XVII INTERNATIONAL SUMMER SPACE SCHOOL

“Future space technologies and experiments in space”

Lecture

Features of the nanosatellite dynamics in LEO

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Features of the nanosatellite dynamics in LEO

Lesson plan

1. *Basics of nanosatellite attitude motion*
2. *Features of nanosatellites' motion in low orbits*
3. *Spatial motion in low orbits of nanosatellite around its center of mass*
4. *Planar motion of nanosatellite around its center of mass under the influence of the gravitational and aerodynamic moments during descent from circular low-altitude orbits*
5. *The selection of the CubeSat design parameters*

Basics of nanosatellite attitude motion

Uncontrolled motion of a satellite around its center of mass

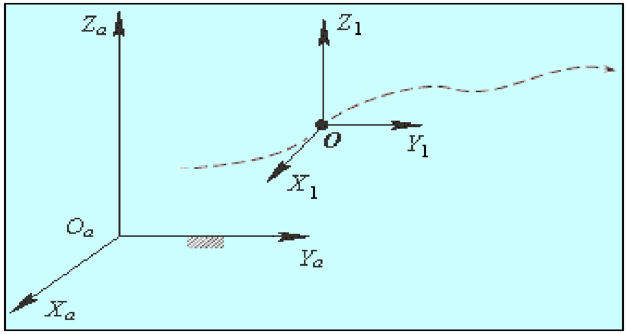


Figure 1.

The motion of system around its center of mass is called motion of points of system relative to the translational moving frame of reference with its origin at the center of mass of the system.

$O_a X_a Y_a Z_a$ is the inertial frame of reference,
$OX_1 Y_1 Z_1$ is the translational moving frame of reference with its origin at the center of mass of the system.

Vector form of Euler's equations of motion

$$\frac{d\vec{K}_o}{dt} + \vec{\omega} \times \vec{K}_o = \vec{M}_o^e \qquad (1)$$

where

$\vec{K}_0 = I\vec{\omega}$ is the kinetic moment (angular momentum) vector about the center of mass,

$\vec{\omega}$ is the absolute angular velocity,

$\vec{M}_0^e$ is the moment of the external forces about the center of mass,

I is the inertia tensor.

Euler's equations of motion

The vector Equation (1) in projections onto the coordinate axes of the frame of reference fixed in the rotating satellite and having its axes parallel to the principal axes of inertia of the satellite:

$$
\begin{aligned}
I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z &= M_x, \\
I_y\dot{\omega}_y + (I_x - I_z)\omega_z\omega_x &= M_y, \\
I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y &= M_z,
\end{aligned}
\qquad (2)
$$

Leonhard Euler (1707 - 1783)

where

$\omega_x, \omega_y, \omega_z$ are the components of the angular velocity vector;

I_x, I_y, I_z are the principal moments of inertia;

M_x, M_y, M_z are the components of the moment of the external forces.

Frames of reference

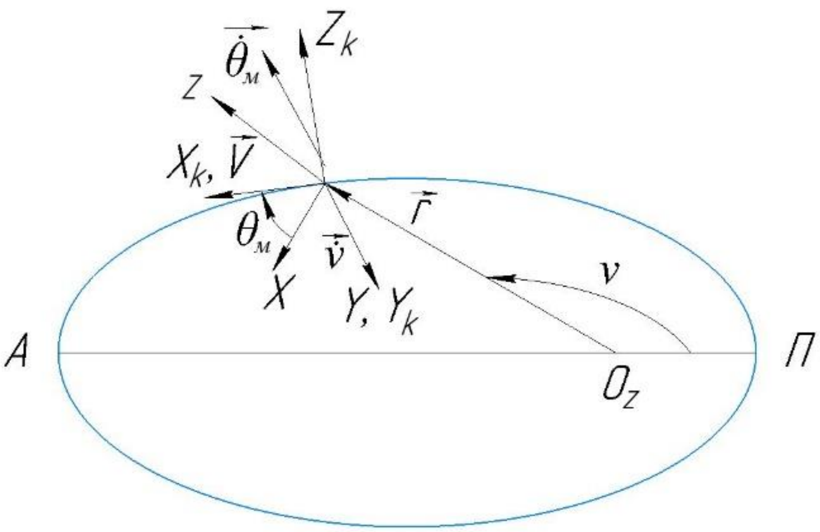


Figure 2.

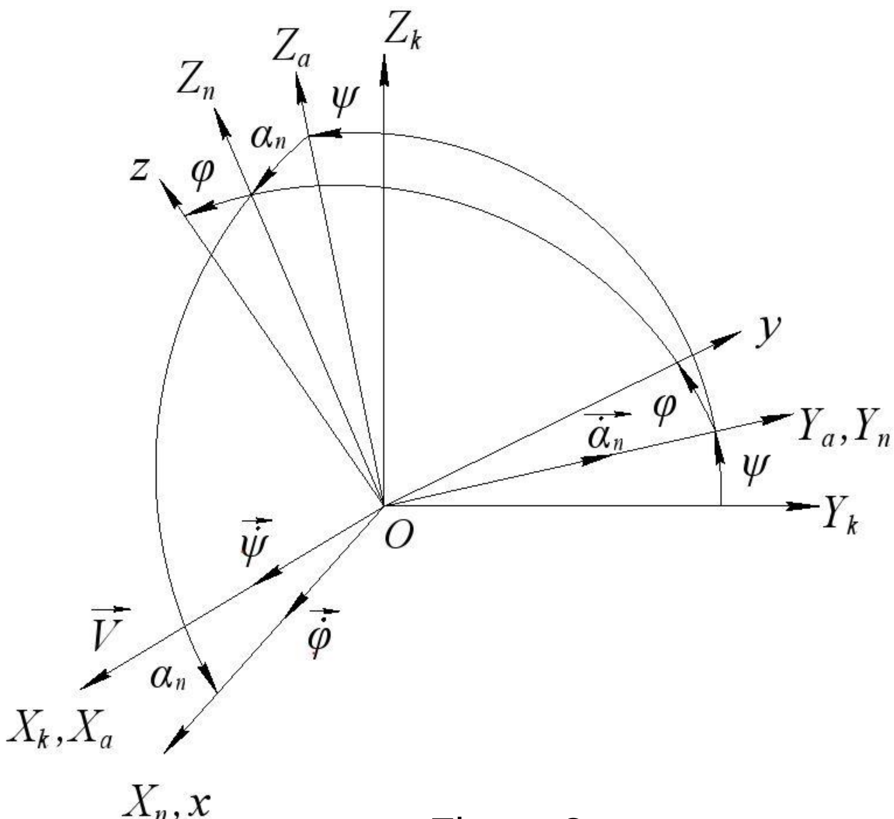


Figure 3.

where
$OXYZ$ is the orbital frame of reference,
$OX_kY_kZ_k$ is the trajectory frame of reference,
θ_m is the inclination angle of trajectory,
ν is the true anomaly.

$$\overline{\omega} = \overline{\dot{\psi}} + \overline{\dot{\varphi}} + \overline{\dot{\alpha}}_n + \overline{\dot{\nu}} + \overline{\dot{\theta}}_m \quad (3)$$

$Oxyz$ is the body-fixed frame of reference;
ψ, θ, φ are Euler angles;
ψ is the angle of precession;
$\theta = \alpha_n$ is the angle of nutation (spatial angle of attack);
φ is the angle of proper rotation.

Equations of kinematics

$$
\begin{aligned}
\omega_x &= \dot{\psi}\cos\alpha_n + \dot{\varphi} + (\dot{v} - \dot{\theta}_m)b_{12}, \\
\omega_y &= \dot{\psi}\sin\varphi\sin\alpha_n + \dot{\alpha}_n\cos\varphi + (\dot{v} - \dot{\theta}_m)b_{22}, \qquad (4) \\
\omega_z &= \dot{\psi}\cos\varphi\sin\alpha_n - \dot{\alpha}_n\sin\varphi + (\dot{v} - \dot{\theta}_m)b_{32},
\end{aligned}
$$

where b_{ij} – the direction cosine matrix of the orthogonal transformation from the trajectory frame of reference to the body-fixed frame of reference.

Moments of the external forces acting on the satellite

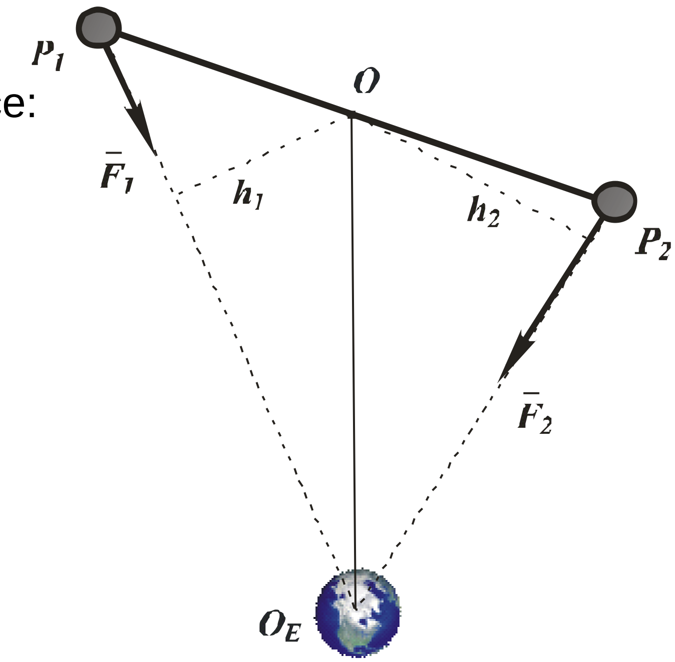
- Gravitational moment
- Aerodynamic moment
- Magnetic moment
- Moment of the pressure of the solar rays
- Reaction moment of gas efflux from the satellite
- Moment of shocks of meteoric particles

Gravitational moment

The projections of the gravitational moment vector onto the coordinate axes of the body-fixed frame of reference:

$$M_{xg} = \frac{3\gamma M}{R^3}(I_z - I_y)c_{23}c_{33}\,,$$
$$M_{yg} = \frac{3\gamma M}{R^3}(I_x - I_z)c_{33}c_{13}\,, \qquad (5)$$
$$M_{zg} = \frac{3\gamma M}{R^3}(I_y - I_x)c_{13}c_{23}\,,$$

where I_x, I_y, I_z are the principal moments of inertia of the satellite;
R is the distance from the center of attraction to the center of mass of the satellite;
γ is the universal gravitational constant;
M is the mass of Earth;
c_{ij} are the direction cosine matrix of the orthogonal transformation from the orbital frame of reference to the body-fixed frame of reference.



$$m_2 = m_1,$$
$$O_E P_2 < O_E P_1\,,$$
$$F_2 > F_1, \qquad h_2 > h_1,$$
$$M_2 = F_2 h_2 \;>\; M_1 = F_1 h_1.$$

Figure 4.

Condition of oscillatory motion of a satellite in a circular orbit

$$h < \left\{ \frac{3}{2} n^2 (I_x - I_z),\ \frac{1}{2} n^2 (I_y - I_x) \right\}, \qquad (6)$$

$$h = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2) + \frac{3}{2} n^2 (I_x c_{31}^2 + I_y c_{32}^2 + I_z c_{33}^2) -$$
$$- n(I_x\omega_x c_{21} + I_y\omega_y c_{22} + I_z\omega_z c_{23}), \qquad (7)$$

where h is the first integral of the equations of motion of the satellite,

$n = \sqrt{\mu / (R_E + H)^3}$ is the orbital angular velocity of the satellite,

R_E is the radius of the spherical Earth,

μ is Earth's gravitational parameter,

H is the altitude of the circular orbit.

Condition of relative stable equilibrium of the satellite in the orbital frame of reference

$$I_y > I_x > I_z, \qquad (8)$$

where I_x, I_y, I_z are the principal moments of inertia.

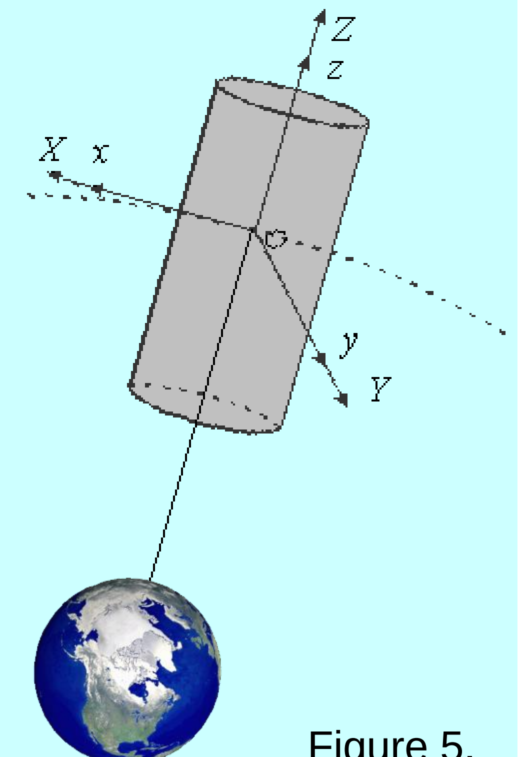


Figure 5.

Aerodynamic moment

$$\vec{M}_a = \vec{r}_d \times \vec{Q}_{xv},$$

where $\vec{r}_d$ is the center of pressure position relative to the center of mass;

$\vec{Q}_{xv} = -c_0 \tilde{S} q S \vec{e}_v$ is aerodynamic drag force

The projections of the gravitational moment vector onto the coordinate axes of the body-fixed frame of reference:

$$M_{xa} = 0,$$
$$M_{ya} = m_a(\alpha, \varphi) qSl \cos\varphi, \quad (9)$$
$$M_{za} = -m_a(\alpha, \varphi) qSl \sin\varphi,$$

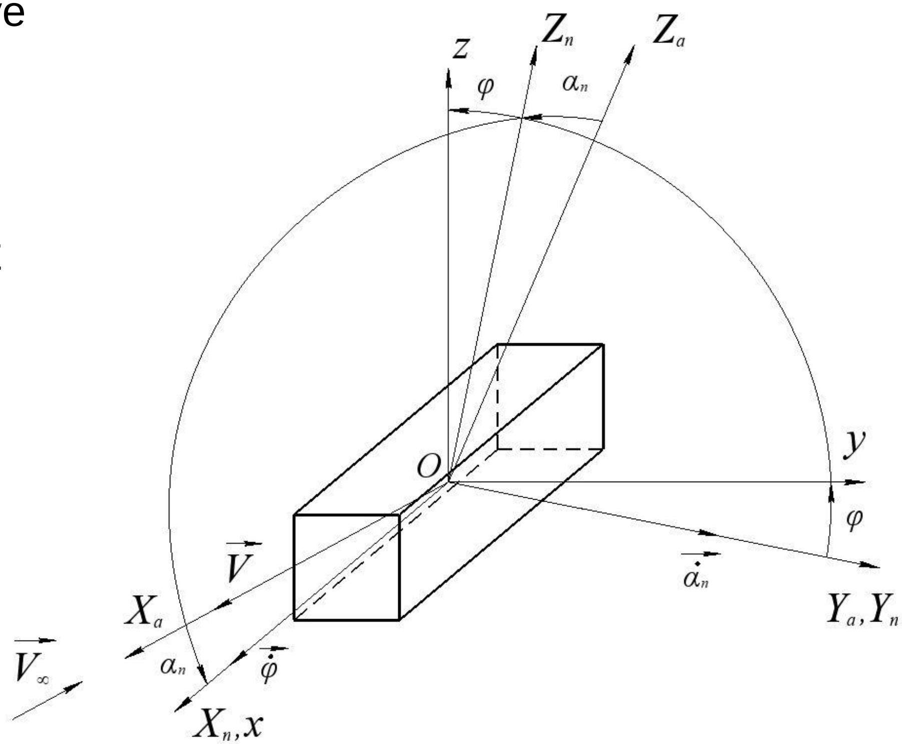


Figure 6.

where

$m_a(\alpha, \varphi)$ is the restoring aerodynamic moment coefficient measured about the nanosatellite center of mass, $\alpha = \alpha_n$ is the spatial angle of attack, φ is the angle of proper rotation;

$q = \rho V^2 / 2$ is velocity head; V is flight speed; ρ is atmospheric density;

S is the characteristic area; l is the characteristic dimension.

The restoring aerodynamic moment coefficient

$$m_\alpha(\alpha,\varphi) = -c_0 \tilde{S}(\alpha,\varphi)\Delta\bar{x}\sin\alpha\,, \qquad (10)$$

where

c_0 = 2.2 is the drag force coefficient;

$\Delta\bar{x} = \Delta x / l$ is the relative static stability margin, Δx is the static stability margin (the distance measured from the center of mass to the geometric center of the nanosatellite, l is the nanosatellite length;

$\tilde{S} = |\cos\alpha| + k\sin\alpha \cdot \left(|\sin\varphi| + |\cos\varphi|\right)$ is the nanosatellite area projected on a plane that is perpendicular to the flow velocity vector divided by the characteristic area of nanosatellite, k is the ratio of the one side surface area to the characteristic area.

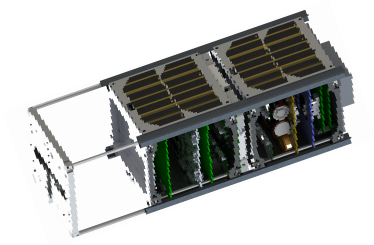
For the analysis of angular motion of the nanosatellite in the case when the angular velocity of proper rotation is close to uniform the restoring aerodynamic moment coefficient can be averaged over the angle of proper rotation:

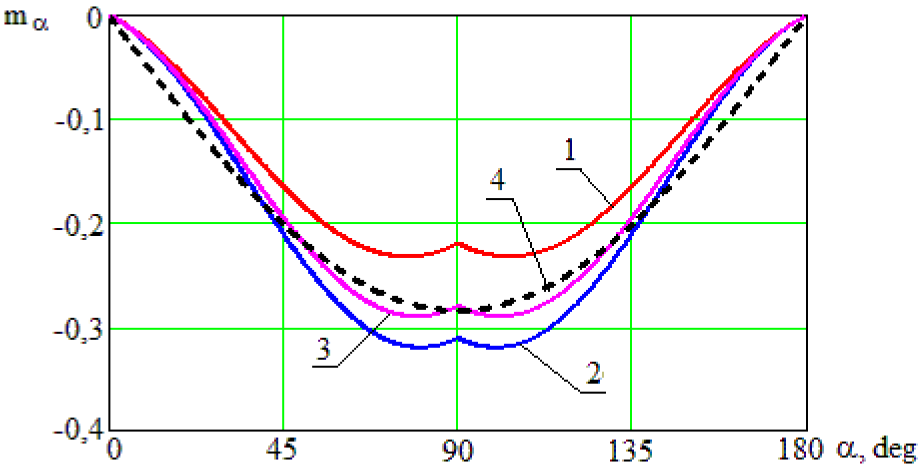
$$m_\alpha(\alpha,\varphi) = -c_0\Delta\bar{x}\sin\alpha\left(|\cos\alpha| + \frac{4k}{\pi}|\sin\alpha|\right). \qquad (11)$$

For approximate analysis of motion parameters, the dependence of the restoring aerodynamic moment coefficient measured about the nanosatellite center of mass $m_\alpha(\alpha)$, averaged over the angle of proper rotation φ, with sufficient accuracy can be approximated by a sinusoidal dependence in the angle of attack:

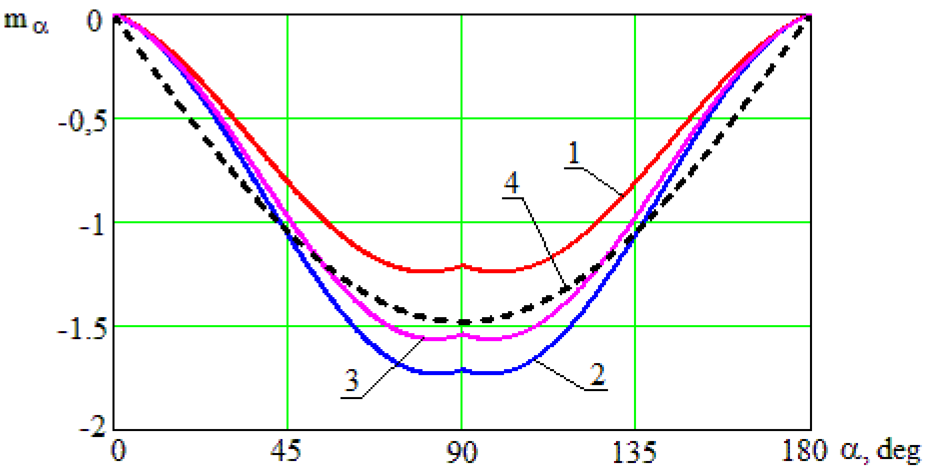
$$m_\alpha(\alpha) = a_0\sin\alpha\,. \qquad (12)$$

The restoring aerodynamic moment coefficient of transforming nanosatellite SamSat-QB50





(a) - before transformation



(b) - after transformation

Figure 7. Dependence of the restoring aerodynamic moment coefficient of SamSat-QB50 on the spatial angle of attack α and the angle of proper rotation φ,
1 - $\varphi = 0$, 2 - $\varphi = 45°$, 3 - averaged over the angle φ,
4 - approximated by sinusoidal dependence $a_0 \sin(\alpha)$.

Before the nanosatellite transformation coefficient $a_0 = -0.28$, after transformation $a_0 = -1.5$

Due to the transformation the aerodynamic moment value is increased in 8 times, while the gravitational moment is increased only in 1.7.

Features of nanosatellites' motion in low orbits

1. The ballistic coefficient of the spacecraft is inversely proportional to the its linear dimension, thus the value of the ballistic coefficient of nanosatellite is greater than for a satellite with large dimensions and mass (with the same values of the relative static stability margin and mass density value), and, therefore, the lifetime in the orbit of nanosatellite is shorter.

$\sigma = \frac{c_0 S}{m}$ is the ballistic coefficient,

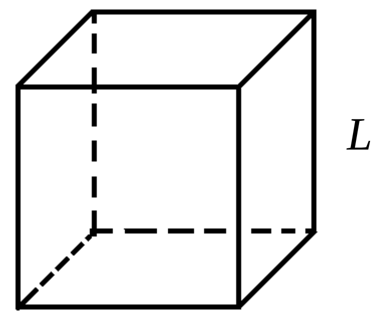
where c_0 = 2.2 is the drag force coefficient, m is the satellite mass,
S is the projection area of the nanosatellite on the plane perpendicular to the velocity vector of the oncoming flow.

Nanosatellite CubeSat 1U (0.1×0.1×0.1 m^3)

$$\sigma_c \sim \frac{c_0 l^2}{m_c} = \frac{c_0 l^2}{\gamma_c l^3} = \frac{c_0}{\gamma_c l},$$

where γ_c is the mass density of the nanosatellite,
l is the rib length of the nanosatellite.

Minisatellite (Cube: 1 × 1 × 1 m^3)



$$\sigma_m \sim \frac{c_0 L^2}{m_m} = \frac{c_0 L^2}{\gamma_m L^3} = \frac{c_0}{\gamma_m L},$$

where γ_m is the mass density of the minisatellite,
L is the rib length of the minisatellite.

$$\frac{\sigma_c}{\sigma_m} = \frac{\gamma_m L}{\gamma_c l} = 10\frac{\gamma_m}{\gamma_c} \quad (13)$$

Features of nanosatellites' motion in low orbits

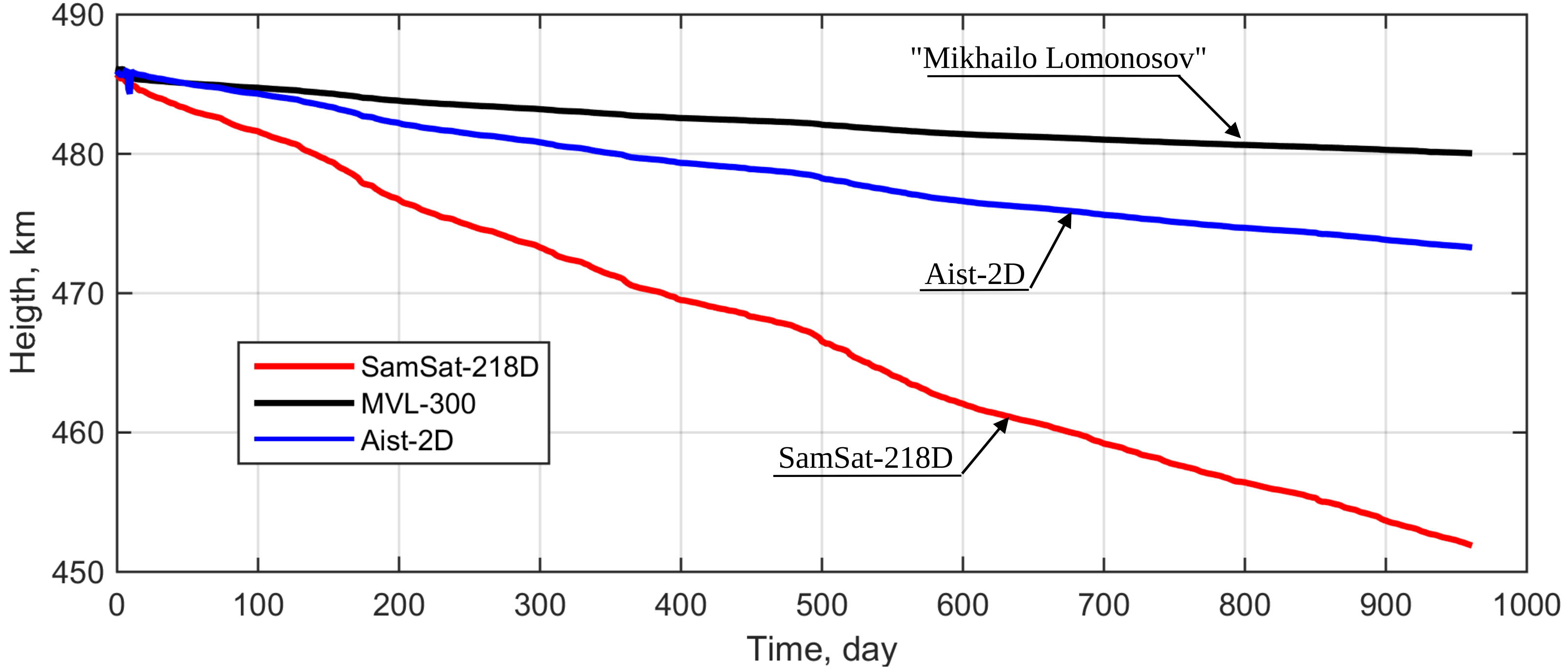


Figure 8. The changes in altitude of the orbit of satellites "Mikhailo Lomonosov", Aist-2D and nanosatellite SamSat-218D within 31 months, which were launched into close to a circular orbit with an average altitude of H = 486 km at 28 of April, 2016, from Vostochny

Features of nanosatellites' motion in low orbits

2. Since the magnitude of the angular acceleration due to the aerodynamic moment of the satellite is inversely proportional to the square of the its linear dimension, then the angular acceleration due to the aerodynamic moment acting on nanosatellite is much higher than for the satellite with large dimensions and mass (with the same values of the relative static stability margin and mass density value). This extends the range of altitudes at which the aerodynamic moment acting on the nanosatellite is significant and it can be used for passively stabilization of the nanosatellite along the velocity vector.

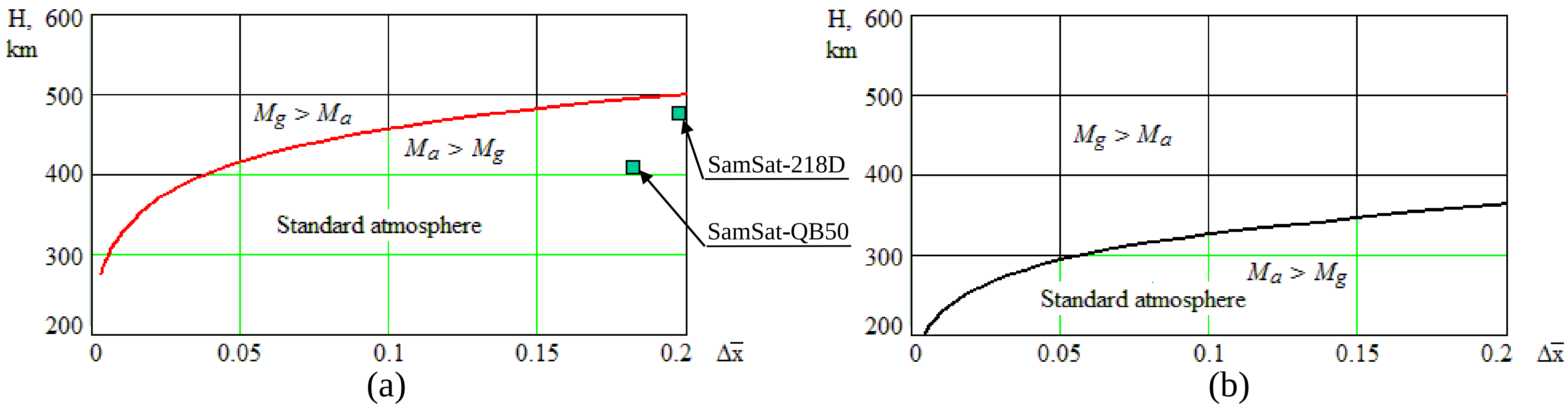


Figure 9. The area of altitudes *H* and the relative margin of static stability, where the aerodynamic moment M_a exceeds the gravitational moment M_g for: (a) - the nanosatellite CubeSat 3U; (b) - the satellite whose dimensions are 10 times larger than the dimensions of the nanosatellite CubeSat 3U.

SamSat-218D: H_0=486km, M_a / M_g = 2.3.

SamSat-QB50: H_0=405km, M_a / M_g = 10.

3. Existing commercial separation systems of nanosatellites generate large initial angular velocity values. In addition, when launching nanosatellites from platforms that perform uncontrolled motion, it is necessary to take into account the random nature of the angular motion of these platforms. These features of the motion of nanosatellites cause the need to apply a probabilistic approach for analysis of motion around its center of mass.

4. It is important to consider the possibility of occurrence of resonant modes of motion. Due to CubeSat nanosatellites have the shape of a rectangular parallelepiped, the aerodynamic moment depends not only on the spatial angle of attack and but also on the angle of proper rotation, and this creates the prerequisites for the appearance of a resonance, which manifests itself in a sharp change of the amplitude of oscillations of the angle of attack, when the linear integer combination of the oscillation frequency of the spatial angle of attack and the average frequency of its proper rotation is close to zero.

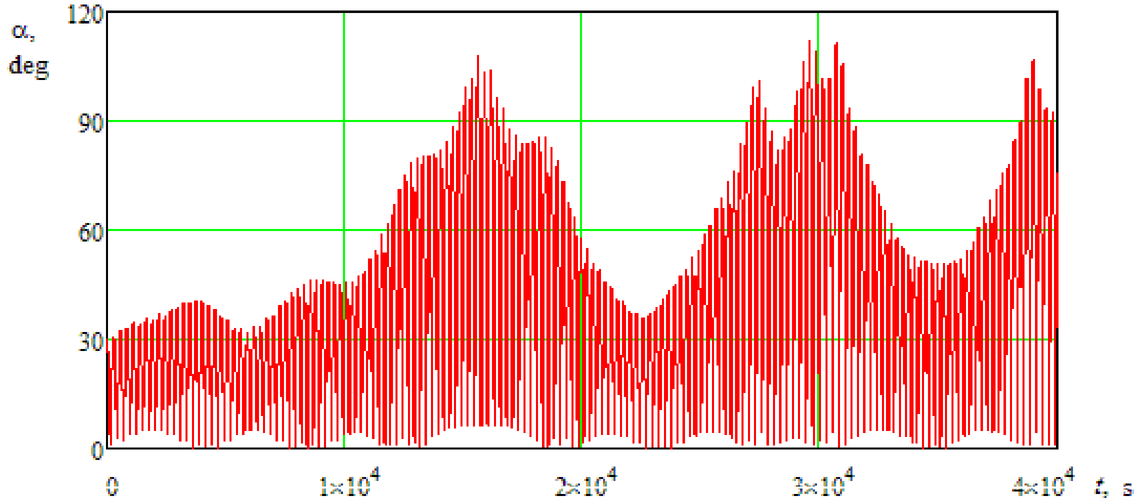


Figure 10. The resonant change in the spatial angle of attack of CubeSat 3U for the following initial conditions of motion: the initial altitude of the flight *H* = 270 km, initial value of the spatial angle of attack $\alpha_0 = 30°$, longitudinal angular velocity $\omega_x = 0.4° / s$.

Regular precession of nanosatellite

$$\frac{d\vec{K}_0}{dt} = \vec{M}_o^e = 0 \quad \Rightarrow \quad \vec{K}_0 = \text{const} \qquad (14)$$

where

$\vec{K}_0 = I\vec{\omega}$ is the kinetic moment (angular momentum) vector about the center of mass,

$\vec{\omega}$ is the absolute angular velocity,

$\vec{M}_0^e$ is the moment of the external forces about the mass center,

I is the inertia tensor matrix,

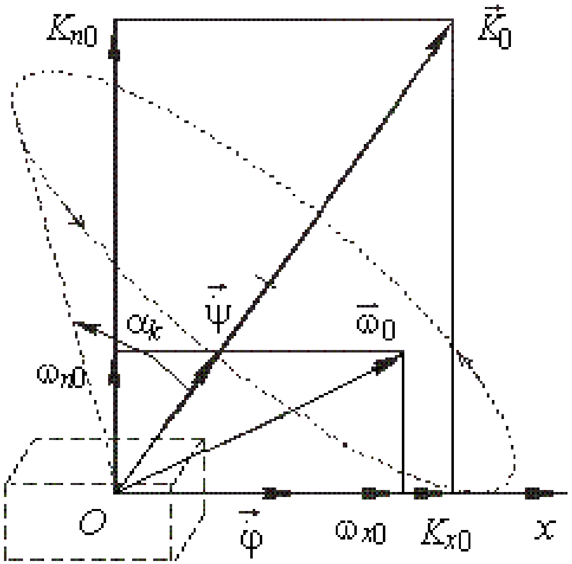


Figure 11.

Angular velocity of precession: $$\dot{\psi} = \frac{I_x \omega_{x0}}{I_n \cos\alpha_k} \qquad (15)$$

Angular velocity of proper rotation: $$\dot{\varphi} = \frac{(I_n - I_x)\omega_{x0}}{I_n} \qquad (16)$$

Angle between the axis of symmetry and the axis about which it precesses (the half-angle cone of precession): $$\alpha_k = \arcsin\left(\frac{K_{n0}}{K_0}\right) \qquad (17)$$

$$K_0 = \sqrt{K_{x0}^2 + K_{n0}^2}$$

$$K_{n0} = I_n \omega_{n0}\,, \quad K_{x0} = I_x \omega_{x0}$$

$$\omega_{n0} = \sqrt{\omega_{y0}^2 + \omega_{z0}^2}$$

$I_y = I_z = I_n$, I_x are transversal and longitudinal moments of inertia.

Regular precession of nanosatellite

Trajectory of the end of the longitudinal axis of nanosatellite SamSat-QB50 on the unit sphere concerning the inertial reference frame
(ω_x = 0.2 deg/s, ω_y = 0, ω_z = 0.2 deg/s, time interval = 2650 s, statical stability factor = 0)

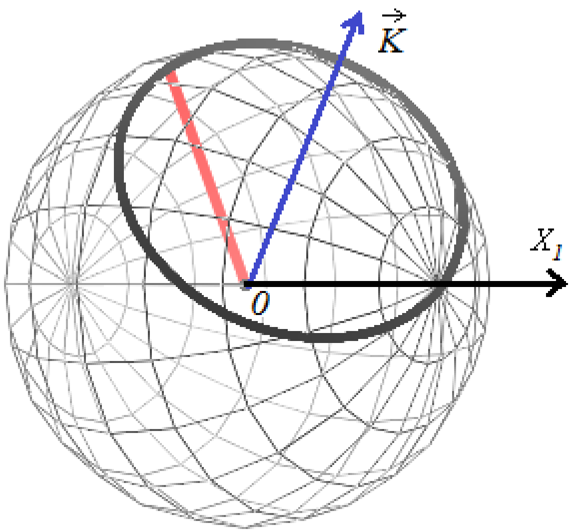
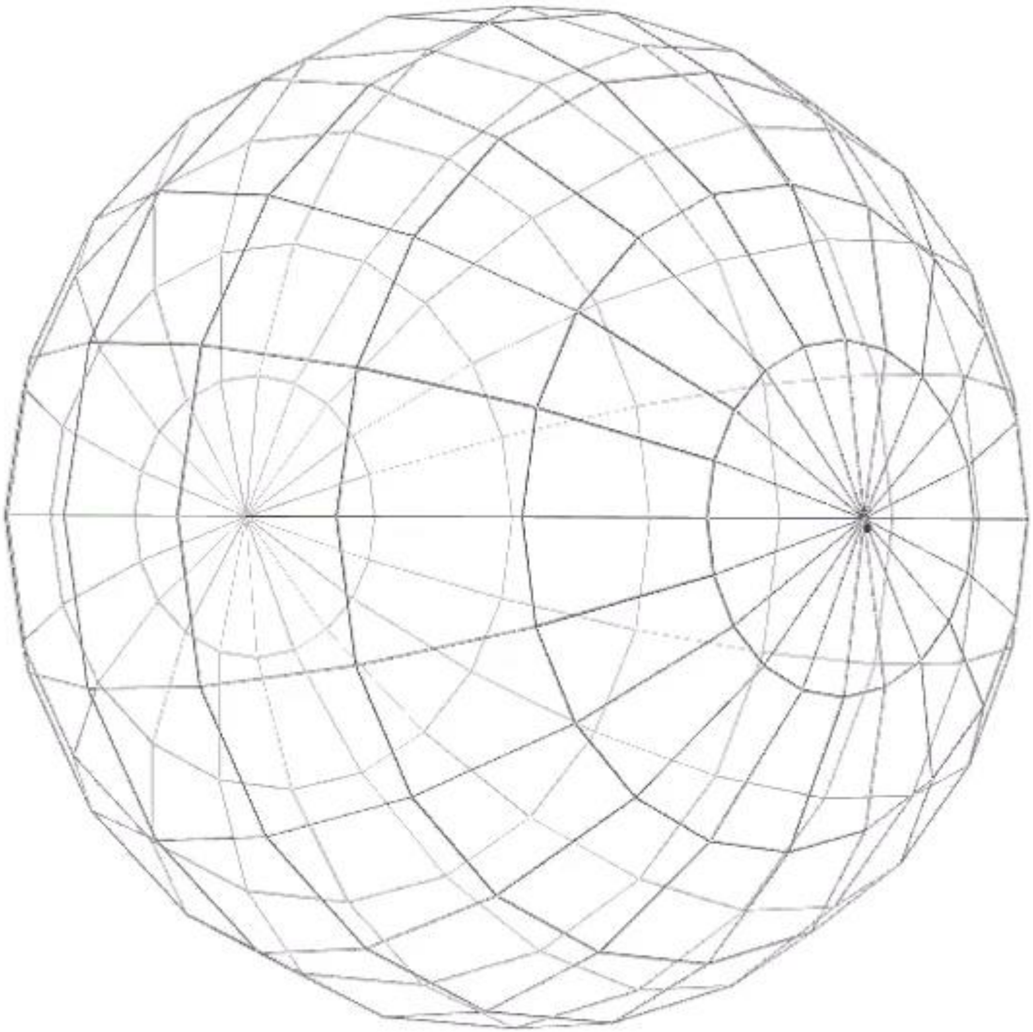


Figure 12.



Forced precession of nanosatellite

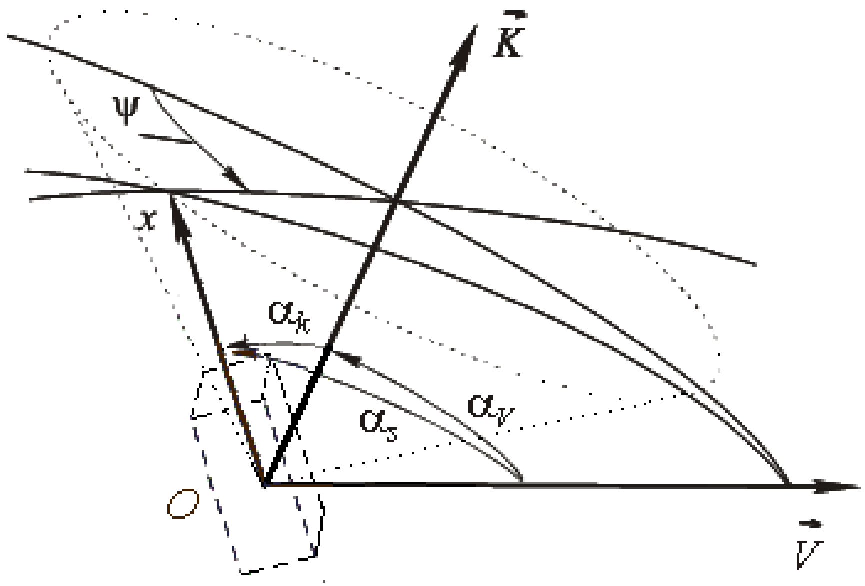


Figure 13.

Vector equation of motion of satellite around its center of mass under the influence of aerodynamic moment:

$$\frac{d\vec{K}_o}{dt} + \vec{\omega} \times \vec{K}_o = \vec{M}^{\,e}_{oA} \qquad (18)$$

Forced precession of nanosatellite

Trajectory of the end of the longitudinal axis of nanosatellite SamSat-QB50 on the unit sphere concerning the trajectory reference frame (flight altitude *H* = 330 km, ω_x = 0.2 deg/s, ω_y = 0, ω_z = 0.2 deg/s, time interval = 2650 s, statical stability factor = 0.06 m)

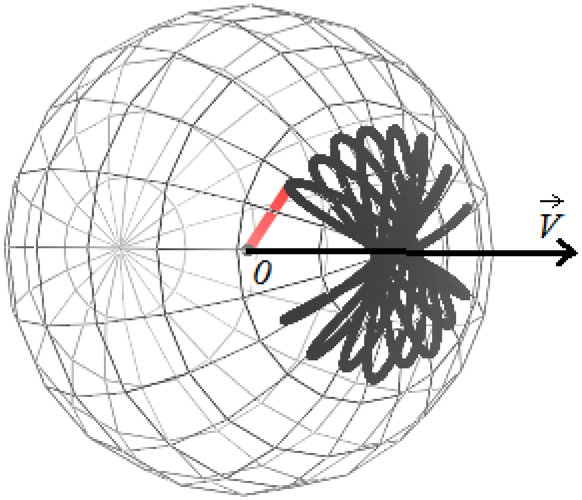
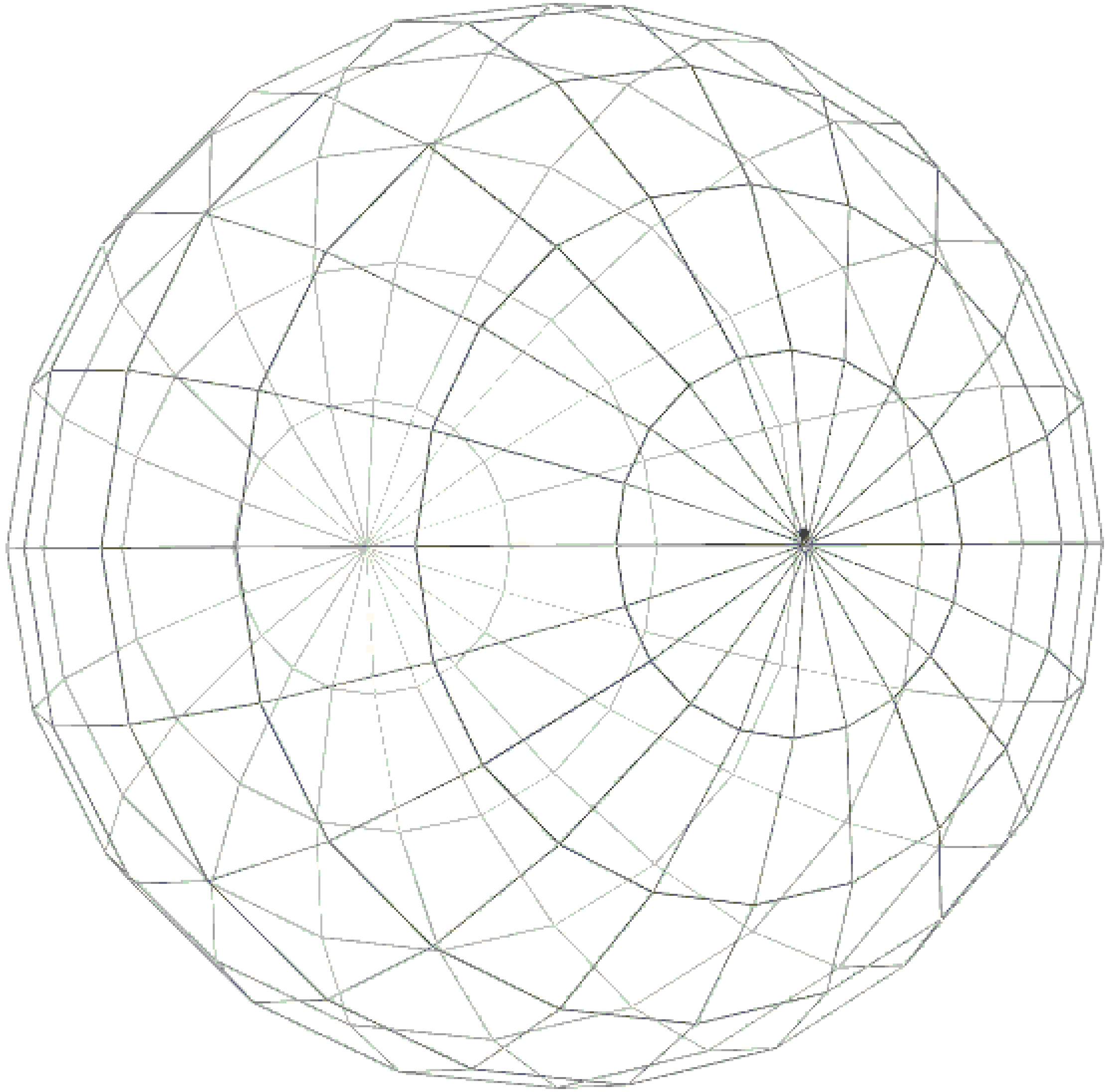


Figure 14.



Planar motion of nanosatellite around its center of mass under the influence of the gravitational and aerodynamic moments during descent from circular low-altitude orbits

Equations of motion

$$\ddot{\alpha} - a(H)\sin\alpha - c(H)\sin 2\alpha = 0,$$
$$\dot{h} = -\frac{2c_0\tilde{S}qS}{mg}V, \qquad (19)$$

where

α is the angle of attack; h is the flight altitude;

$c(h)\sin 2\alpha = \frac{M_g}{I}$ is the gravitational moment, normalized with respect to transversal moment of inertia I;

$c(h) = \frac{3(I - I_x)n^2}{2I}$; $n = \sqrt{k/R^3}$ is the orbital angular velocity of the satellite;

$a(h)\sin\alpha = \frac{M_a}{I}$ is the restoring aerodynamic moment, normalized with respect to transversal moment of inertia;

I, I_x are transversal and longitudinal moments of inertia of the satellite;

$k = 398600\ km^3/s^2$ is the standard gravitational parameter for the Earth;

$R=R_E+h$; $R_E = 6371000\ m$ is the Earth radius;

$q = \frac{\rho V^2}{2}$ is the velocity head; $g = g_0 \left(\frac{R_0}{R}\right)^2$ is the gravitational acceleration;

$g_0 = 9.820\ m/s^2$ is the gravitational acceleration on the Earth; ρ is the atmospheric density;

V is the flight velocity; m is the satellite mass;

S is the characteristic area; l is the characteristic dimension.

2-U CubeSat

$$m_\alpha(\alpha) = -2.2(|\cos(\alpha)| + \frac{8}{\pi}|\sin(\alpha)|)\frac{\Delta x}{l}\sin(\alpha)$$

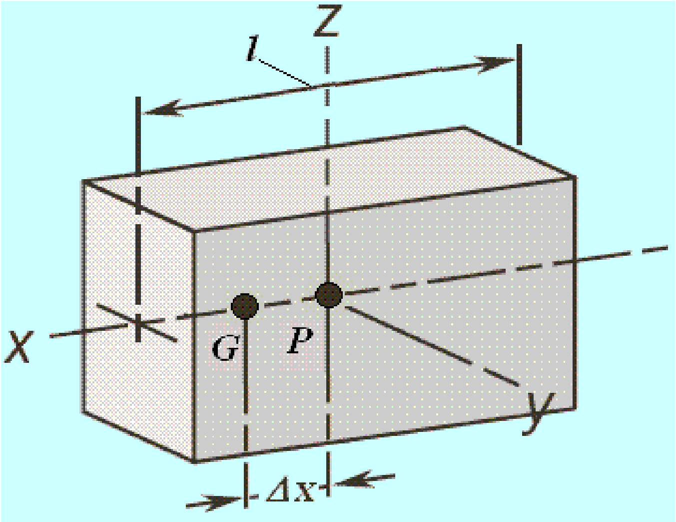
$m = 2$ kg,

$l = 0.2$ m,

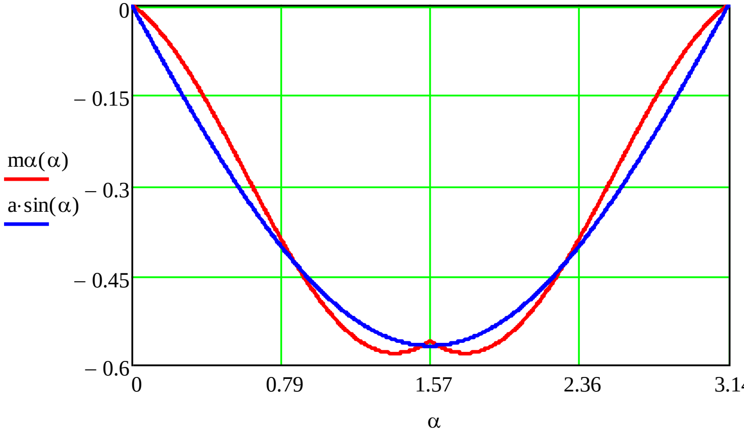
$S = 0.01$ m^2,

$I_x = 0.00333$ kg·m^2,

$I = 0.00833$ kg·m^2



$$m_\alpha(\alpha) = a_0 \sin(\alpha) \qquad (20)$$



Energy integral of system (19) for *h=const*

$$\frac{\dot{\alpha}^2}{2} + a\cos\alpha + c\cos^2\alpha = const = E_0 \qquad (21)$$

Phase portraits of the planar motion

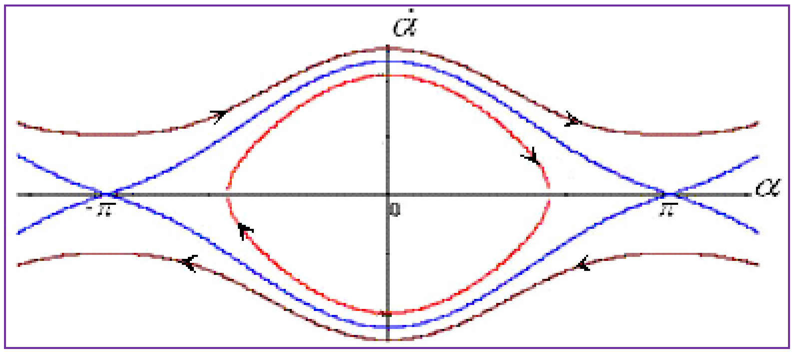


Figure 15. $|a| \geq 2|c|$, $a < 0$.

Rotational motion: $E_0 > -a+c$.

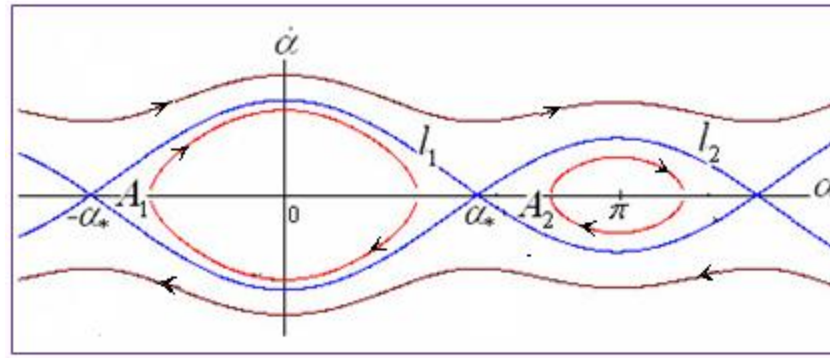
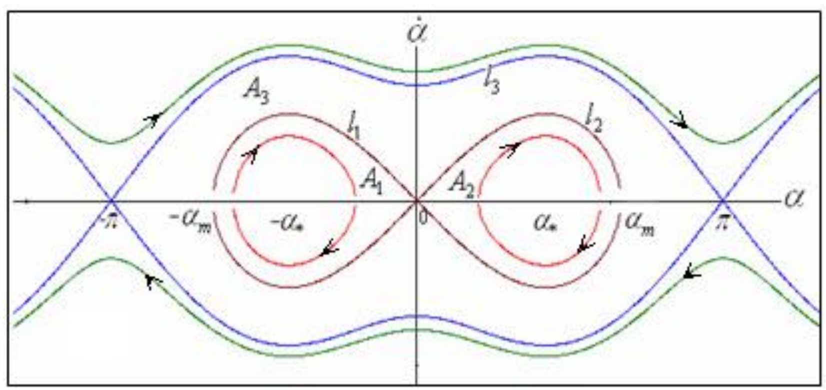


Figure 16. $|c| > 0.5|a|, c < 0.$

Rotational motion: $E_0 > -a^2/(4c)$.



$\alpha_* = \arccos(-0.5a/c)$.

Figure 17. $c > 0.5|a|,\ c > 0, a < 0.$ Rotational motion: $E_0 > -a+c$.

Oscillates with respect to the equilibrium position $\alpha=0$: $-a+c > E_0 > a+c$.

The results of numerical simulation

Initial condition of motion of 2-U CubeSat

flight altitude: 380 km, angle of attack: 8 deg, angular velocity: 0.4522 deg/s, statical stability factor: 0.02m

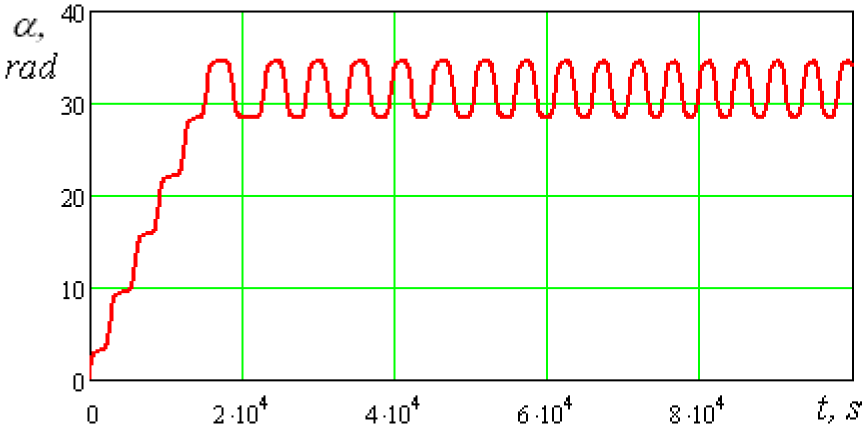


Figure 18. The change in the spatial angle of attack.

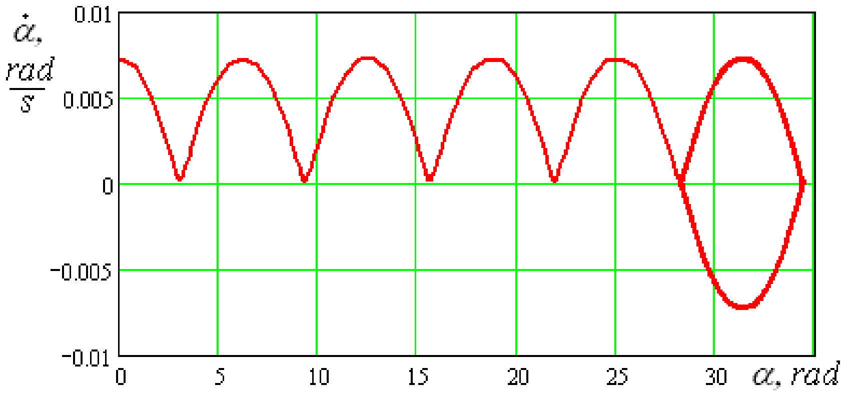
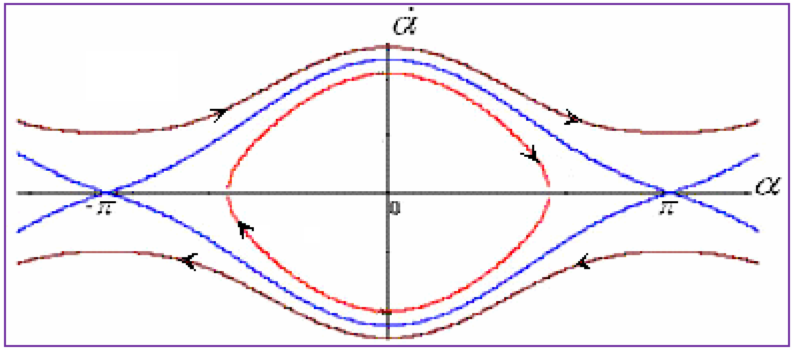


Figure 19. Dependence of the angular velocity on the angle of attack



$$|a| \geq 2|c| \text{ , } a < 0.$$

Initial conditions of motion

flight altitude:380 km, angle of attack:55 deg, angular velocity: 0.0346deg/s, statical stability factor:0.002m

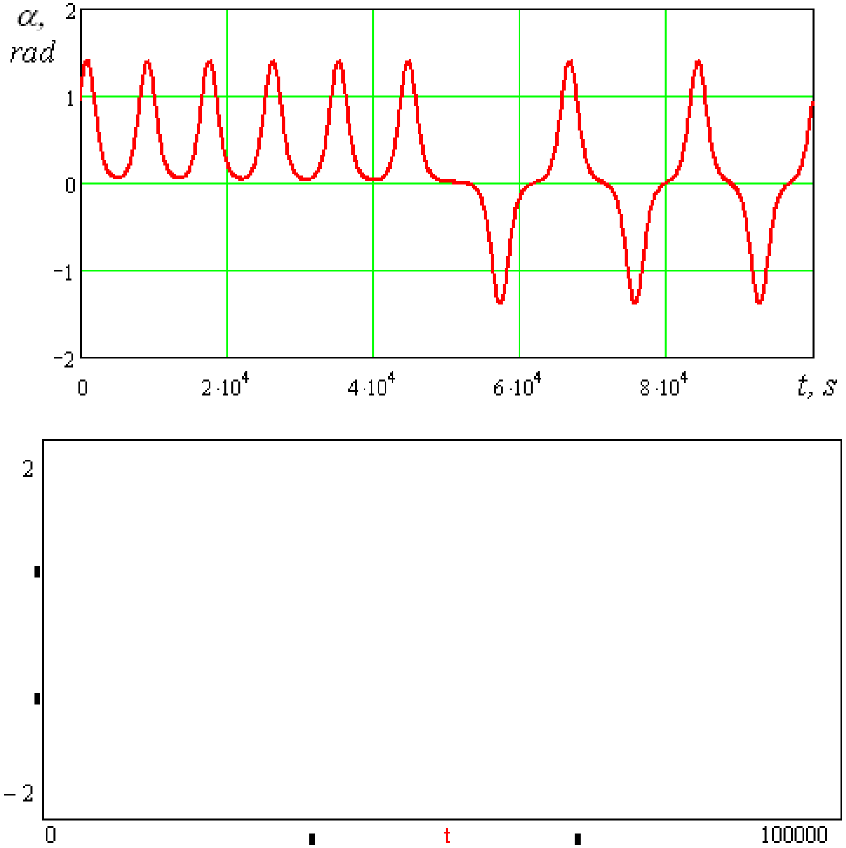


Figure 20. The change in the spatial angle of attack

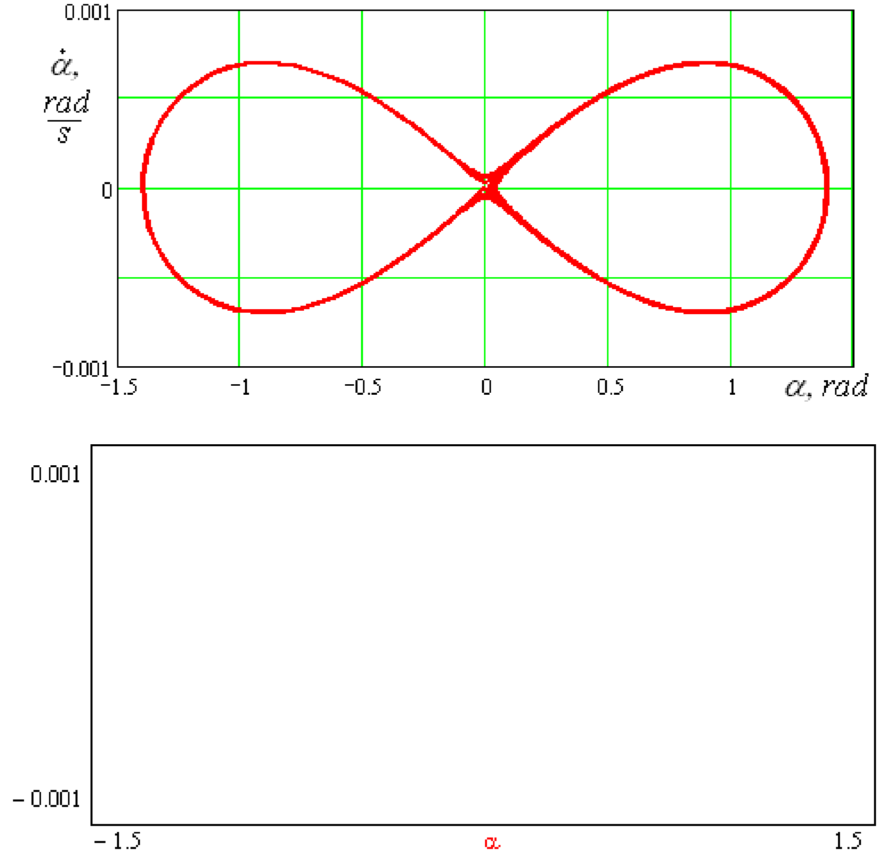
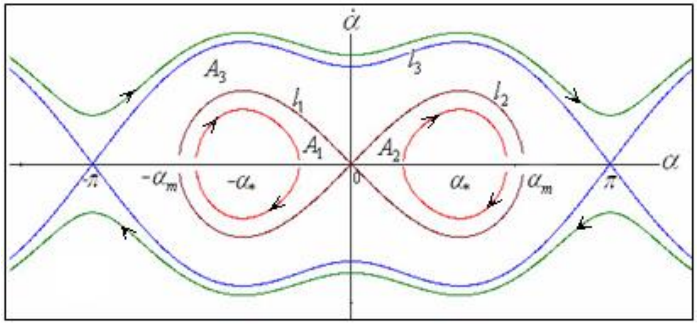


Figure 21. Dependence of the angular velocity on the angle of attack.

$c > 0.5|a|,\ c > 0,\ a < 0.$



$\alpha_* = \arccos(-0.5a/c).$

Passive Stabilization System

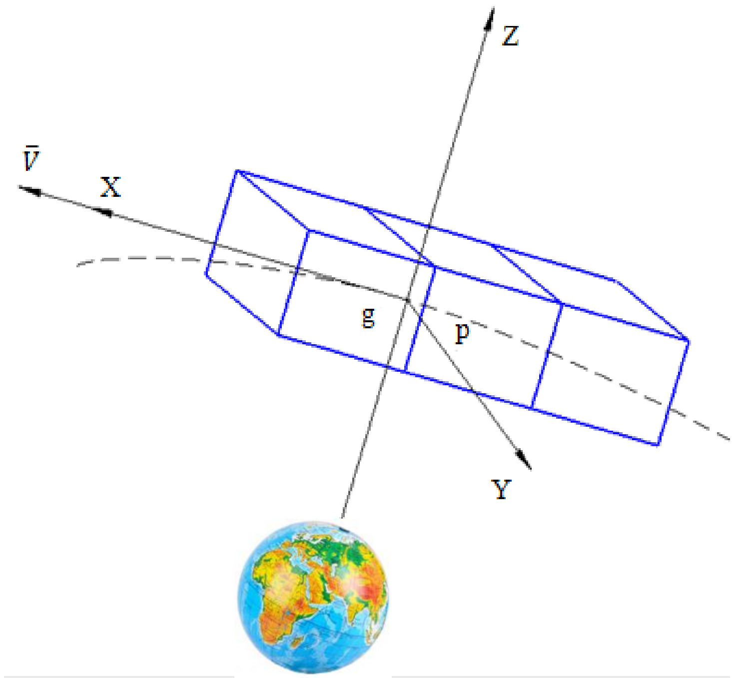


Figure 22. Aerodynamic stabilization

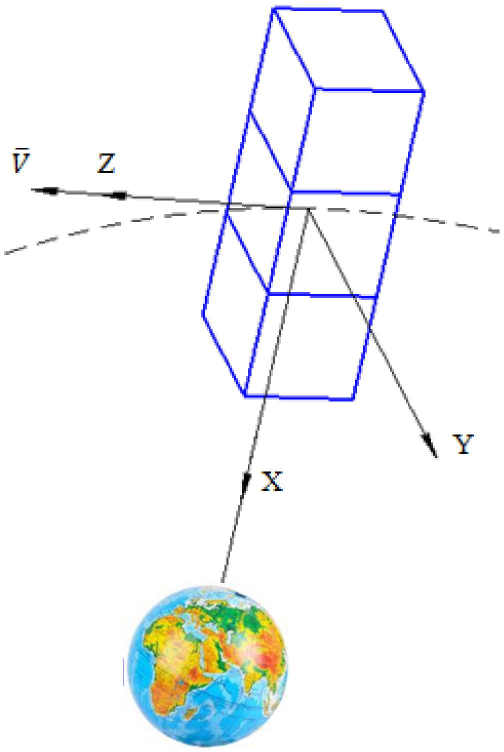


Figure 23. Gravitational stabilization

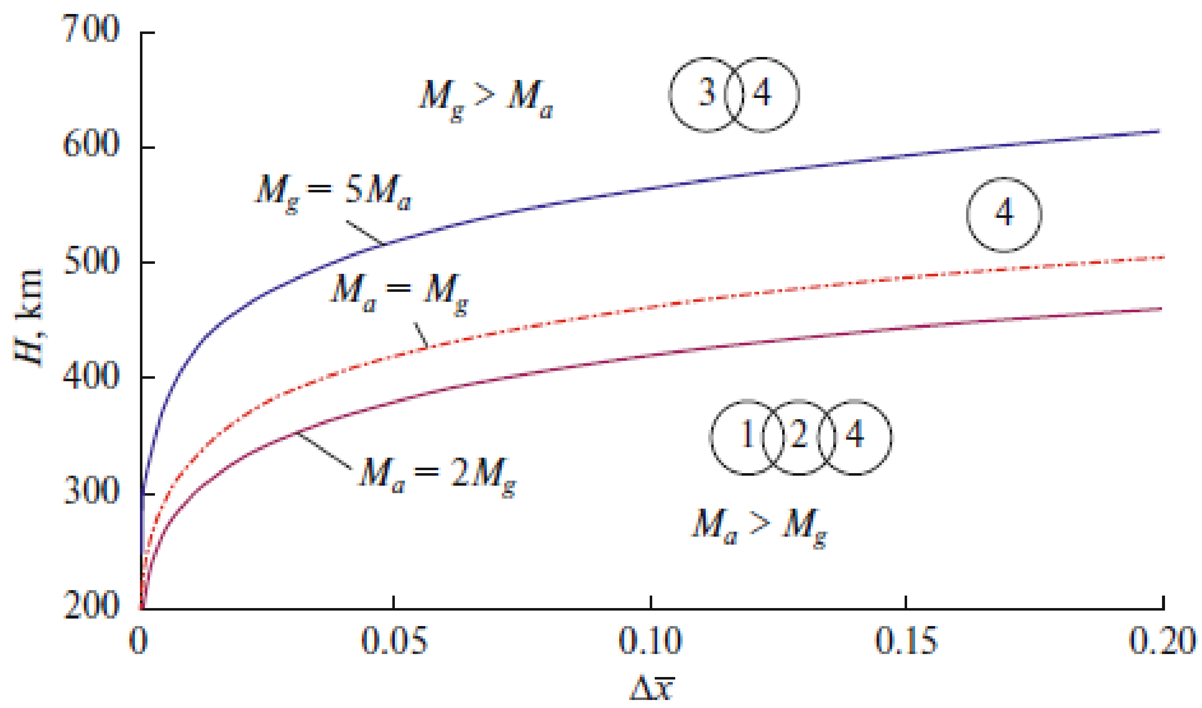


Figure 24. Regions of preferred application of passive stabilization types for CubeSat 3U nanosatellite depending on the altitudes *H* and relative static margin

1—single-axis aerodynamic stabilization system along the velocity vector (region where the aerodynamic moment is greater than the gravitational one *Ma* > *Mg);*

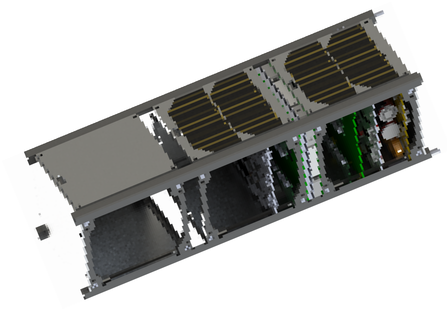
2—three-axis aerodynamic-gravitational stabilization system (region where *Ma* > *Mg*);

3—single-axis and three-axis gravitational stabilization systems (region where *Ma* < *Mg*);

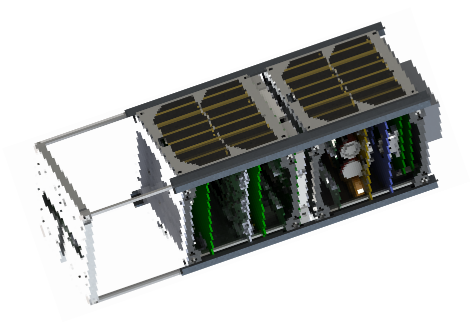
4—three-axis gravitational-aerodynamic stabilization system (regions with any proportion of aerodynamic and gravitational moments).

The selection of the design parameters of the aerodynamically stabilized nanosatellite of the CubeSat standard

SamSat-218Д



SamSat-QB50



Maximum value of the angle of attack is determined by the equation:

$$\cos\alpha_{max} = -\frac{a}{2c} - \sqrt{\left(\frac{a}{2c}\right)^2 + \frac{a}{c}\cos\alpha_0 + \cos^2\alpha_0 + \frac{\dot{\alpha}_0^2}{2c}} \quad . \qquad (22)$$

Cumulative distribution function of the maximum angle of attack

If the value of the initial transverse angular velocity $\dot{\alpha}_0$ is distributed according to the Rayleigh law: $f(\dot{\alpha}_0) = \frac{\dot{\alpha}_0}{\sigma^2} \exp\left(-\frac{\dot{\alpha}_0^2}{2\sigma^2}\right)$

The cumulative distribution function:

$$F(\alpha_{\max}) = 1 - \exp\left(\frac{-a(\cos\alpha_{\max} - \cos\alpha_0) - c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}{\sigma^2}\right) \qquad (23)$$

If the value $\dot{\alpha}_0$ is distributed according to the uniform law: $f(\dot{\alpha}_0) = \begin{cases} \frac{1}{\dot{\alpha}_{0\max}}, & \dot{\alpha}_0 \in [0, \dot{\alpha}_{0\max}] \\ 0, & \dot{\alpha}_0 \notin [0, \dot{\alpha}_{0\max}] \end{cases}$

The cumulative distribution function :

$$F(\alpha_{\max}) = \frac{\sqrt{2a(\cos\alpha_{\max} - \cos\alpha_0) + 2c(\cos^2\alpha_{\max} - \cos^2\alpha_0)}}{\dot{\alpha}_{0\max}} \qquad (24)$$

where a is coefficient associated with aerodynamic restoring moment;

c is coefficient associated with the gravitational moment;

Formulas for the selection of design parameters of aerodynamically stabilized nanosatellite standard CubeSat

If the value $\dot{\alpha}_0$ is distributed according to the Rayleigh law:

$$d = \frac{\Delta x}{I_n} lb \geq d_r = \frac{\pi\sigma^2 \ln(1 - p^*)}{4c_0\left(\cos\alpha^*_{\max} - \cos\alpha_0\right)q(H)} \qquad (25)$$

If the value $\dot{\alpha}_0$ is distributed according to the uniform law:

$$d = \frac{\Delta x}{I_n} lb \geq d_r = \frac{\pi\left(\dot{\alpha}_{0\max} p^*\right)^2}{8c_0\left(\cos\alpha_0 - \cos\alpha^*_{\max}\right)q(H)} \qquad (26)$$

where Δx is the static stability factor (the distance measured from the center of mass to the nanosatellite (NS) geometric center), l is the NS length, b is the NS width, α_0 is the initial value of spatial angle of attack (the angle between the longitudinal axis and velocity vector), $I_n = I_y = I_z$ is the inertia transverse moment, $q(H) = V^2\rho(H)/2$ is the velocity head, V is the flight speed, H is the orbit altitude, $\rho(H)$ is the atmospheric density, c_0 = 2.2 is the drag force coefficient.

Selection of design parameters of aerodynamically stabilized nanosatellite CubeSat 3U

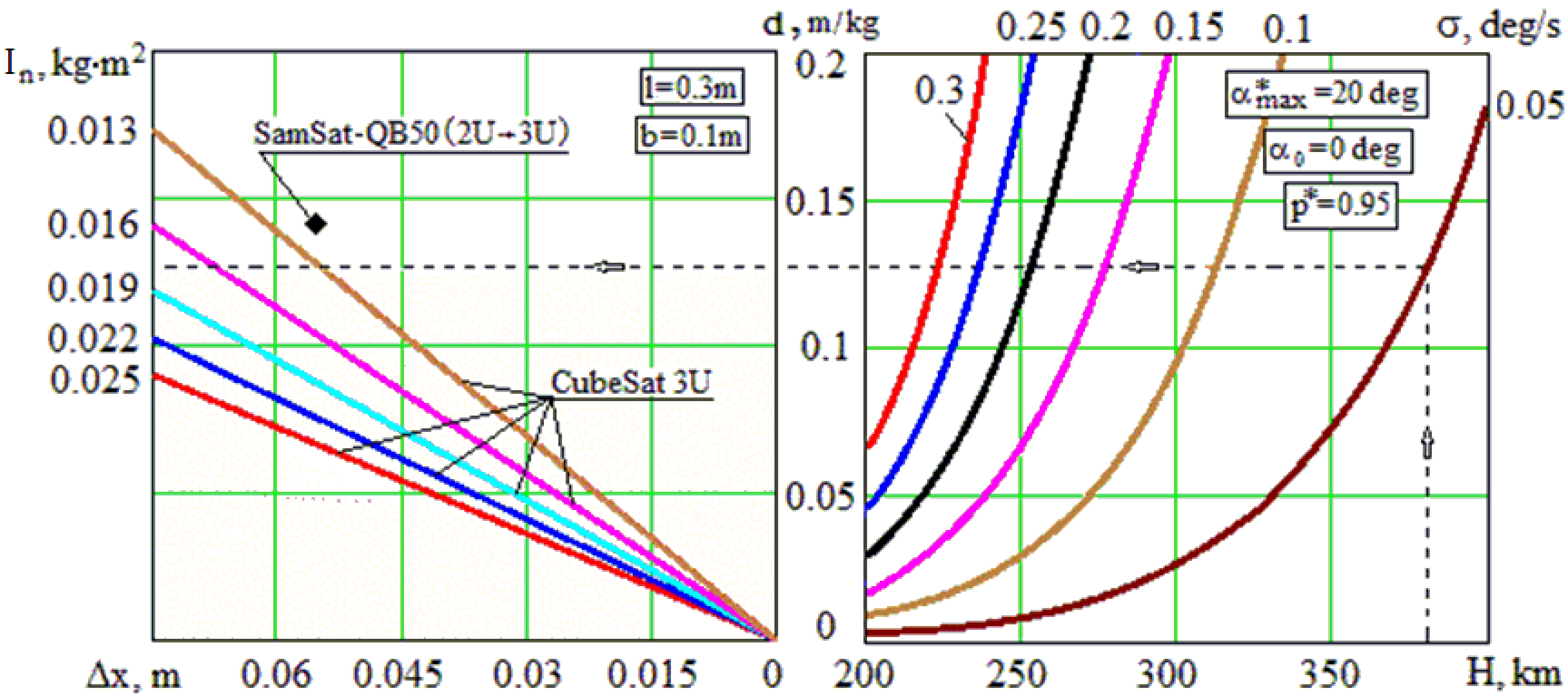


Figure 25. Nomogram to select structural parameter of nanosatellite depending on the altitude *H* and the parameter values σ at $\alpha^*_{max} = 20$ deg, $p^* = 0.95$, $\alpha_0 = 0$.

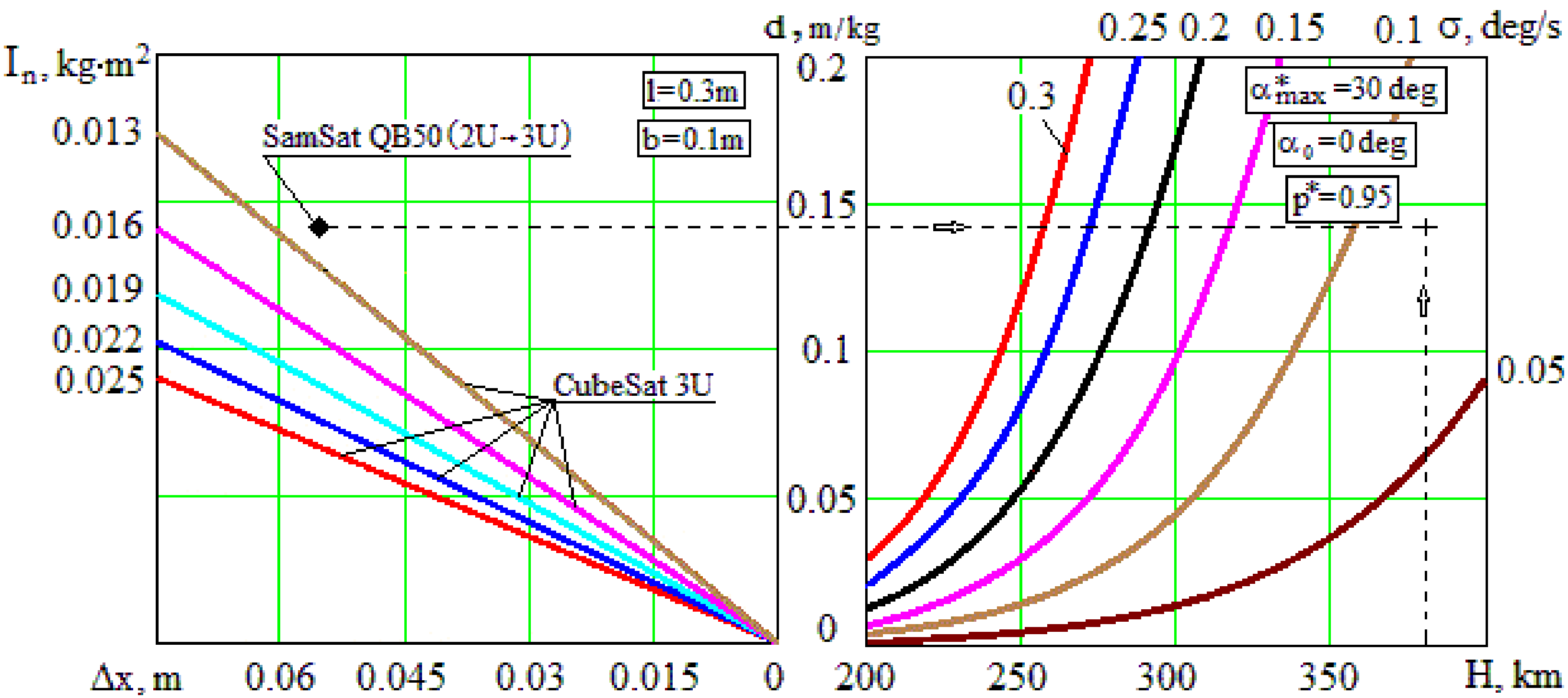


Figure 25. Nomogram to select structural parameter of nanosatellite depending on the altitude *H* and the parameter values σ at $\alpha^*_{\max} = 30$ deg , $p^* = 0.95$, $\alpha_0 = 0$.

The selection of the design parameters for aerodynamic-gravitational three-axis stabilization

Approximate model of the angular motion of a nanosatellite with respect to the angle of roll

$$\ddot{\delta} - \frac{2\mu}{(R_E + H)^3}\left(\frac{I_z - I_y}{I_x}\right) sin\, 2\,\delta = 0 \qquad (27)$$

where δ is angle of the transverse axis deviation from the flight plane

$\omega_{x0} = \dot{\delta}_0$ (initial longitudinal angular velocity) has a largest spread

Laws of distribution of the maximum angle of roll

- If the value ω_{x0} have Gaussian distribution with zero mathematical expectation and with standard deviation σ

$$F(\delta_{max}) = 2\Phi_0\left(\frac{\sqrt{2\frac{\mu}{(R_E + H)^3}\left(\frac{I_z - I_y}{I_x}\right)(cos\, 2\,\delta_{max} - cos\, 2\,\delta_0)}}{\sigma}\right) \qquad (28)$$

Here $\Phi_0(w) = \frac{1}{\sqrt{2\pi}}\int_0^w e^{-w^2/2}\, dw$ is Laplace function

- If the modulus of ω_{x0} is distributed according to the uniform law in the interval $[0,\ \omega_{x0max}]$

$$F(\delta_{max}) = \frac{\sqrt{2\frac{\mu}{(R_E + H)^3}\left(\frac{I_z - I_y}{I_x}\right)(cos\, 2\,\delta_{max} - cos\, 2\,\delta_0)}}{\omega_{x0max}} \qquad (29)$$

The selection of the design parameters for aerodynamic-gravitational three-axis stabilization

- If the value ω_{x0} have <u>Gaussian distribution</u> with zero mathematical expectation and with standard deviation σ

$$d_k = \frac{I_y - I_z}{I_x} \geq \frac{(R_E + H)^3}{2\mu} \frac{\sigma^2 (w^*)^2}{(cos\, 2\, \delta_0 - cos\, 2\, \delta^*_{max})} \qquad (30)$$

Here w^* is Laplace function argument with the given probability $\Phi_0(w^*) = p^*/2$

- If the modulus of ω_{x0} is distributed according to the <u>uniform law</u> in the interval $[0,\ \omega_{x0max}]$

$$d_k = \frac{I_y - I_z}{I_x} \geq \frac{(R_E + H)^3}{2\mu} \frac{(\omega_{x0max} p^*)^2}{(cos\, 2\, \delta_0 - cos\, 2\, \delta^*_{max})} \qquad (31)$$

The selection of the design parameters for aerodynamic-gravitational three-axis stabilization

$\overline{I_x} = \frac{I_x}{I_y}$, $\overline{I_z} = \frac{I_z}{I_y}$ are ratios of the moments of inertia

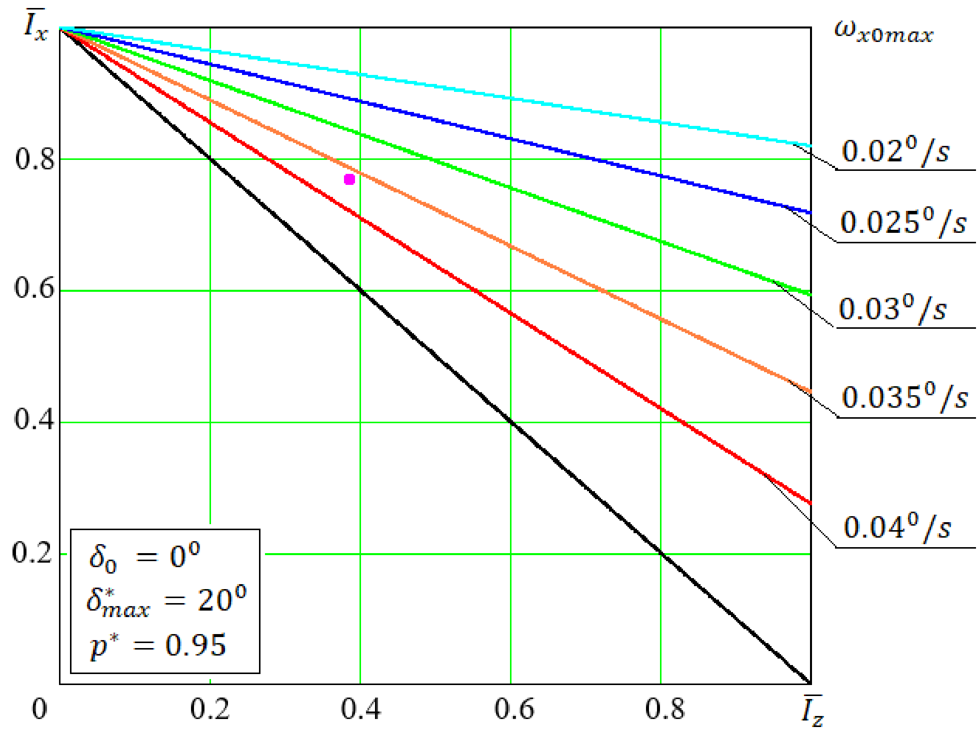


Figure 26. Example of a nomogram for choosing the ratios of the moments of inertia for various values of the modulus of the initial longitudinal angular velocity ω_{x0max}

The selection of the design parameters for gravitational three-axis stabilization

$\beta = \alpha - \pi/2$ is the angle of deviation from the local vertical in the plane of the angle of attack

Requirement for the design parameters

- In case the initial angular velocity ω_{n0} has Rayleigh distribution with parameter σ
- In case the initial angular velocity ω_{n0} is distributed according to the uniform law within the interval $[0, \ \omega_{n0max}]$

when considering motion with regard to the angle β

$$\frac{I_z - I_x}{I_y} \geq \frac{-2(R_E + H)^3}{3\mu} \frac{\sigma^2 \, ln(1 - p^*)}{(sin^2 \beta^*_{max} - sin^2 \beta_0)} \qquad (32)$$

$$\frac{I_z - I_x}{I_y} \geq \frac{(R_E + H)^3}{3\mu} \frac{(\omega_{n0max} p^*)^2}{(sin^2 \beta^*_{max} - sin^2 \beta_0)} \qquad (35)$$

when considering motion with regard to the angle ψ

$$\frac{I_y - I_x}{I_z} \geq \frac{-(R_E + H)^3}{2\mu} \frac{\sigma^2 \, ln(1 - p^*)}{(sin^2 \psi^*_{max} - sin^2 \psi_0)} \qquad (33)$$

$$\frac{I_y - I_x}{I_z} \geq \frac{(R_E + H)^3}{4\mu} \frac{(\omega_{n0max} p^*)^2}{(sin^2 \psi^*_{max} - sin^2 \psi_0)} \qquad (36)$$

when considering motion with regard to the angle φ

- If the value ω_{x0} has Gaussian distribution with zero mathematical expectation and with standard deviation σ
- If the modulus of ω_{x0} is distributed according to the uniform law in the interval $[0, \ \omega_{x0max}]$

$$\frac{I_y - I_z}{I_x} \geq \frac{(R_E + H)^3}{\mu} \frac{\sigma^2 (t^*)^2}{(cos^2 \varphi_0 - cos^2 \varphi^*_{max})} \qquad (34)$$

$$\frac{I_y - I_z}{I_x} \geq \frac{(R_E + H)^3}{\mu} \frac{(\omega_{x0max} p^*)^2}{(sin^2 \varphi^*_{max} - sin^2 \varphi_0)} \qquad (37)$$

The selection of the design parameters for gravitational three-axis stabilization

$\overline{I_x} = \frac{I_x}{I_y}$, $\overline{I_z} = \frac{I_z}{I_y}$ are ratios of the moments of inertia

$p^* = 0.95,\ H = 500\ km$

$\beta^*_{max} = \psi^*_{max} = \varphi^*_{max} = 20^0$

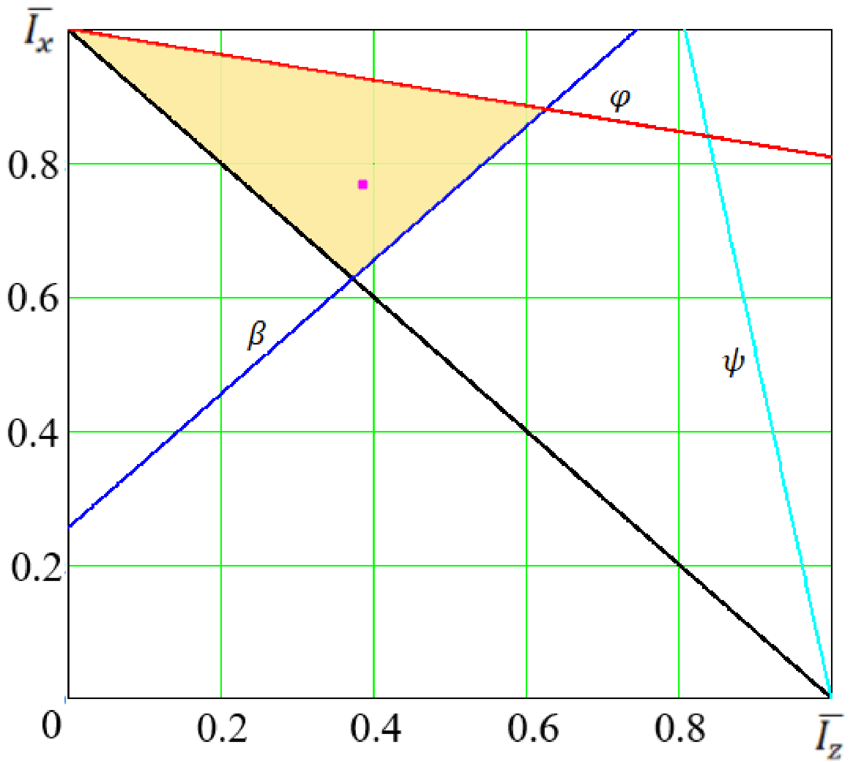


Figure 27. Restrictions on the ratio of the moments of inertia to ensure gravitational three-axis stabilization at $\omega_{n0max} = 0.02^0/s,\ \ \omega_{x0max} = 0.01^0/s$

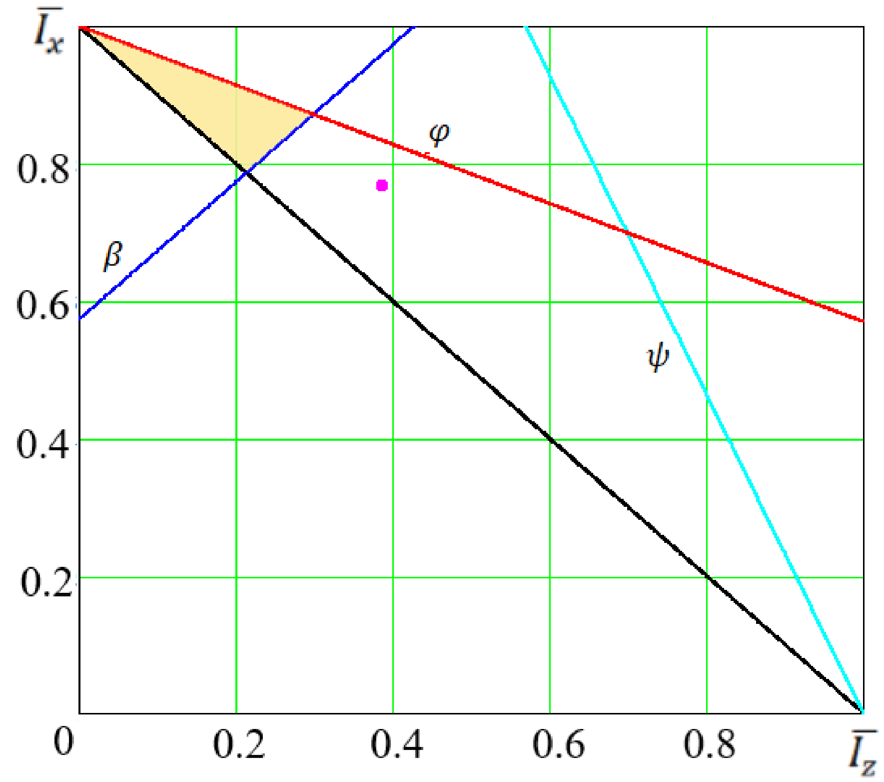
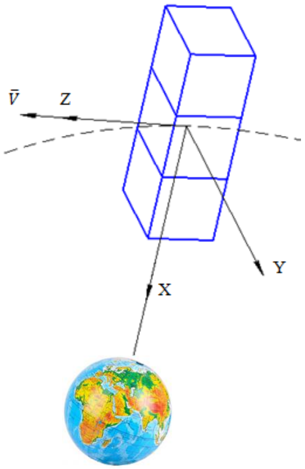


Figure 27. Restrictions on the ratio of the moments of inertia to ensure gravitational three-axis stabilization at $\omega_{n0max} = 0.03^0/s,\ \ \omega_{x0max} = 0.015^0/s$

The selection of the design parameters for gravitational-aerodynamic three-axis stabilization



The technique for choosing design parameters of a CubeSat 3U

1. The values Δx, Δy must be kept as small as possible.
2. Select design parameters that satisfy the given restrictions on the angle of precession ψ using formulas (33) or (36) depending on the selected distribution law
3. Choose the static stability margin along the z-axis for the chosen moments of inertia ratios, while the restriction on the value of the initial transverse angular velocity is taken the same as when considering the motion along the angle of precession ψ

- In case the initial angular rate ω_{n0} has <u>Rayleigh distribution</u> with parameter σ

$$\Delta z \geq I_y[ln(1-p^*)\sigma^2 + c\,(cos^2\alpha^*_{max} - cos^2\alpha_0) + a_x(u(\alpha^*_{max}) - u(\alpha_0))]/[c_0 S_x q(H)(\nu(\alpha^*_{max}) - \nu(\alpha_0))] \tag{38}$$

- In case the initial angular rate ω_{n0} is distributed according to the <u>uniform law within</u> the interval $[0,\ \omega_{n0max}]$

$$\Delta z \geq I_y[(\omega_{n0max}p^*)^2 - 2c(cos^2\alpha^*_{max} - cos^2\alpha_0) - 2a_x(u(\alpha^*_{max}) - u(\alpha_0))]/\left[-2c_0 S_x q(H)\left(\nu(\alpha^*_{max}) - \nu(\alpha_0)\right)\right] \tag{39}$$

Here $u(\alpha) = \frac{1}{2}sign(\cos\alpha)\cos^2\alpha + \frac{k}{2}sign(\sin\alpha)\left(\frac{\sin 2\alpha}{2} - \alpha + 2\pi\left\lfloor\frac{\alpha+\pi}{2}\right\rfloor\right)$

$$\nu(\alpha) = \frac{1}{2}sign(cos(\alpha))\left(\frac{sin\,2\alpha}{2} + \alpha - \frac{\pi}{2} - 2\pi\cdot\left\lfloor\frac{\alpha+\frac{\pi}{2}}{2\pi}\right\rfloor\right) + \frac{k}{2}sign(sin(\alpha))\,sin^2\alpha$$

The selection of the design parameters for gravitational-aerodynamic three-axis stabilization

4. Determine the limitation on the value of the initial nanosatellite longitudinal angular velocity ω_{x0}

- If the value ω_{x0} have Gaussian distribution with zero mathematical expectation and with standard deviation σ

$$\sigma \le \frac{\sqrt{\frac{-2\Delta z c_0 l_x l_y q(H)}{I_x}(cos\varphi_{max} - cos\varphi_0) + \frac{\mu}{(R_E + H)^3}\left(\frac{I_z - I_y}{I_x}\right)\left(cos^2\varphi_{max} - cos^2\varphi_0\right)}}{w^*} \quad (40)$$

- If the modulus of ω_{x0} is distributed according to the uniform law in the interval $[0,\ \omega_{x0max}]$

$$\omega_{x0max} \le \frac{\sqrt{\frac{-2\Delta z c_0 l_x l_y q(H)}{I_x}(cos\varphi_{max} - cos\varphi_0) + \frac{\mu}{(R_E + H)^3}\left(\frac{I_z - I_y}{I_x}\right)\left(cos^2\varphi_{max} - cos^2\varphi_0\right)}}{p^*} \quad (41)$$

Thank you for your attention!