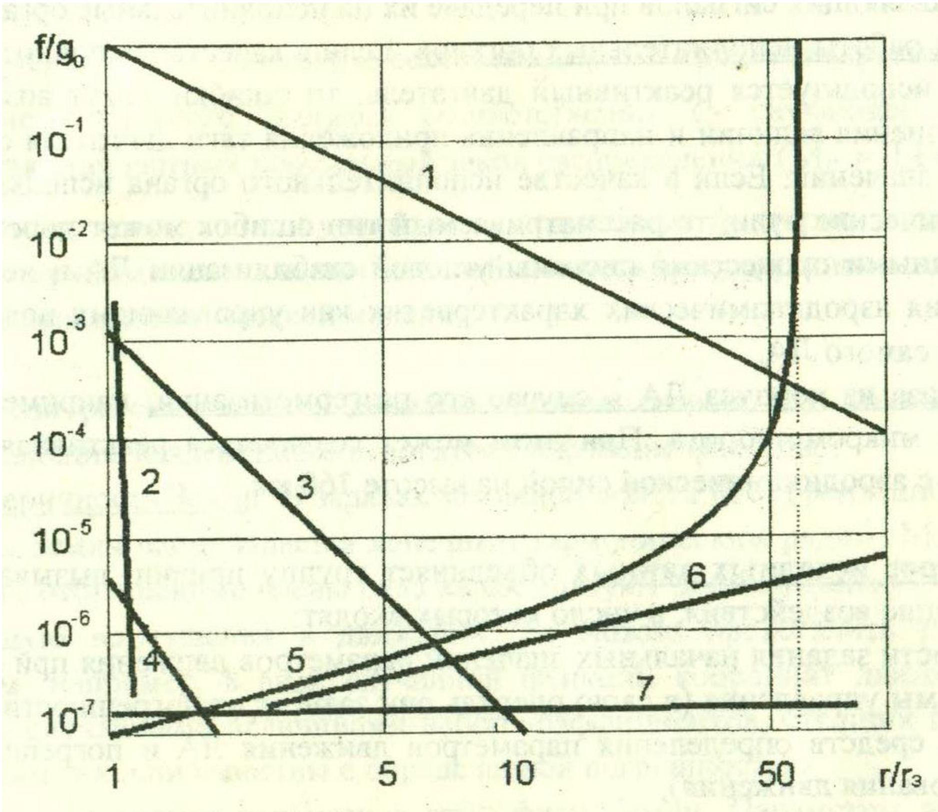


What forces act on the the a spacecraft in flight



- $\vec{F}_s$ – gravity of the Sun
- $\vec{F}_a$ – aerodynamic drag
- $\vec{F}_M$ – gravity of the Moon
- $\vec{F}_g$ – gravity of the Earth
- $\vec{F}_{sp}$ – light pressure of the Sun

Forces and accelerations that act on the spacecraft in flight



1- g

2- perturbation for aerodynamic drag

3- the second harmonic of the gravitational field of the Earth

4- fourth harmonic of the gravitational field of the Earth

5- force of gravity of the Moon

6- force of gravity of the Sun

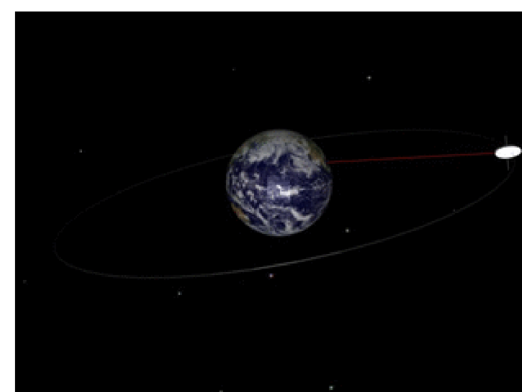
7-force of light pressure of the Sun

1
2
3
4

LEO is an orbit around Earth with an altitude between 160 kilometers and 2000 kilometers

1
5
6
7

GEO

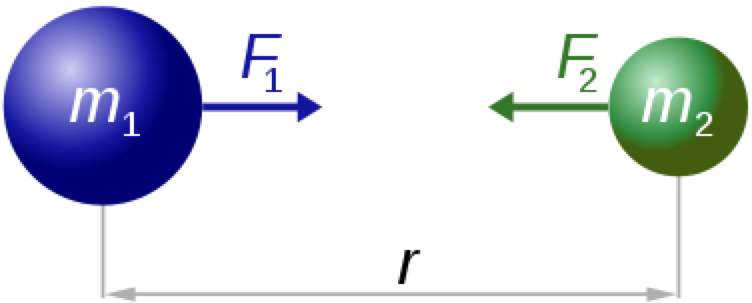


f - perturbing acceleration

g_0 -acceleration of central gravity field (zero spherical harmonics)

$r_э$ -Earth's radius

r-radius vector of a spacecraft



Newton's law of universal gravitation

$$F_1 = F_2 = G\frac{m_1 m_2}{r^2}$$

1. The equation for the magnitude of the force caused by gravity

$$\vec{F} = -\frac{\mu m}{r^2}\vec{r}_o = -\frac{\mu m}{r^3}\vec{r}$$

μ=GM – gravitational parameter

$\vec{r}_o = \frac{\vec{r}}{r}$ – unit vector

r – distance between the center of mass of the Earth and the center of mass of the spacecraft

$\vec{r}$ – radius-vector of the spacecraft

m – mass of a spacecraft

2.Combining Newton's second law with universal gravitation law, we obtain an equation for the acceleration vector of a satellite:

$$\vec{F} = m\vec{a} = -\frac{\mu m}{r^3}\vec{r}$$

Motion equation in inertial coordinate system

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = 0 \qquad (1)$$

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Motion equation in
inertial coordinate system

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = 0$$

$$\dot{X} = V_X,$$

$$\dot{Y} = V_Y,$$

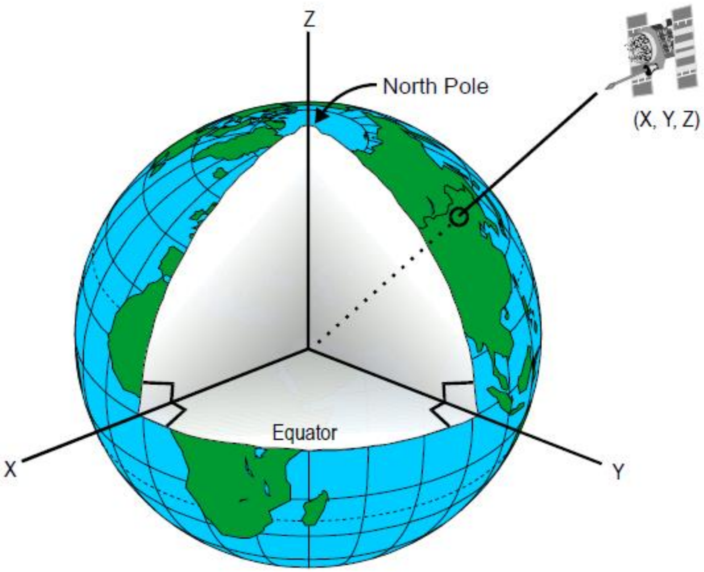
$$\dot{Z} = V_Z,$$

$$\dot{V}_X = -\frac{\mu}{r^3}X$$

$$\dot{V}_Y = -\frac{\mu}{r^3}Y$$

$$\dot{V}_Z = -\frac{\mu}{r^3}Z$$

$\mu = 398602\ km^3/s^2$



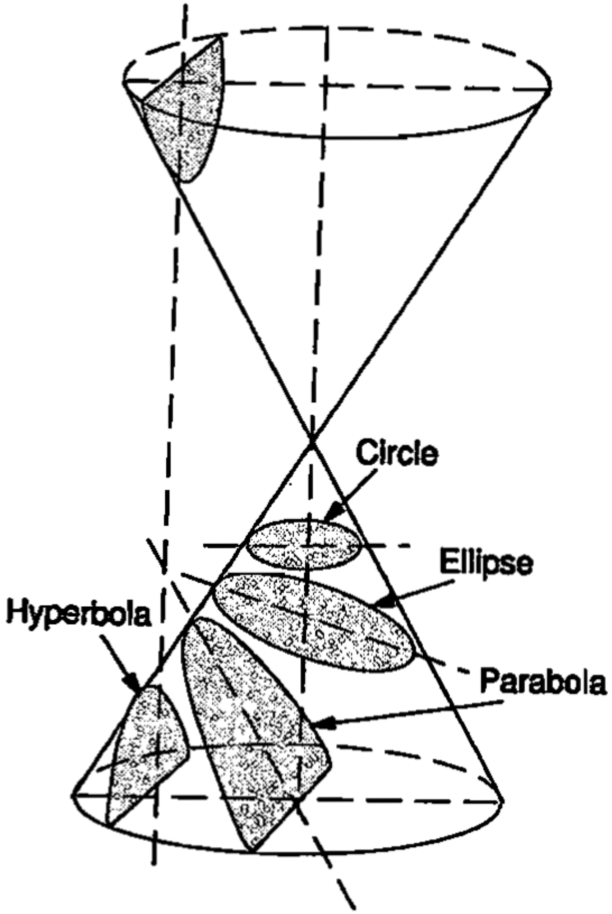
Assumptions:

1) Central gravity field
2) No other forces
3) Mass of a Satellite << Mass of the Earth

$$r = \frac{p}{1 + e\cos(\nu)} \quad (5)$$

<u>Kepler's laws №1 of spacecraft motion</u>

Motion of a spacecraft in the central gravitational field is made along a conical section. One of the focuses is located in the attracting center (the Earth), and the main focal axis coincides with the direction of the Laplace vector.



There is the following classification of orbits depending on the magnitude eccentricity:

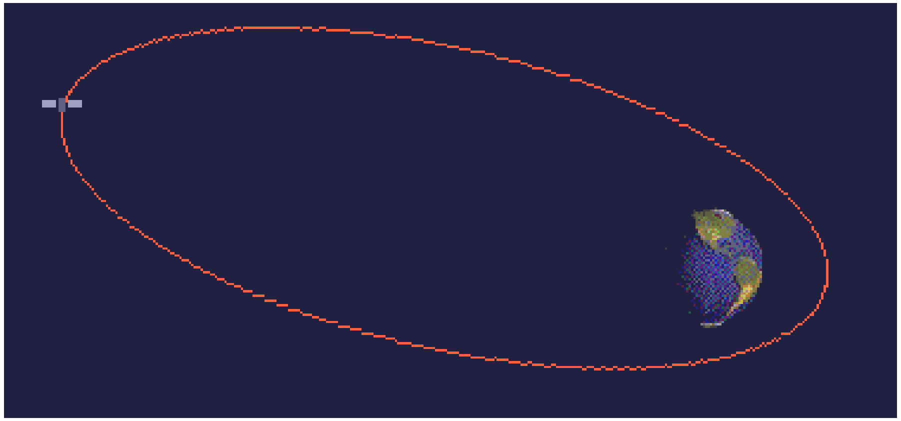
e=0 – orbit is a circle
0<e<1 – orbit is a ellipse
e=1 – orbit is a parabola
e>1 – orbit is a hyperbola

Elliptic orbits are the most common in nature

The equation of the elliptical orbit is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$0<e<1 \qquad r = \dfrac{p}{1 + e\cos\nu}$



a) If e,p are given then

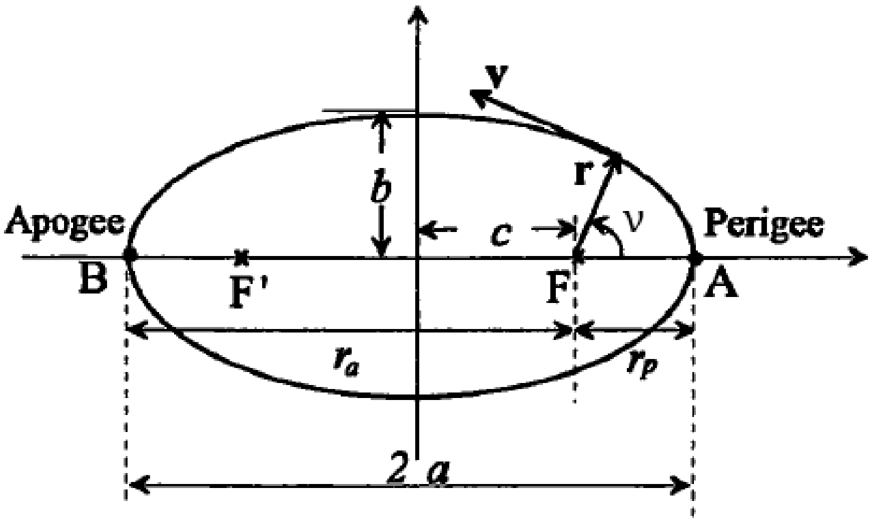
$$r_\pi = \frac{p}{1+e}, r_\alpha = \frac{p}{1-e},$$

$$a = \frac{r_\alpha + r_\pi}{2} = \frac{p}{1-e^2},$$

$$c = \frac{r_\alpha - r_\pi}{2} = ae,$$

$$b = \sqrt{a^2 - c^2} = a\sqrt{1-e^2} = \frac{p}{\sqrt{1-e^2}},$$

$$e = \frac{c}{a}.$$



b) If r_π, r_α are given then

$$r_\pi = R + H_\pi, r_\alpha = R + H_\alpha,$$

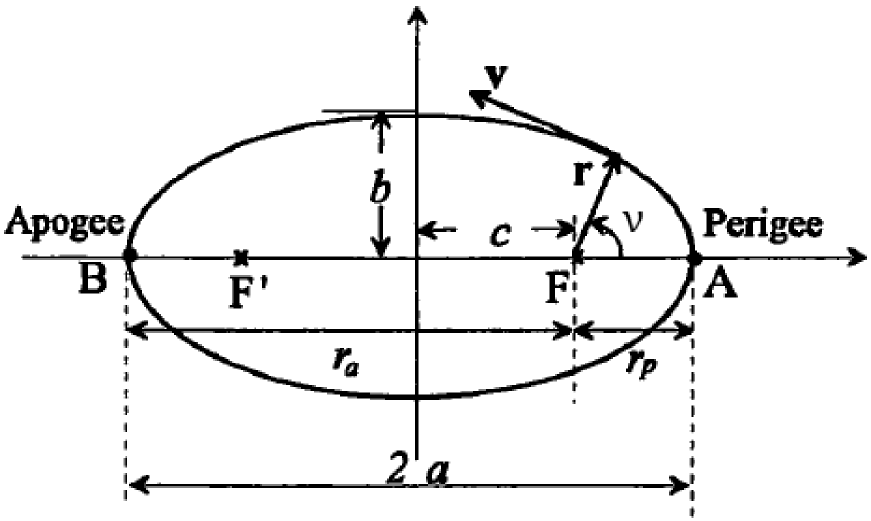
$$p = r_\pi(1 + e) = r_\alpha(1 - e),$$

$$b = \sqrt{2 r_\alpha r_\pi}$$

$$p = \frac{2 r_\alpha r_\pi}{r_\alpha + r_\pi}$$

$$e = \frac{r_\alpha - r_\pi}{r_\alpha + r_\pi}$$

$$V_{el} = \sqrt{\frac{\mu}{r}\left(2 - \frac{r}{a}\right)}$$



The time of motion along an elliptical orbit describes the Kepler’s equation

$$t-\tau=\frac{a^{3/2}}{\sqrt{\mu}}(E-\mathrm{e}\sin E)$$

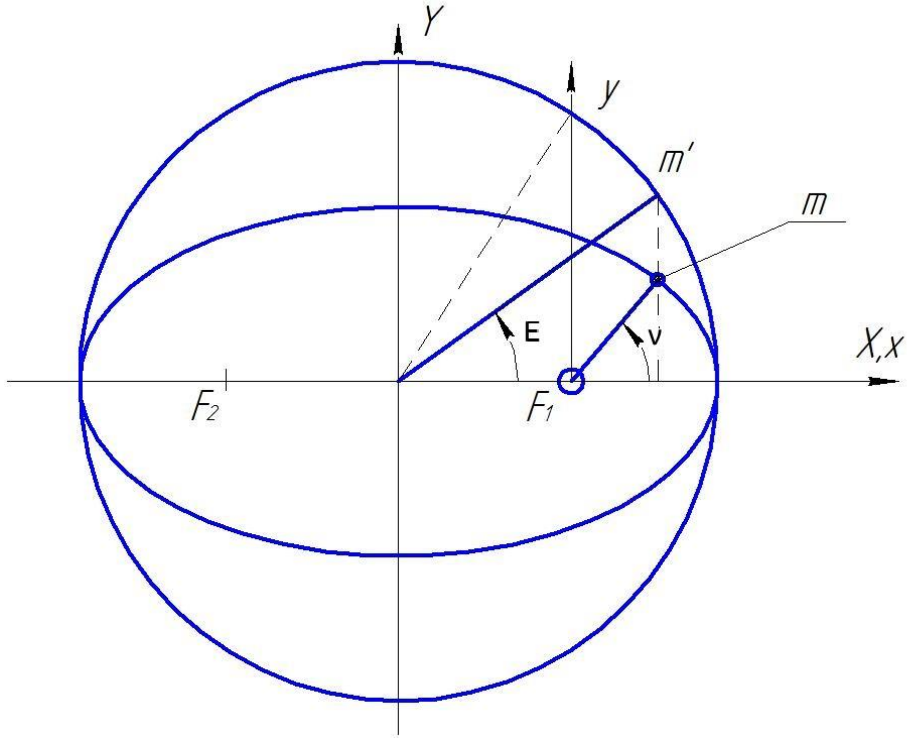
τ - time of passage of pericentre
E - eccentric anomaly

Period of revolution for elliptical orbit

$$T=\frac{2\pi a^{3/2}}{\sqrt{\mu}}$$

Kepler's laws №3 of planetary motion

$$\frac{T_1^{\,2}}{T_2^{\,2}}=\frac{a_1^{\,3}}{a_2^{\,3}}$$



To predict the motion, the Kepler’s equation is represented in the form of the transcendental equation:

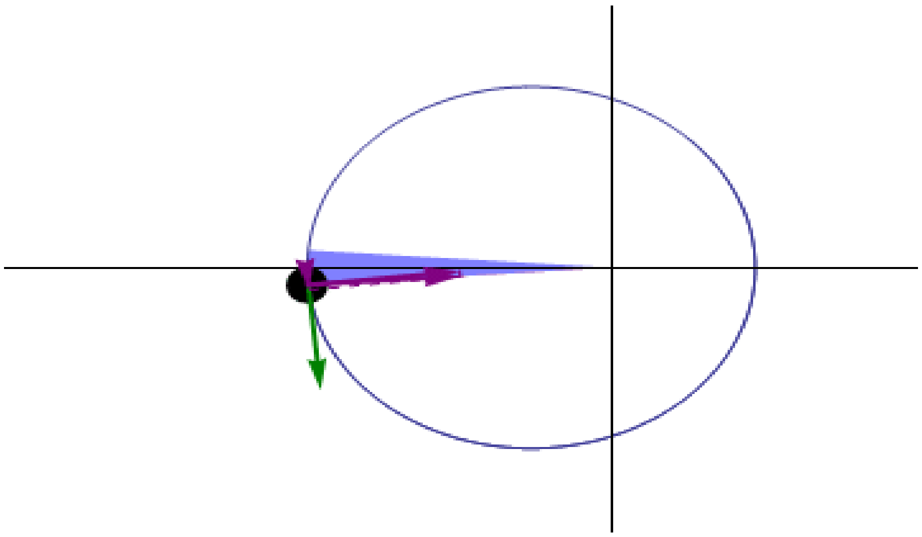
$$E-\mathrm{e}\sin E=M$$

$$M=\sqrt{\frac{\mu}{a^3}}(t-\tau)$$

Kepler's laws №2 of spacecraft motion

$$\vec{r} \times \overrightarrow{V} = \overrightarrow{C}$$

A line segment joining a spacecraft and the Earth sweeps out equal areas during equal intervals of time



The same (blue) area is swept out in a fixed time period. The green arrow is velocity. The purple arrow directed to the Earth is the acceleration. The other two purple arrows are components of acceleration. They are parallel and perpendicular to the velocity.

Velocity in circular orbit $V_1 = \sqrt{\frac{\mu}{r}}$

The Escape velocity $V_e = \sqrt{\frac{2\mu}{r}} = \sqrt{2}V_1$ – is the minimum speed needed for a free, non-propelled object to escape from the gravitational influence of a massive body

	1^{st}, *km/s*	2^{nd}, *km/s*
Earth	7,91	11,2
Moon	1,68	2,38
Mars	3,55	5

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Elements of the orbit in space

The main **two elements that define the shape and size** of the ellipse:

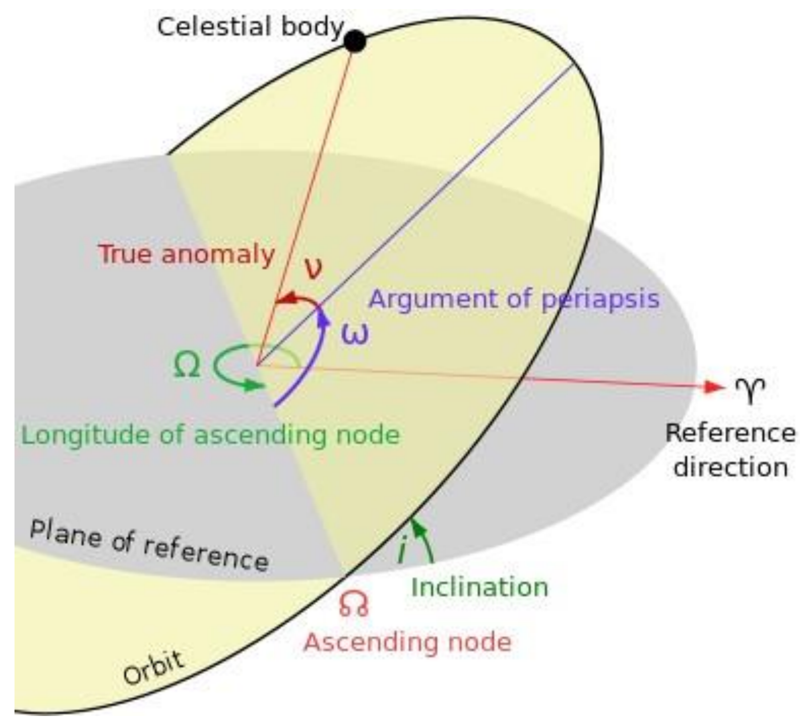
<u>Eccentricity (e)</u>—shape of the ellipse, describing how much it is elongated compared to a circle (not marked in diagram).

<u>Semimajor axis (a)</u> —the sum of the periapsis and apoapsis distances divided by two.

Two elements define the orientation of the orbital plane in which the ellipse is embedded:

<u>Inclination (i)</u>—vertical tilt of the ellipse with respect to the reference plane, measured at the ascending node (where the orbit passes upward through the reference plane, the green angle i in the diagram).

<u>Longitude of the ascending node</u> (☊ or Ω)—horizontally orients the ascending node of the ellipse (where the orbit passes upward through the reference plane) with respect to the reference frame's vernal point (the green angle Ω in the diagram).



<u>Argument of periapsis (ω)</u> defines the orientation of the ellipse in the orbital plane, as an angle measured from the ascending node to the periapsis (the closest point the satellite object comes to the primary object around which it orbits, the blue angle ω in the diagram).

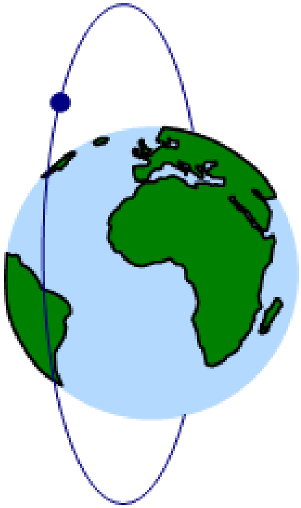
<u>True anomaly</u> (ν, θ, or f) at epoch (M0) defines the position of the orbiting body along the ellipse at a specific time (the "epoch").

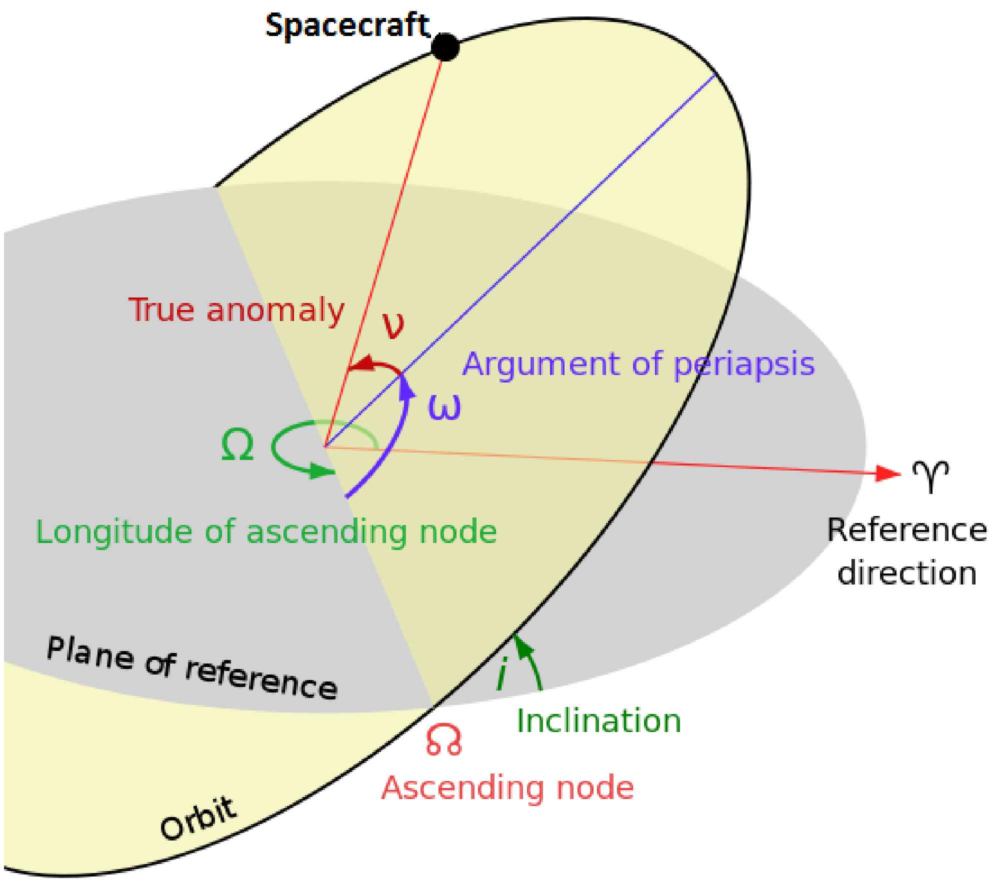
Classification of orbits of spacecraft by inclination of the orbit

Inclination belongs to the range $0^\circ < i < 90^\circ$

1. Equatorial orbit $i = 0^\circ$

2. Polar orbit $i = 90^\circ$





$$X = r(\cos u \cos\Omega - \sin u \cos i \sin\Omega),$$

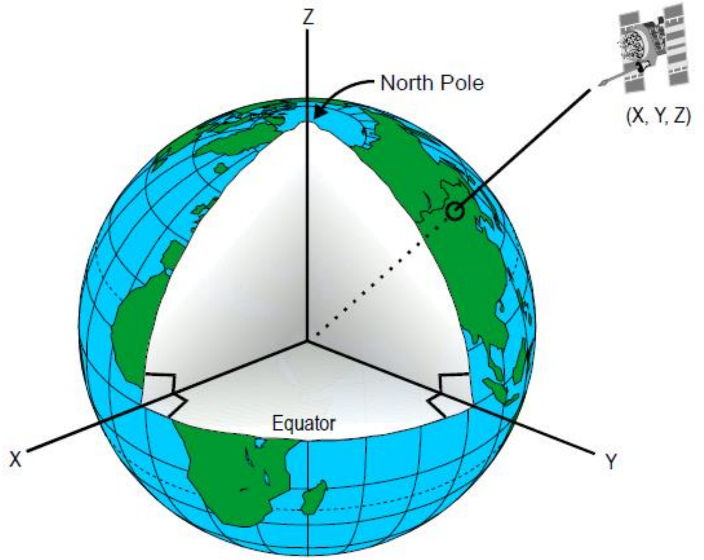
$$Y = r(\cos u \sin\Omega + \sin u \cos i \cos\Omega),$$

$$Z = r \sin u \sin i\,.$$

$$V_x = V_r(\cos u \cos\Omega - \sin u \cos i \sin\Omega) - V_n(\cos\Omega \sin u + \sin\Omega \cos u \cos i),$$

$$V_y = V_r(\cos u \sin\Omega + \sin u \cos i \cos\Omega) - V_n(\sin\Omega \sin u - \cos\Omega \cos u \cos i),$$

$$V_z = V_r \sin u \sin i + V_n \cos u \sin i\,,$$



$$\dot{X} = V_X, \qquad \dot{Y} = V_Y, \qquad \dot{Z} = V_Z,$$

$$\dot{V}_X = -\frac{\mu}{r^3}X$$

$$\dot{V}_Y = -\frac{\mu}{r^3}Y$$

$$\dot{V}_Z = -\frac{\mu}{r^3}Z$$

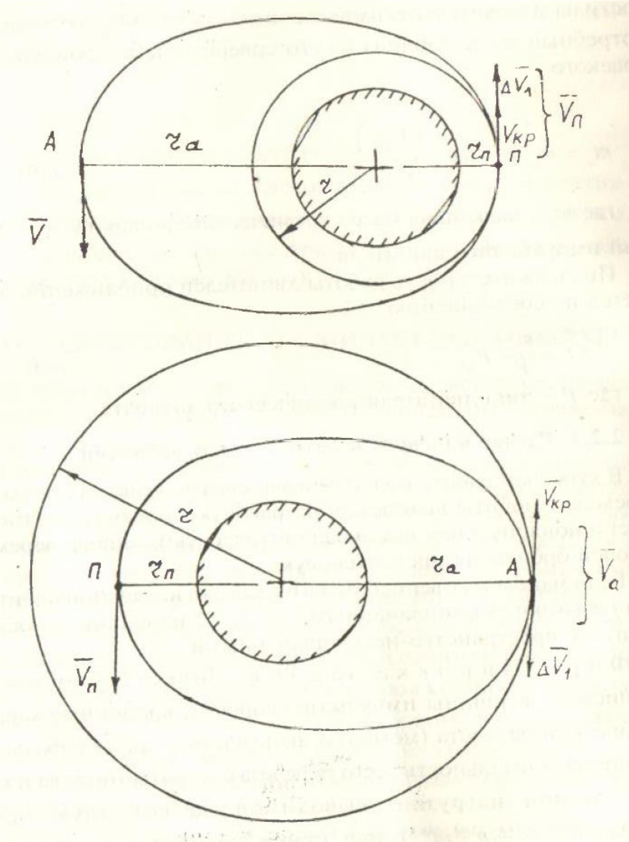
$\mu = 398602\ km^3/s^2$

Coplanar maneuvers

- Transition from a circular orbit to an elliptical orbit.
- Transition from a circular orbit to a hyperbolic orbit.
- Two-burn transition between circular orbits.

Noncoplanar maneuvers

- Transition between noncoplanar circular orbits of equal radius.
- Transition between noncoplanar circular orbits of different radii.



If two orbits are in the same plane, they are coplanar.

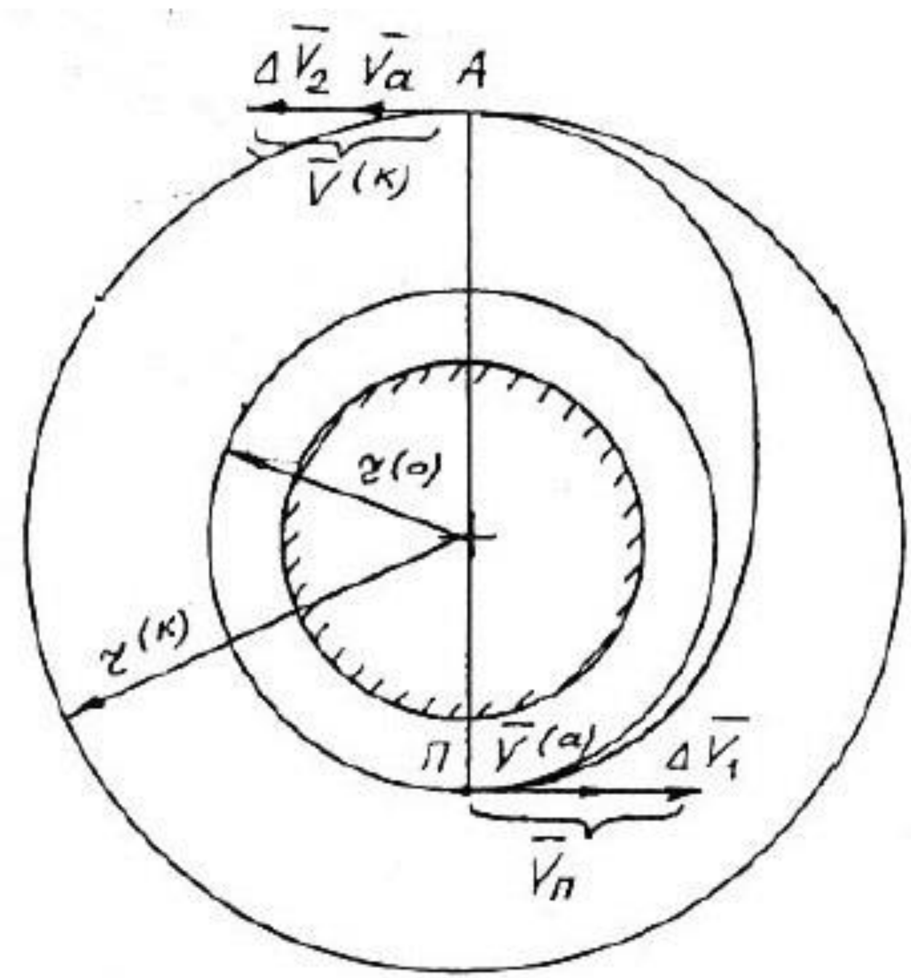
The transfer from a circular orbit to an elliptical or from an elliptical orbit to a circular orbit if they have common point

$$V_p = \sqrt{\frac{2\mu r_a}{r_\mathrm{p}(r_a + r_\mathrm{p})}}, \qquad V_a = \sqrt{\frac{2\mu r_\mathrm{p}}{r_a(r_a + r_\mathrm{p})}}.$$

$$\Delta V_1 = V_\mathrm{p} - V_\mathrm{cir} = \sqrt{\frac{\mu}{r_\mathrm{p}}}\left(\sqrt{\frac{2r_\mathrm{a}}{r_\mathrm{a} + r_p}} - 1\right).$$

$$\Delta V_1 = V_\mathrm{cir} - V_a = \sqrt{\frac{\mu}{r_\mathrm{a}}}\left(1 - \sqrt{\frac{2r_\mathrm{p}}{r_\mathrm{a} + r_\mathrm{p}}}\right).$$

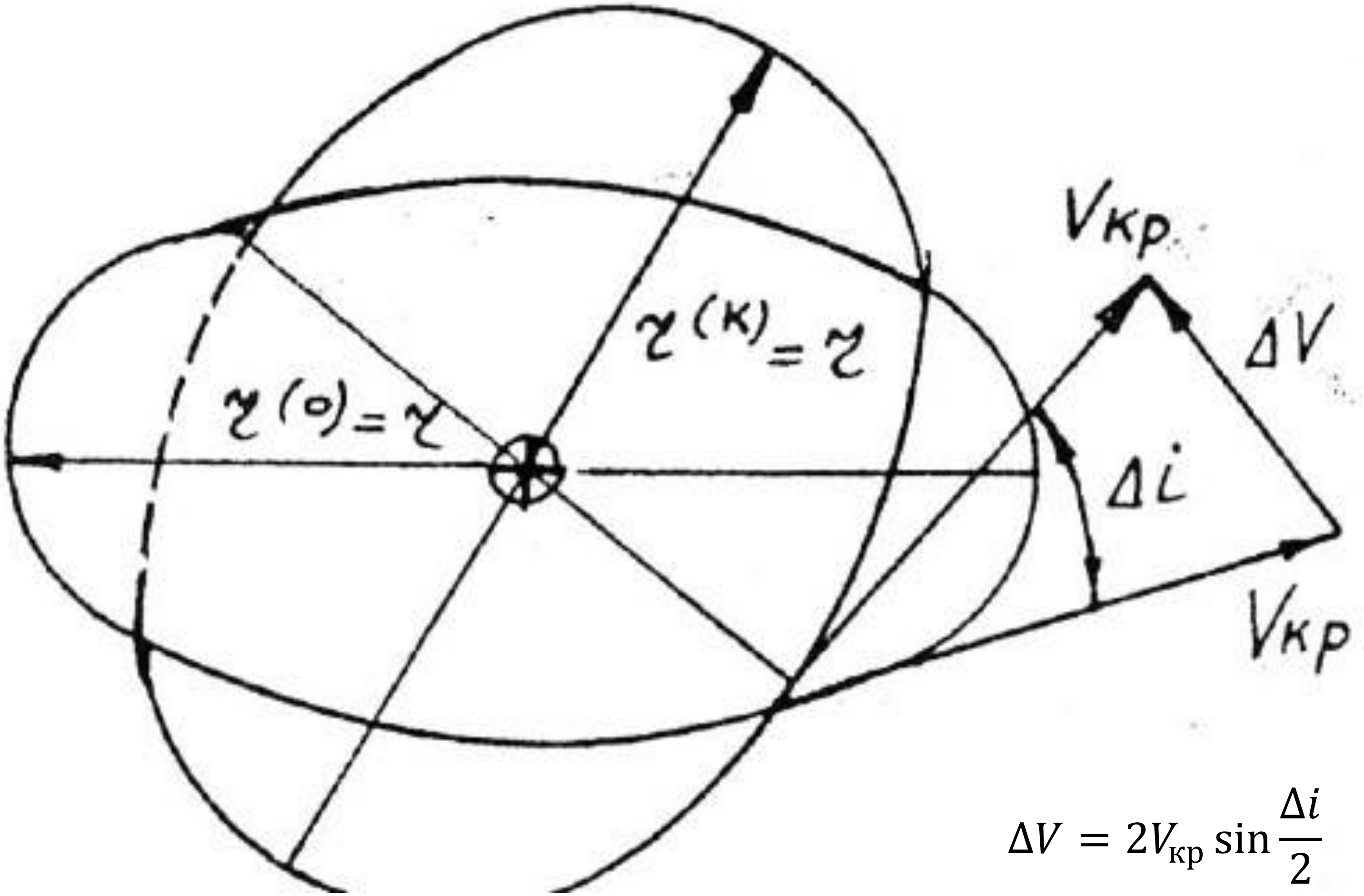
- Two-burn transition between circular orbits.



$$\Delta V_1 = \sqrt{\frac{\mu}{r^{(0)}}}\left(\sqrt{\frac{2r^{(k)}}{r^{(0)}+r^{(k)}}}-1\right)$$

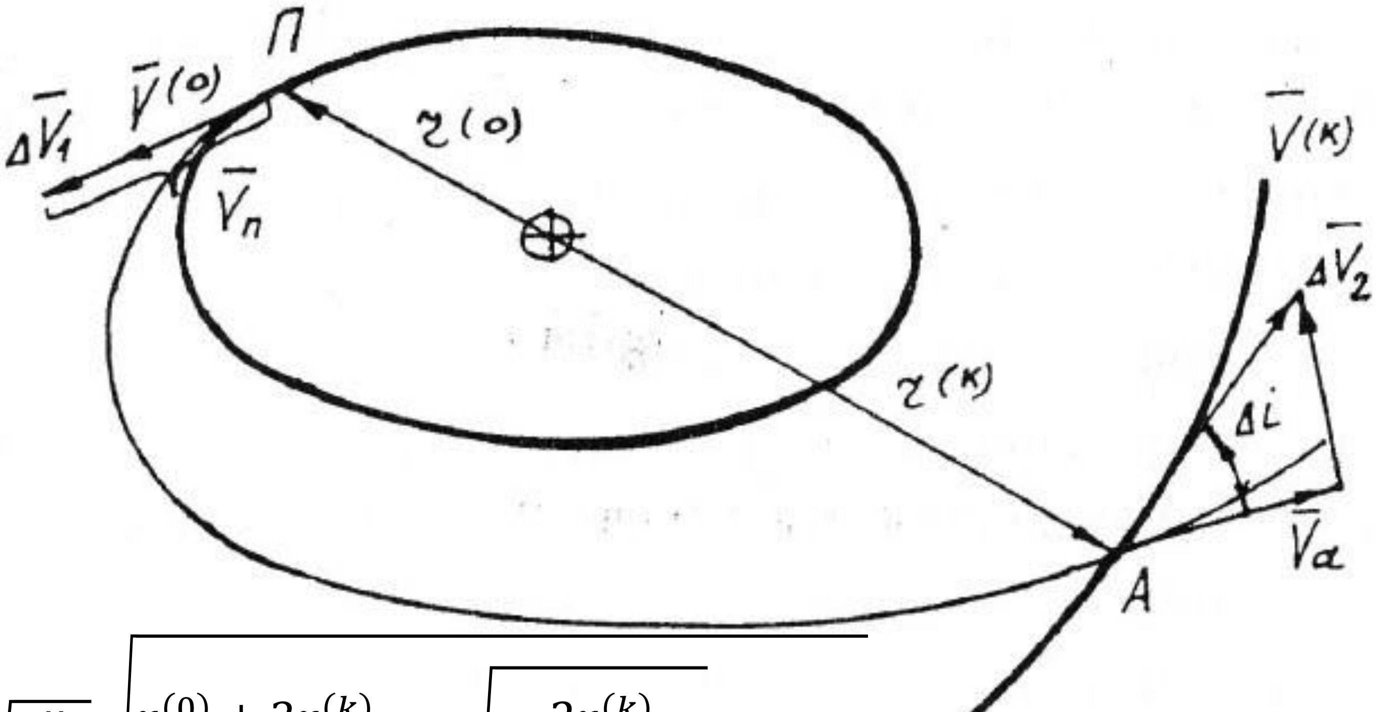
$$\Delta V_2 = \sqrt{\frac{\mu}{r^{(k)}}}\left(1-\sqrt{\frac{2r^{(0)}}{r^{(0)}+r^{(k)}}}\right)$$

- Transition between noncoplanar circular orbits of equal radius.



$$\Delta V = 2V_{кр}\sin\frac{\Delta i}{2}$$

- Transition between noncoplanar circular orbits of different radius.



$$\Delta V_1 = \sqrt{\frac{\mu}{r^{(0)}}}\sqrt{\frac{r^{(0)}+3r^{(k)}}{r^{(0)}+r^{(k)}} - 2\sqrt{\frac{2r^{(k)}}{r^{(0)}+r^{(k)}}}\cos(\Delta i)}$$

$$\Delta V_2 = \sqrt{\frac{\mu}{r^{(k)}}}\sqrt{\frac{3r^{(0)}+r^{(k)}}{r^{(0)}+r^{(k)}} - 2\sqrt{\frac{2r^{(0)}}{r^{(0)}+r^{(k)}}}\cos(\Delta i - \Delta i_1)}$$

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