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SAMARA UNIVERSITY

Methods and Algorithms for Nanosatellite Attitude Determination

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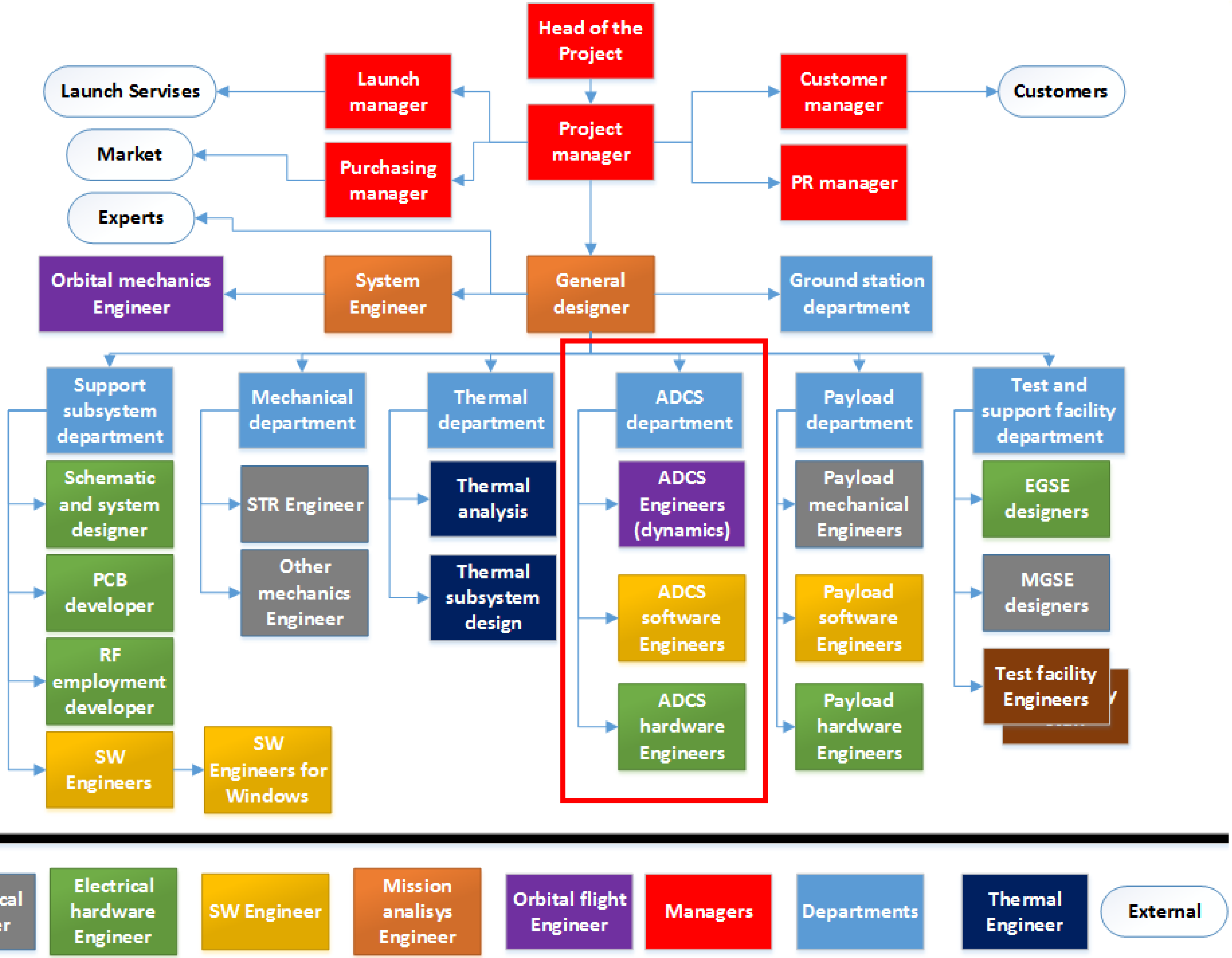
Samara 2023

Nanosatellite Development



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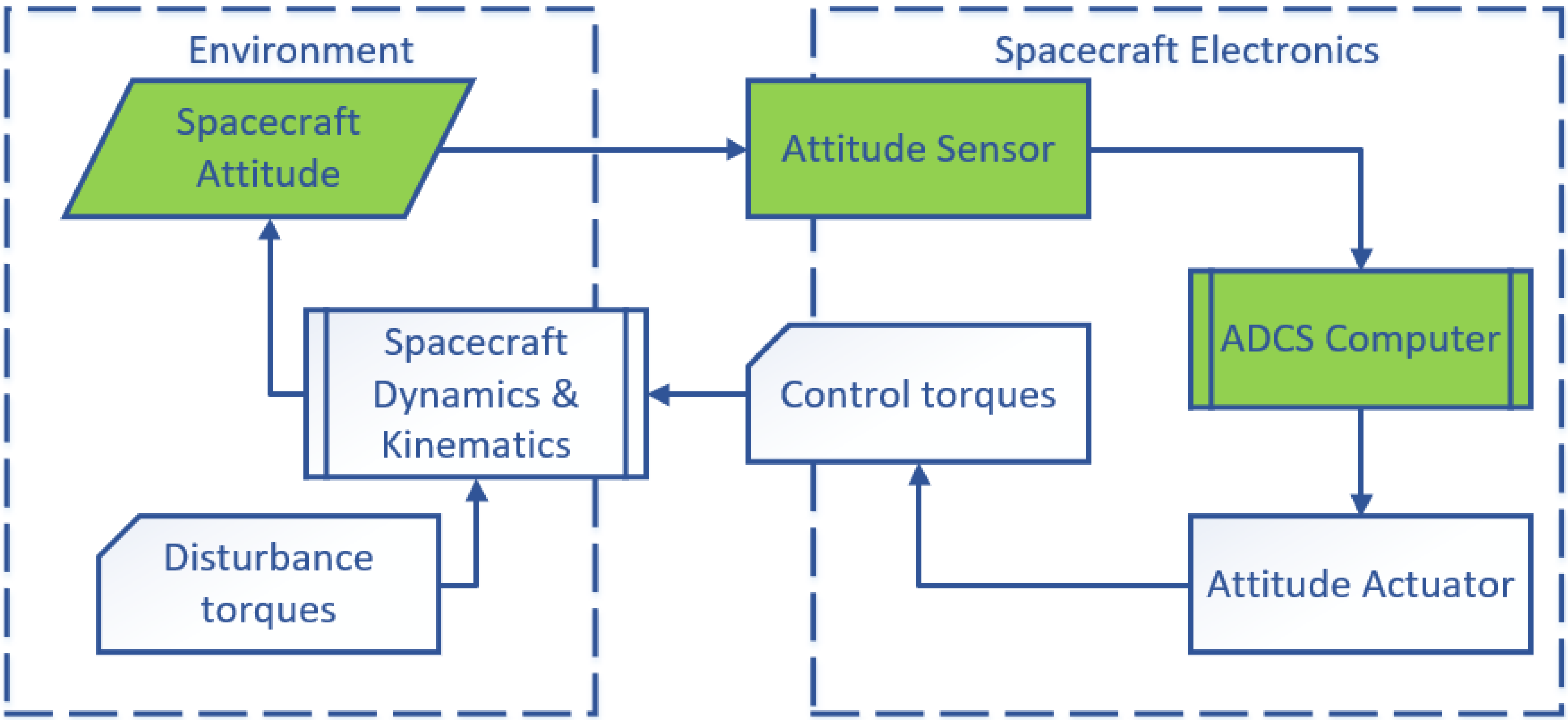

ADCS closed-loop control system

The main frames of reference:

- the **body frame** of reference (BFR)
- the **orbital frame** of reference (OFR);
- the **geocentric frame** of reference (GFR).

Attitude matrix:

$$M_{X_1X_2} = \begin{cases} f_1(\vartheta, \psi, \varphi), \\ f_2(q_0, q_1, q_2, q_3), \\ f_3(m_{ij}, \quad i,j = \overline{1,3}). \end{cases}$$

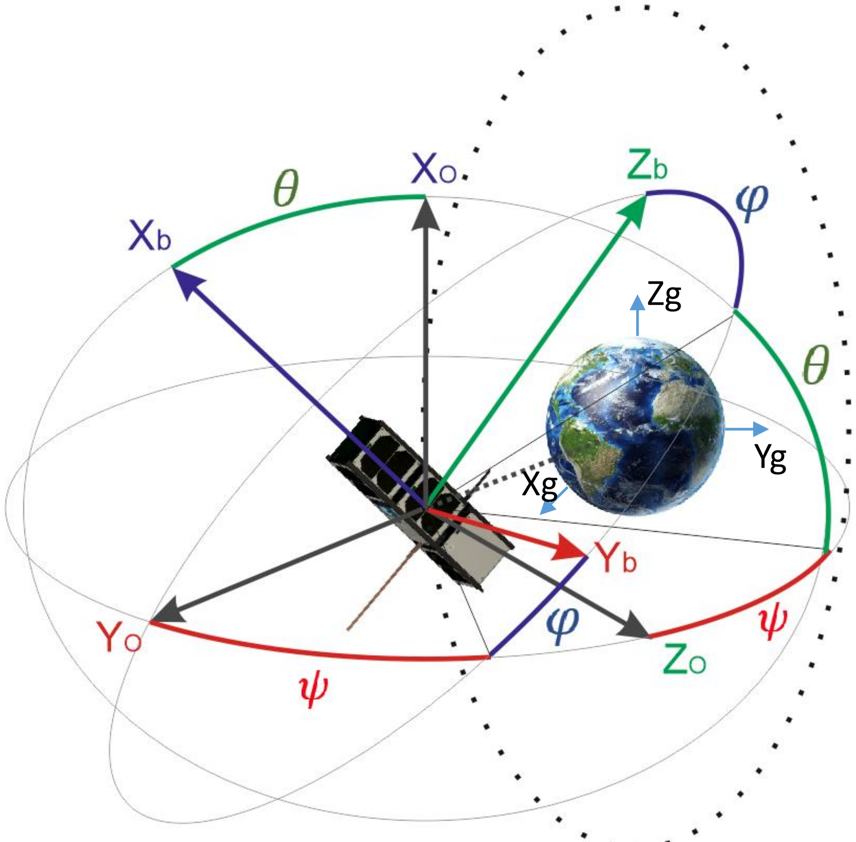

Fig. 1.1 – The frames of reference

$$\mathbf{M}_{X_1X_2} = [m_{ij}]_{i,j=\overline{1,3}} = \begin{bmatrix} \cos\vartheta\cdot\cos\psi & \cos\vartheta\cdot\sin\psi & -\sin\vartheta \\ \sin\varphi\cdot\sin\vartheta\cdot\cos\psi - \cos\varphi\cdot\sin\psi & \sin\varphi\cdot\sin\vartheta\cdot\sin\psi + \cos\varphi\cdot\cos\psi & \sin\varphi\cos\vartheta \\ \cos\varphi\cdot\sin\vartheta\cdot\cos\psi + \sin\varphi\cdot\sin\psi & \cos\varphi\cdot\sin\vartheta\cdot\sin\psi - \sin\varphi\cdot\cos\psi & \cos\varphi\cos\vartheta \end{bmatrix}$$

Representation of Attitude

Representation	Par.	Characteristic	Application
Rotation matrix	9	• Inherently nonsingular • Intuitive representation • Difficult to mantain ortogonality • Expensive to store • Six redundant parameter	Analytical studies and transformation of vectors.
Euler angles	3	• Minimal set • Clear physical interpretation • Trigometric functions in rotation matrix • No simple composition rule • Singular for certain rotations • Trigonometric functions in kinematic relation	Theoretical physics, spinning spacecraft and attitude maneuvers. Used in analytical studies.
Axis-azimuth	3	• Minimal set • Clear physical interpretation • Often computed directly from observations • No simple composition rule • Computation of rotating matrix very difficult • Singular for certain rotation • Trigonometric functions in kinematic relation	Primarily spinning spacecraft.
Rodriguez (Gibbs)	3	• Minimal set • Clear physical interpretation • Singular for rotations near $\theta = \pm\pi$ • Simple kinematic relations	Often interpreted as incremental rotation vector.
Quaternions	4	• Easy orthogonality of rotation matrix • Bilinear composition rule • Not singular at any rotation matrix • Linear kinematic equations • No clear physical interpretation • One redundant parameter • Simple kinematic relation	Widely used in smulations and data processing. Preferred attitude representation for attitude control systems.

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Rotation matrix depending on the Euler angles

$$A_{yzy} = \begin{bmatrix} \cos\varphi\cos\alpha\cos\psi - \sin\varphi\sin\psi & \cos\varphi\sin\alpha & -\cos\varphi\cos\alpha\sin\psi - \sin\varphi\cos\psi \\ -\sin\alpha\cos\psi & \cos\alpha & \sin\alpha\sin\psi \\ \sin\varphi\cos\alpha\cos\psi + \cos\varphi\sin\psi & \sin\varphi\sin\alpha & -\sin\varphi\cos\alpha\sin\psi + \cos\varphi\cos\psi \end{bmatrix}$$

Rotation matrix depending on the quaternion

$$A = \begin{bmatrix} 1 - 2(q_2^2 + q_3^2) & 2(q_1 q_2 - q_3 q_0) & 2(q_1 q_3 + q_2 q_0) \\ 2(q_1 q_2 + q_3 q_0) & 1 - 2(q_1^2 + q_3^2) & 2(q_2 q_3 - q_1 q_0) \\ 2(q_1 q_3 - q_2 q_0) & 2(q_2 q_3 + q_1 q_0) & 1 - 2(q_1^2 + q_2^2) \end{bmatrix}$$

Quaternion depending on the Euler angles

$$q_0 = \cos\frac{\alpha}{2}\cos\frac{\psi + \varphi}{2}\ ;\ q_1 = \sin\frac{\alpha}{2}\sin\frac{\psi - \varphi}{2}\ ;\ q_3 = \cos\frac{\alpha}{2}\sin\frac{\psi + \varphi}{2}\ ;\ q_4 = \sin\frac{\alpha}{2}\cos\frac{\psi - \varphi}{2}$$

Two main categories of attitude sensors

Reference Sensors	Inertial Sensors
Sun Sensor	Gyroscope
Star Tracker	Accelerometer
Magnetometer	

1

2

3

4

1. NanoSSOC-D60 Digital Sun Sensor
2. MAI-SS Space Sextant
3. HMR2300R-485 3-AXIS Magnetometer
4. DSP-1750 Optical Sensor (gyro)

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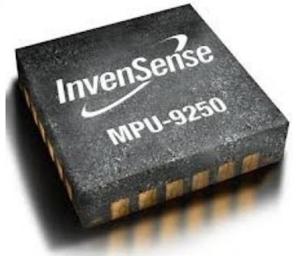
MPU-9250 microchip



Location of MPU-9250 sensors
on OBC of **SamSat** platform

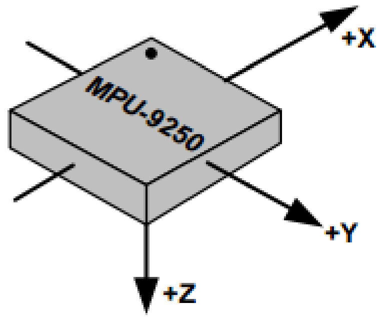
Gyroscope

range of measuring ±250 °/s
sensitivity scale factor 131 LSB/(º/s)
digitally-programmable low-pass filter
total RMS Noise 0.1 º/s-rms
rate noise spectral density 0.01 º/s/√Hz
zero shift of the gyroscope measurements has a nonlinear temperature characteristic.

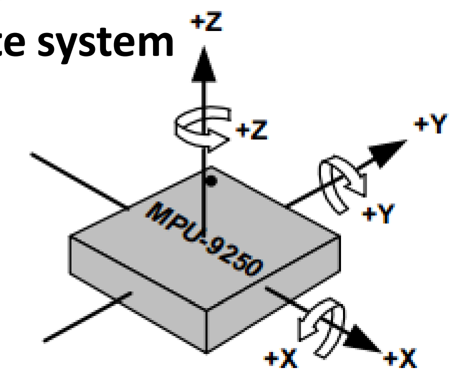
Magnetometer

range of measuring ± 4800μT
sensitivity scale factor 0.6μT/LSB
zero shift of the gyroscope measurements has a linear temperature characteristic.

MPU-9250 coordinate system



Magnetometer



Accelerometer & Gyro

* https://www.invensense.com/wp-content/uploads/2015/02/PS-MPU-9250A-01-v1.1.pdf

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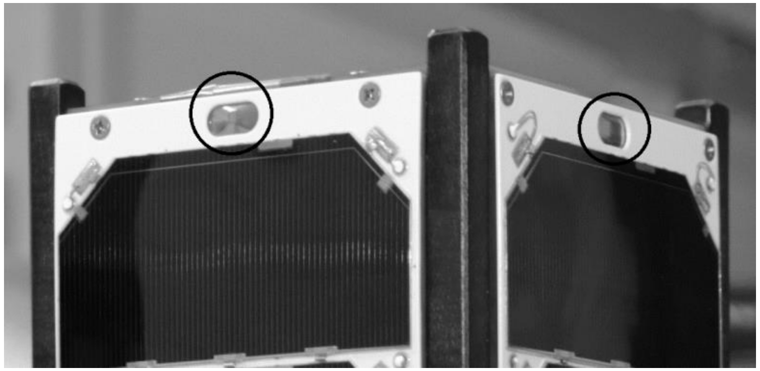
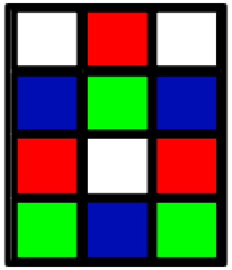
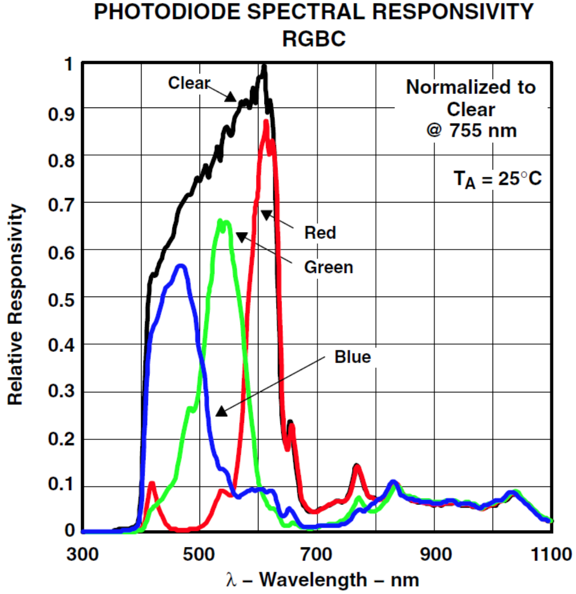


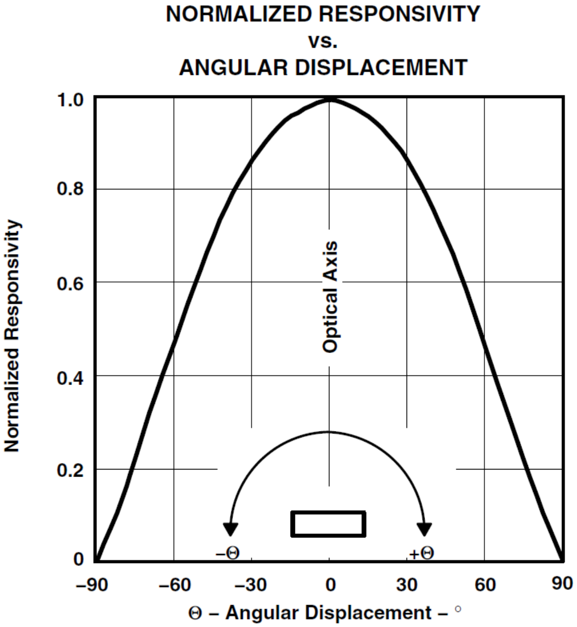
Location of light sensors on SamSat platform

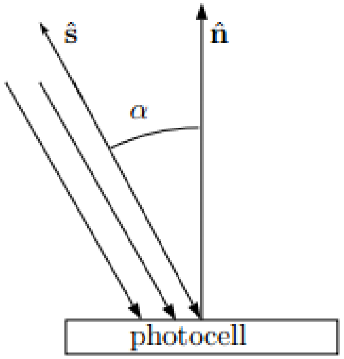
TCS34725 Color (Sun) Sensor



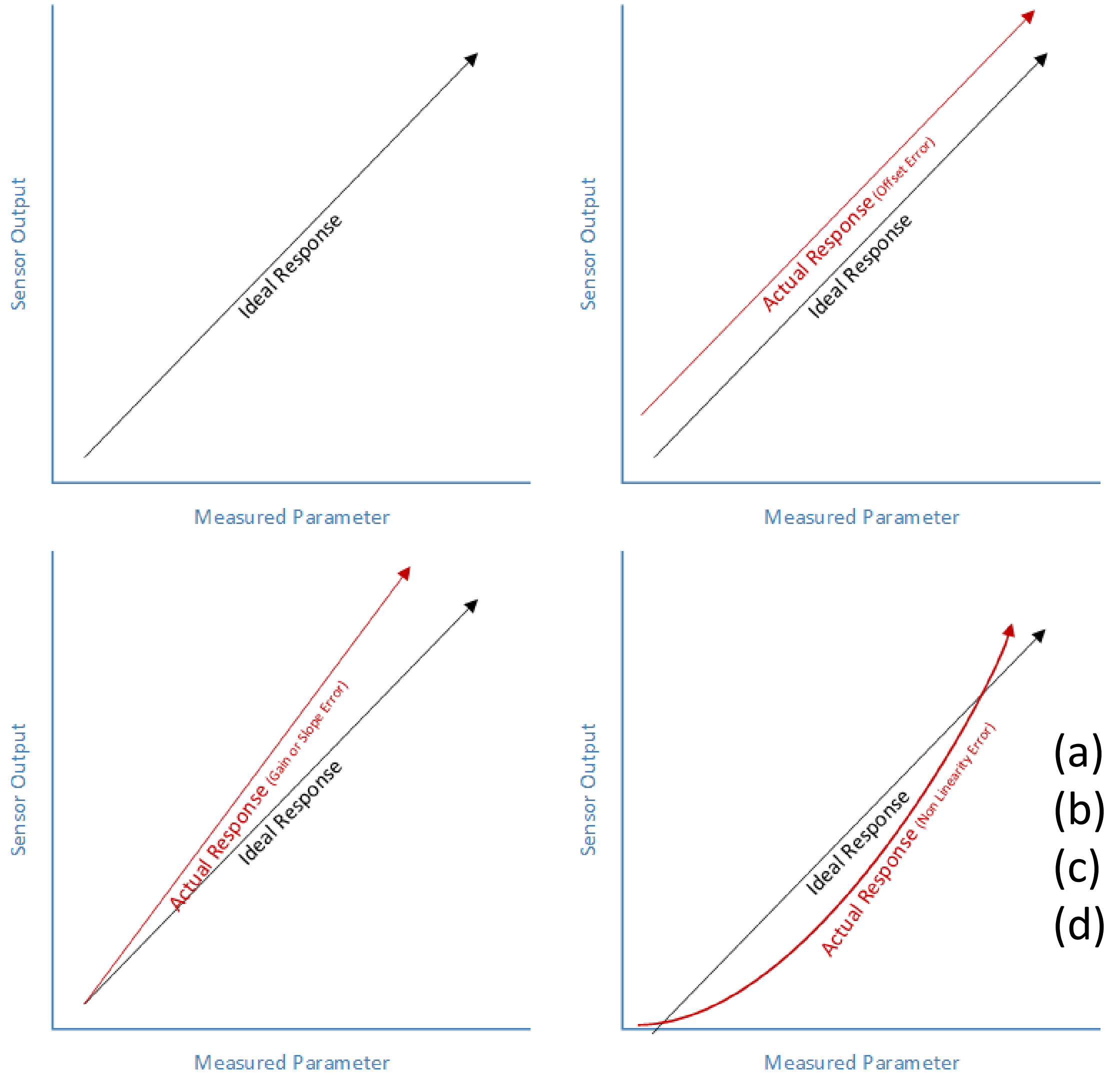
Red-filtered, green-filtered, blue-filtered, and clear (unfiltered) photodiodes







* https://cdn-shop.adafruit.com/datasheets/TCS34725.pdf



Types of errors:

- Bias;
- Scale factor;
- Nonlinearity;
- Noise;
- Depending from temperature etc.

(a) Ideal Response;
(b) Actual Response (bias error);
(c) Actual Response (scale factor error);
(d) Actual Response (non linearity error).

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<u>Example</u>: Calibrating the accelerometer

$$\begin{bmatrix} A_{x1} \\ A_{y1} \\ A_{z1} \end{bmatrix} = [A\_m]_{3x3} \begin{bmatrix} 1/A\_SC_x & 0 & 0 \\ 0 & 1/A\_SC_y & 0 \\ 0 & 0 & 1/A\_SC_z \end{bmatrix} \cdot \begin{bmatrix} A_x - A\_OS_x \\ A_y - A\_OS_y \\ A_z - A\_OS_z \end{bmatrix}$$

$$= \begin{bmatrix} ACC_{11} & ACC_{12} & ACC_{13} \\ ACC_{21} & ACC_{22} & ACC_{23} \\ ACC_{31} & ACC_{32} & ACC_{33} \end{bmatrix} \cdot \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \begin{bmatrix} ACC_{10} \\ ACC_{20} \\ ACC_{30} \end{bmatrix}$$

where [A_m] is the 3 x 3 **misalignment matrix** between the accelerometer sensing axes and the device body axes, A_SCi (i = x, y, z) is **the sensitivity (or scale factor)** and A_OSi is the zero-g level (or **offset**).

The goal of accelerometer calibration is to determine **12 parameters** from ACC_{10} to ACC_{33}, so that with any given raw measurements at arbitrary positions.

Example: Calibrating the accelerometer

Table 1. Sign definition of sensor raw measurements

Stationary position	Accelerometer (signed integer)		
	A_x	A_y	A_z
Z_b down	0	0	+1 *g*
Z_b up	0	0	-1 *g*
Y_b down	0	+1 *g*	0
Y_b up	0	-1 *g*	0
X_b down	+1 *g*	0	0
X_b up	-1 *g*	0	0

$$[A_{x1} \quad A_{y1} \quad A_{z1}] = [A_x \quad A_y \quad A_z \quad 1] \cdot \begin{bmatrix} ACC_{11} & ACC_{21} & ACC_{31} \\ ACC_{12} & ACC_{22} & ACC_{32} \\ ACC_{13} & ACC_{23} & ACC_{33} \\ ACC_{10} & ACC_{20} & ACC_{30} \end{bmatrix}$$

or $Y = w \cdot X$

where:

- Matrix **X** is the 12 calibration parameters that need to be determined
- Matrix **w** is sensor raw data LSBs collected at 6 stationary positions
- Matrix **Y** is the known normalized Earth gravity vector

Therefore, the calibration parameter matrix X can be determined by the **least square method** as:

$$X = \left[w^T \cdot w\right]^{-1} \cdot w^T \cdot Y$$

Attitude determination uses a combination of sensors and mathematical models to collect vector components in the body and inertial reference frames. These components are used in one of several different algorithms to determine the attitude, typically in the form of a quaternion, Euler angles, or a rotation matrix. It takes at least two vectors to estimate the attitude.

In general, the attitude determination solutions fall into two groups:
– **Deterministic (point-by-point)** solutions, where the attitude is found based on two or more vector observations from a single point in time,
– **Filters**, **recursive stochastic estimators** that statistically combine measurements from several sensors and often dynamic and/or kinematic models in order to achieve an estimate of the attitude.

The objective function

$$J\left(\mathbf{M}_{X_1X_2}\right)=\sum_{i=1}^{n}\alpha_i\left(\mathbf{U}_1^i-\mathbf{M}_{X_1X_2}\cdot\mathbf{U}_2^i\right)^T\left(\mathbf{U}_1^i-\mathbf{M}_{X_1X_2}\cdot\mathbf{U}_2^i\right) \quad (2.1)$$

where $\mathbf{M}_{X_1X_2}$ is the matrix describing the connection between the OFR and the BFR, parameterized by quaternions;

$\mathbf{U}_1^i, \mathbf{U}_2^i$ are the using vectors in the BFR and the OFR respectively;

n is the number of measuring vectors; min I = 2

α_i is the weight coefficient ($\alpha_i \neq 0$), considering the relative significance of measurements.

Four-dimensional symmetric matrix

$$\mathbf{B}=\sum_{i=1}^{n}\alpha_i\begin{bmatrix}\mathbf{I}\left(\left(\mathbf{U}_1^i\right)^T\mathbf{U}_2^i\right)-\mathbf{U}_2^i\left(\mathbf{U}_1^i\right)^T-\mathbf{U}_1^i\left(\mathbf{U}_2^i\right)^T & -\left(\mathbf{U}_1^i\times\mathbf{U}_2^i\right) \\ -\left(\mathbf{U}_1^i\times\mathbf{U}_2^i\right)^T & -\left(\mathbf{U}_1^i\right)^T\mathbf{U}_2^i\end{bmatrix}. \quad (2.2)$$

Example

Measurements

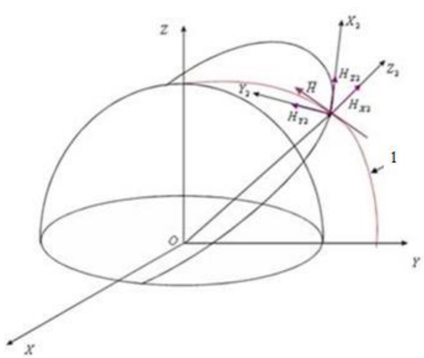
BFR

Earth magnetizing force

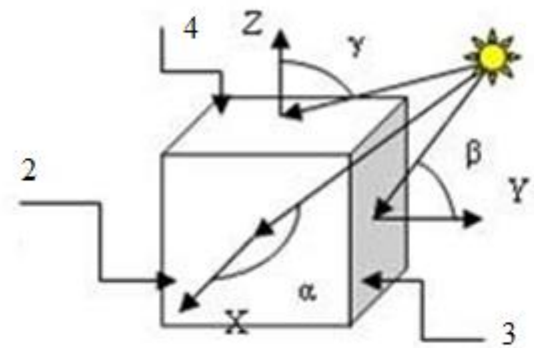
Current from solar battery panels

- measured Earth magnetic vector in the body frame of reference

- current in the body frame of reference



Algorithm QUEST



1 – power line of the Earth magnetic field;
2,3,4 – solar battery panels

OFR

Model of the Earth magnetic field in the orbital frame of reference

Model of the Sun motion in the orbital frame of reference

Models

3.1 Linear problem. Basic concepts

Model	Continuous time	Discrete time
System	$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t) + \boldsymbol{\omega}(t)$	$\boldsymbol{x}_k = \boldsymbol{\Phi}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{\omega}_k$
Measurements	$\boldsymbol{z} = \boldsymbol{H}(t)\boldsymbol{x}(t) + \boldsymbol{v}(t)$	$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k$
System noise	$E\left\langle \boldsymbol{\omega}(t) \right\rangle = 0$ $E\left\langle \boldsymbol{\omega}(t)\boldsymbol{\omega}^T(s) \right\rangle = \delta(t-s)\boldsymbol{Q}(t)$	$E\left\langle \boldsymbol{\omega}_k \right\rangle = 0$ $E\left\langle \boldsymbol{\omega}_k\boldsymbol{\omega}_i^T \right\rangle = \Delta(k-i)\boldsymbol{Q}_k$
Measurement noise	$E\left\langle \boldsymbol{v}(t) \right\rangle = 0$ $E\left\langle \boldsymbol{v}(t)\boldsymbol{v}^T(s) \right\rangle = \delta(t-s)\boldsymbol{R}(t)$	$E\left\langle \boldsymbol{v}_k \right\rangle = 0$ $E\left\langle \boldsymbol{v}_k\boldsymbol{v}_i^T \right\rangle = \Delta(k-i)\boldsymbol{R}_k$

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3.1 Linear problem. Basic concepts

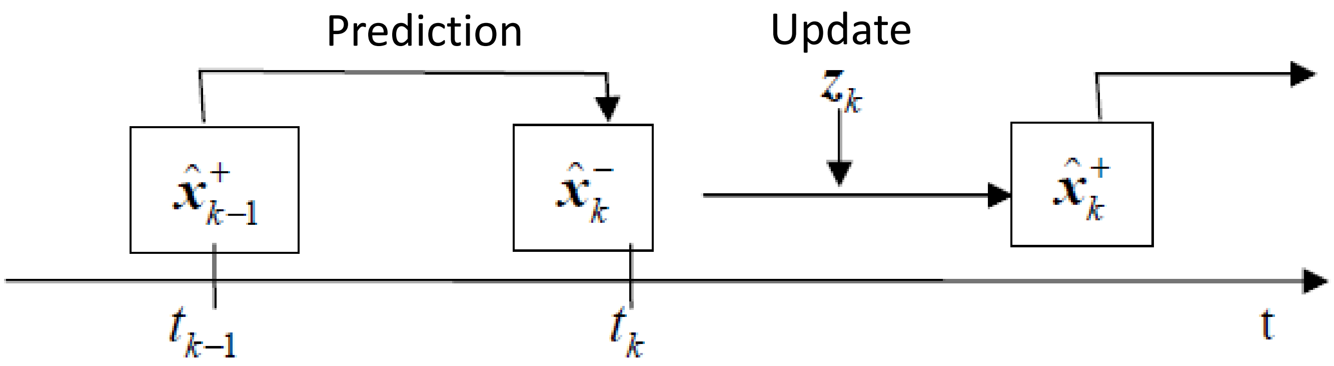


Fig. 3.1 - Kalman filter operation principle

The second moment of the random process can be described in terms of the covariance matrix

$$\boldsymbol{P}(t) = E\left\langle \left[\boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)\right]\left[\boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)\right]^T \right\rangle \tag{3.1}$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t) \qquad \boldsymbol{x}(0) = \hat{\boldsymbol{x}}_{k-1}^{+}$$

Linear Models. Summary

Continuous linear process model and a discrete observation model:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t) + \boldsymbol{\omega}(t)$$

$$\boldsymbol{z}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{\upsilon}_k .$$

The Kalman filter prediction equations:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t)$$

$$\dot{\boldsymbol{P}}(t) = \boldsymbol{F}(t)\boldsymbol{P}(t) + \boldsymbol{P}(t)\boldsymbol{F}^T(t) + \boldsymbol{Q} .$$

The observational update equations:

$$\boldsymbol{K}_{\mathrm{k}}^1 = \boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k ,$$

$$\overline{\boldsymbol{K}}_k = \boldsymbol{P}_k^- \boldsymbol{H}_k^T \left[\boldsymbol{H}_k \boldsymbol{P}_k^- \boldsymbol{H}_k^T + \boldsymbol{R}_k \right]^{-1} .$$

$$\hat{\boldsymbol{x}}_k^+ = \boldsymbol{K}_k^1 \hat{\boldsymbol{x}}_k^- + \overline{\boldsymbol{K}}_k \boldsymbol{z}_k .$$

$$\boldsymbol{P}_k^+ = (\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k)\boldsymbol{P}_k^- .$$

3.4. Kalman filter for nonlinear systems (the expanded filter)

We will assume that the continuous or discrete stochastic system can be presented by the nonlinear dynamic equation and the model equation describing measurements

Model	Continuous time	Discrete time
System	$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), t) + \boldsymbol{\omega}(t)$	$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}, k-1) + \boldsymbol{\omega}_{k-1}$
Measurements	$\boldsymbol{z}(t) = \boldsymbol{h}(\boldsymbol{x}(t), t) + \boldsymbol{v}(t)$	$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_k, k) + \boldsymbol{v}_k$

The applied method of linearization demands that functions **f** and **h** were twice continuously differentiable. We will designate a symbol **δ** the small deviation from the estimated trajectory:

$$\delta \boldsymbol{x}_k = \boldsymbol{x}_k - \hat{\boldsymbol{x}}_k^-,$$

$$\delta \boldsymbol{z}_k = \boldsymbol{z}_k - \boldsymbol{h}\left(\hat{\boldsymbol{x}}_k^-, k\right),$$

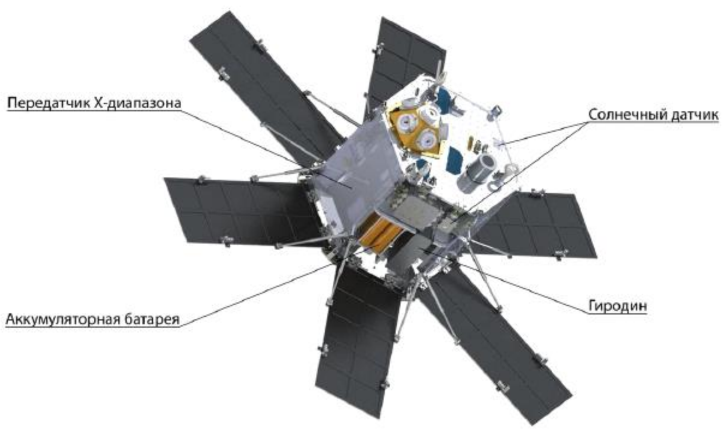
1. ***Problem of setting the initial approximations of attitude parameters***.
For effective operation of the filter it is necessary to have rather good initial state vector. For certain initial conditions the filter can not converge.
2. ***Linearization problem.*** Kalman filter for the work uses the linearized motion model. In case of rather slow motion (or in case of rather frequent measurements) the filter gives a satisfactory estimation of the state vector. Otherwise the filter will give the constant and growing error in the state vector estimation.
3. ***Setting problem***. The filter uses in the work the covariance matrices of errors which setting strongly influences the main characteristics of the filter: the convergence speed and the estimated state vector error after convergence.

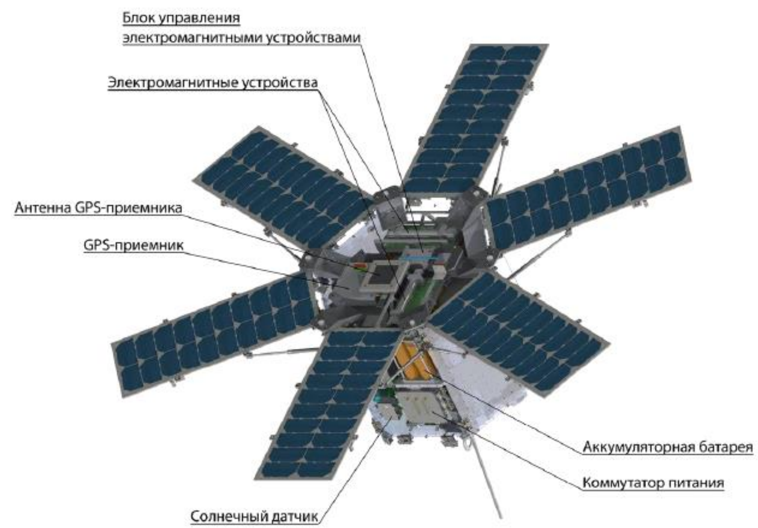
(Source: Ivanov D. S., Ivlev N. A., Karpenko S.O., Ovchinnikov M. Y. Attitude determination algorithms investigation for microsatellites of 'TabletSat' series)

4.1 Characteristics of measuring data

Table 5.1

Characteristic	Magnetometer (MAG)	Sun sensor (SS)	Angular rate sensor (GYR)	Star tracker (ST)
Measurement range	±200 000 ntesla	±45 deg	±250 deg/c	±2 deg
Random deviation (σ)	250 ntesla	0,1 deg	0,005 deg/c	0,001 deg





Numerical simulations:

1. ST+GYR+MAG+SS
2. ST+GYR+MAG
3. ST+MAG+SS
4. ST+MAG
5. MAG+SS+GYR
6. ST+GYR
7. MAG+SS
8. MAG+GYR
9. ST

(~10^{-4} deg; ~10^{-4} deg/s) — items 1–4, 6, 9

(~10^{-2} deg; ~10^{-3} deg/s) — item 5

(~10^{-1} deg; ~10^{-2} deg/s) — item 7

(~10^{-1} deg; ~10^{-3} deg/s) — item 8

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4.3 Research of angular motion determination algorithms

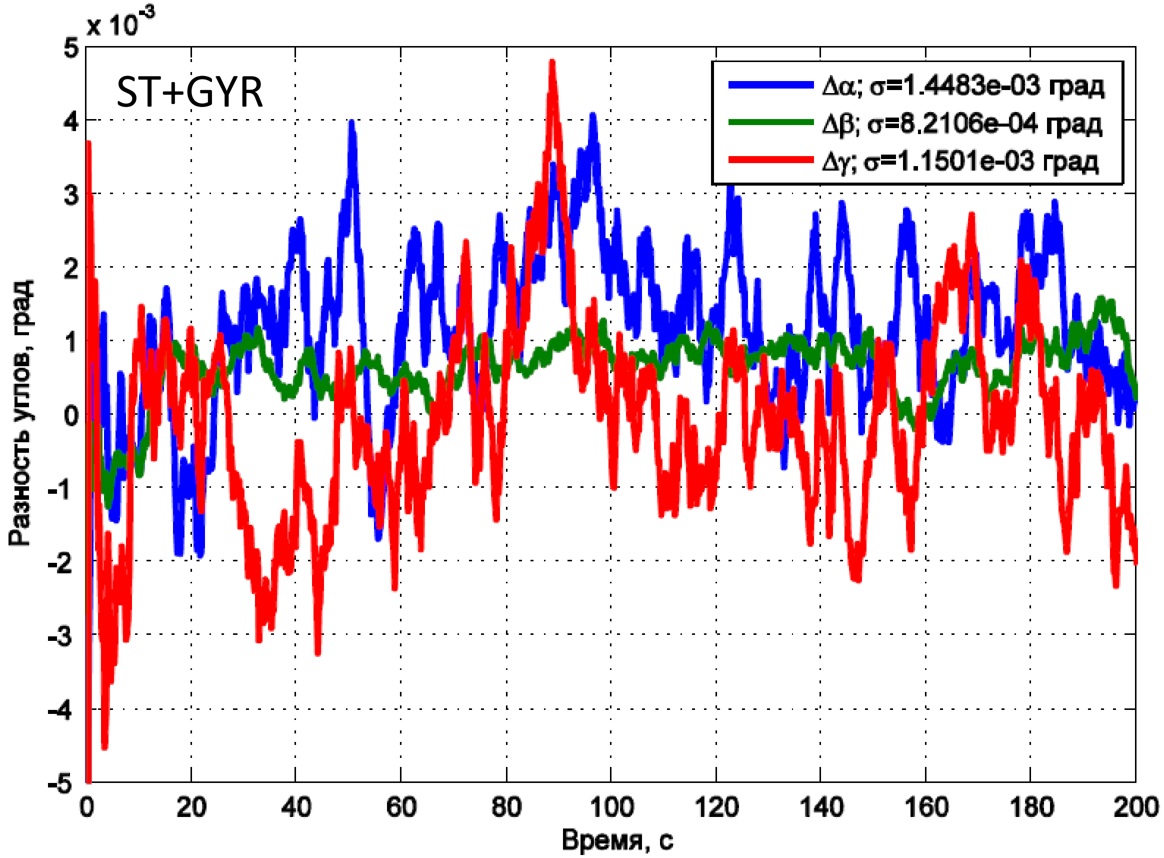

Fig. 4.2 - The graph of the difference of estimates of attitude angles and their real value

4.3 Research of angular motion determination algorithms

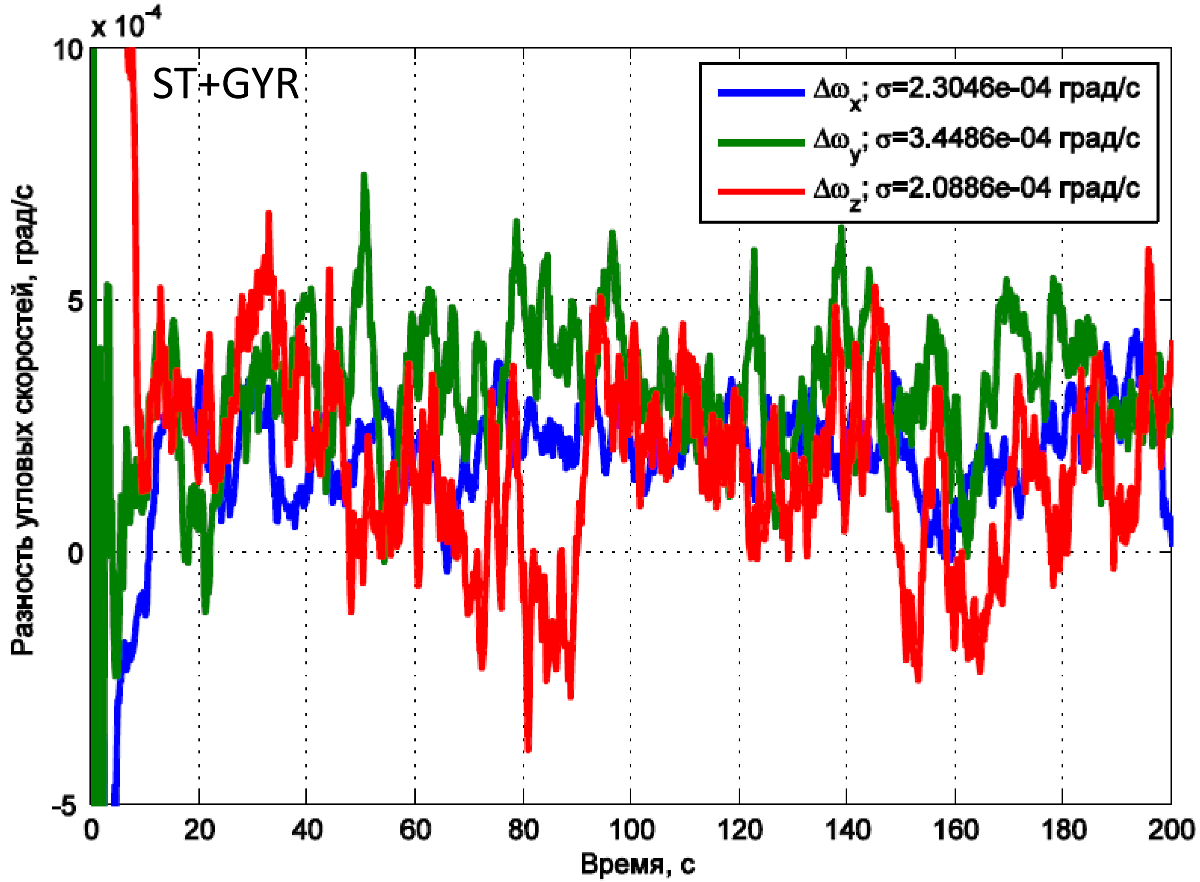

Fig 4.3 - - The graph of the difference of angular rate component estimates and their real value

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