САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

Formation Flying

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The definition, proposed by NASA's Goddard Space Flight Center (GSFC):

Spacecraft Formation Flying **is the tracking or maintenance of a desired relative separation, orientation or position between or among spacecraft.**

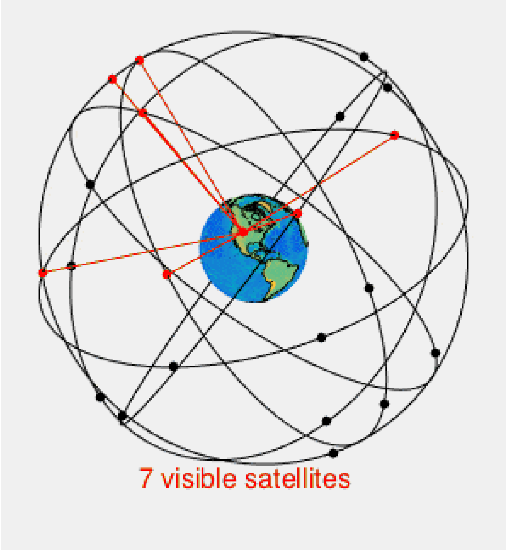
Formation flying spacecraft are therefore a particular case of a more general category, termed distributed space systems

Constellation **is a collection of space vehicles that constitutes the space element of a distributed space system.**

Virtual platform **is a spatially distributed network of individual vehicles collaborating as a single functional unit, and exhibiting a common system-wide capability to accomplish a shared objective.**

Definitions

Constellation is composed of two or more spacecraft in similar orbits with no active control by either to maintain their relative position. Is only necessary that spacecraft maintain themselves within their own prespecified boxes without collisions or changing the overall coverage of the Earth significantly.



Formation is two or more spacecraft that use an active control scheme to maintain the relative positions between spacecraft. It has to be a direct control on the relative position and orientation between one spacecraft and another one (typically its neighbor) or many other.

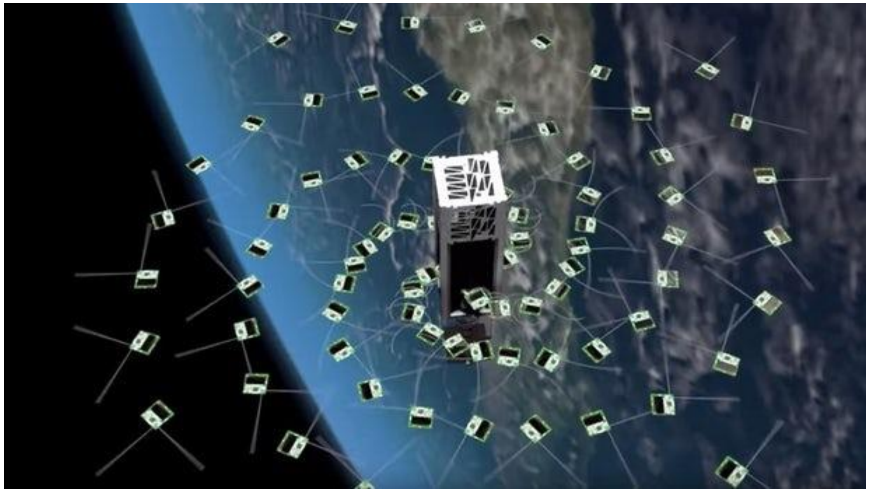
Coordination methods

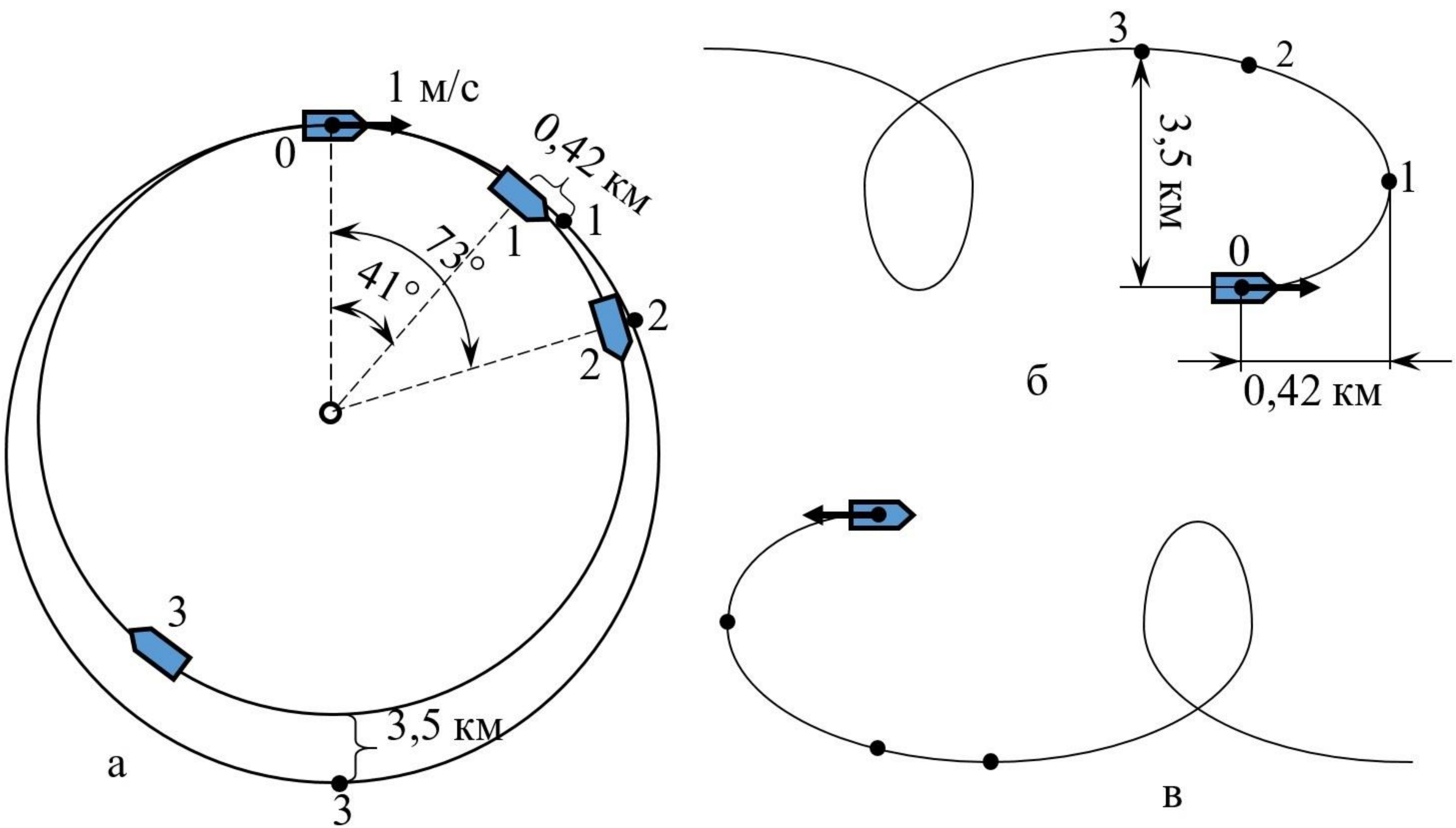
Leader/follower

In **leader/follower** coordination methods one leader spacecraft is controlled to a reference orbit and the other follower spacecraft in the formation control their relative states to that leader.

Swarming

A number of researchers have proposed simple heuristic control laws for arranging arbitrarily large numbers of vehicles into regular arrangements based on local information. These swarming methods have the advantage that they easily scale to large numbers of vehicles without incurring large communication or computation burdens.




1 м/с
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1
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3
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0,42 км
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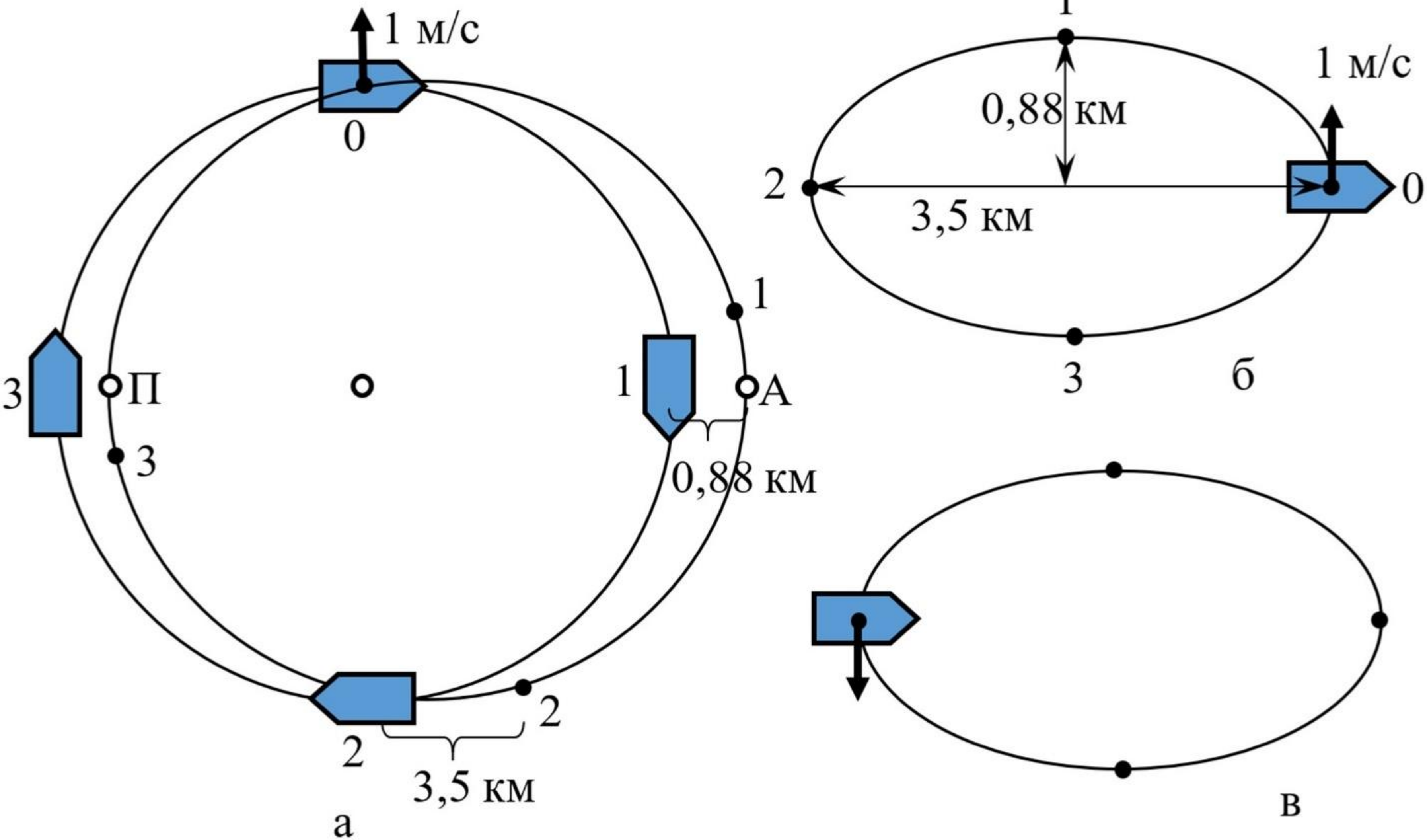

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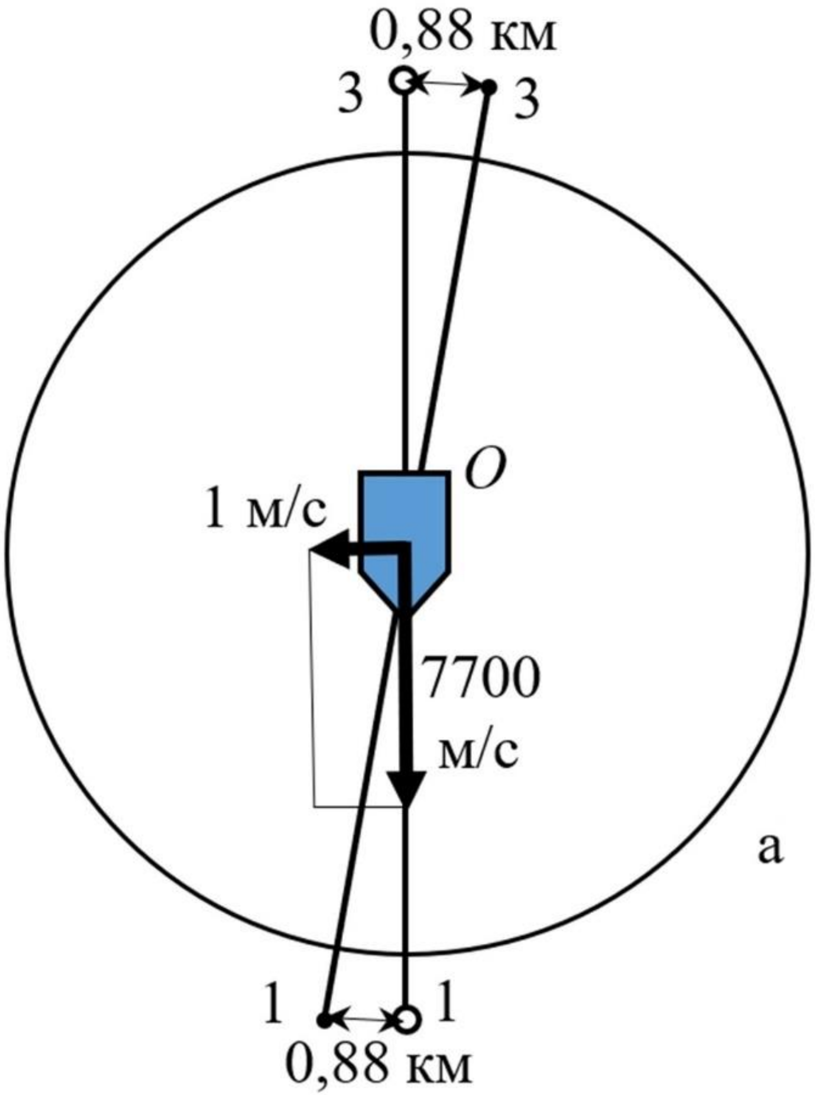
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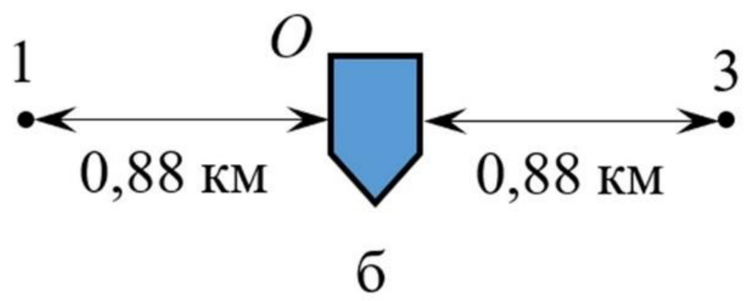
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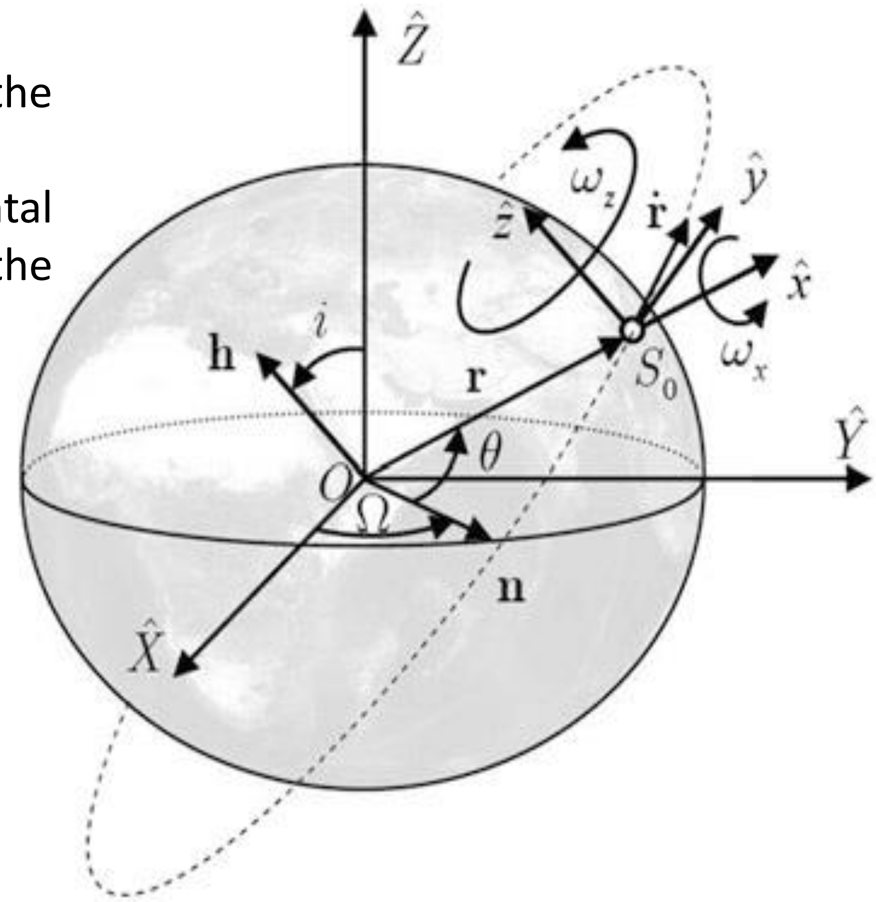
A **local-vertical, local-horizontal (LVLH)** rotating coordinate system, centered at the spacecraft,

- the fundamental plane is the orbital plane
- the unit vector **x** is directed from the spacecraft radially outward,
- the unit vector **z** is normal to the fundamental plane, positive in the direction of the (instantaneous) angular momentum vector,
- the unit vector **y** completes the setup.



The equation of spacecraft motion in a geocentric inertial coordinate system

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = \vec{f}_p$$

where $\vec{f}_p$ – perturbing acceleration.

The equations of motion of leader and follower, respectively

$$\ddot{\vec{r}}_f + \frac{\mu}{r_f^3}\vec{r}_f = \vec{f}_{pf}, \qquad \ddot{\vec{r}}_l + \frac{\mu}{r_l^3}\vec{r}_l = \vec{f}_{pl},$$

$\vec{d} = \vec{r}_f - \vec{r}_l$ is a relative distance vector between spacecraft in inertial coordinate system

$\ddot{\vec{d}} = \ddot{\vec{r}}_f - \ddot{\vec{r}}_l$ is a relative acceleration vector between spacecraft in inertial coordinate system

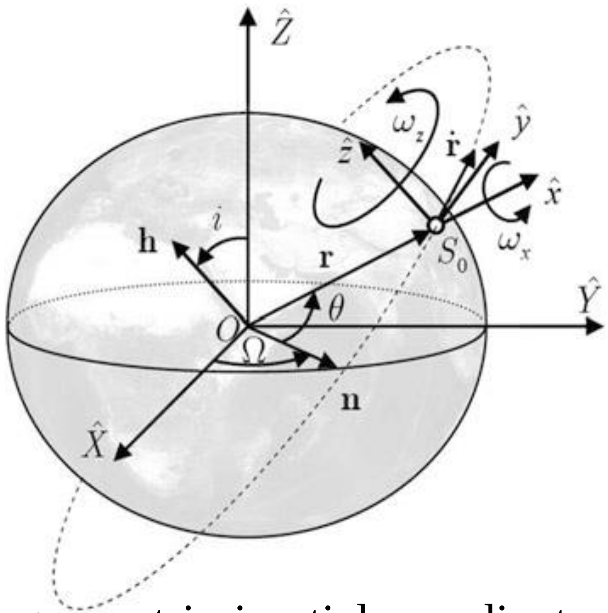
$$\ddot{\vec{r}}_f + \frac{\mu}{r_f^3}\vec{r}_f - \ddot{\vec{r}}_l - \frac{\mu}{r_l^3}\vec{r}_l = \ddot{\vec{d}} + \frac{\mu}{\left|\vec{r}_l + \vec{d}\right|^3}\left(\vec{r}_l + \vec{d}\right) - \frac{\mu}{r_l^3}\vec{r}_l = \vec{f}_p,$$

Relative motion

The relative distance vector between spacecraft, the velocity vector of this distance, the geocentric radius vector of the passive spacecraft and its angular velocity vector have the following components in the LVLH:

$$\vec{\rho} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \dot{\vec{\rho}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}, \vec{r} = \vec{r}_l = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}, \vec{\omega} = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix}.$$

LVLH



The dependence between the relative accelerations in the geocentric inertial coordinate system and LVLH consists of the acceleration in the active spacecraft in the LVLH, the Coriolis acceleration and the centripetal acceleration that the active aircraft receives if it rests in the LVLH:

$$\ddot{\vec{d}} = \ddot{\vec{\rho}} + 2\vec{\omega} \times \dot{\vec{\rho}} + \dot{\vec{\omega}} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}),$$

$$\ddot{\vec{d}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + 2\begin{bmatrix} -\omega\dot{y} \\ \omega\dot{x} \\ 0 \end{bmatrix} + \begin{bmatrix} -\dot{\omega}y \\ \dot{\omega}x \\ 0 \end{bmatrix} + \begin{bmatrix} -\omega^2 x \\ -\omega^2 y \\ 0 \end{bmatrix}.$$

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Relative motion

Considering $r_l + d = r + \rho = \sqrt{(r+x)^2 + y^2 + z^2}$, we have component-wise equations for relative motion:

$$\ddot{x} - 2\omega\dot{y} - \dot{\omega}y - \omega^2 x + \frac{\mu(r+x)}{\left(\sqrt{(r+x)^2 + y^2 + z^2}\right)^3} - \frac{\mu}{r^2} = f_{px},$$

$$\ddot{y} + 2\omega\dot{x} + \dot{\omega}x - \omega^2 y + \frac{\mu y}{\left(\sqrt{(r+x)^2 + y^2 + z^2}\right)^3} = f_{py},$$

$$\ddot{z} + \frac{\mu z}{\left(\sqrt{(r+x)^2 + y^2 + z^2}\right)^3} = f_{pz}$$

where f_{px}, f_{py}, f_{pz} – vector $\vec{f}_p$ projections to LVLH.

Relative motion

Suppose that the leader moves in a circular orbit and LVLH rotates with the constant angular velocity of the orbital motion $n = \sqrt{\frac{\mu}{r^3}}$ then $\mu = n^2 r^3$ and $\dot{n} = 0$.

$$\ddot{x} - 2n\dot{y} - n^2 x + \frac{\mu(r+x)}{\left(\sqrt{(r+x)^2 + y^2 + z^2}\right)^3} - \frac{\mu}{r^2} = f_{px},$$

$$\ddot{y} + 2n\dot{x} - n^2 y + \frac{\mu y}{\left(\sqrt{(r+x)^2 + y^2 + z^2}\right)^3} = f_{py},$$

$$\ddot{z} + \frac{\mu z}{\left(\sqrt{(r+x)^2 + y^2 + z^2}\right)^3} = f_{pz},$$

Relative motion

Assuming that the components of the relative distance vector $\vec{\rho}$ small compared to r, we expand of the expression $\left((r+x)^2+y^2+z^2\right)^{-3/2}$ into a Taylor series about the origin:

$$\left((r+x)^2+y^2+z^2\right)^{-3/2}=\frac{1}{r^3}\left[1-3\frac{x}{r}-\frac{3}{2r^2}(y^2+z^2-4x^2)+\ldots\right].$$

Taking only the first-order terms, we have the linearized equations of motion are called the Clohessy–Wiltshire equations (CW) or the Hill–Clohessy–Wiltshire equations (HCW).

$$\ddot{x}-2n\dot{y}-3n^2x=f_{px},$$

$$\ddot{y}+2n\dot{x}=f_{py},$$
$$\ddot{z}+n^2z=f_{pz}.$$

The analytical solution of the system of equations when perturbing acceleration projections are constant

$$x = 4x_0 + \frac{2\dot{y}_0}{n} + \frac{f_{px}}{n^2} - \left(\frac{2\dot{y}_0}{n} + 3x_0 + \frac{f_{px}}{n^2}\right)\cos(nt) + \left(\frac{\dot{x}_0}{n} - \frac{2f_{py}}{n^2}\right)\sin(nt) + \frac{2f_{py}}{n}t,$$

$$y = y_0 - \frac{2\dot{x}_0}{n} + \frac{4f_{py}}{n^2} - 3\dot{y}_0 t - 6x_0 nt - \frac{2f_{px}}{n}t + \left(\frac{2\dot{x}_0}{n} - \frac{4f_{py}}{n^2}\right)cos(nt) +$$

$$+\left(\frac{4y_0}{n} + 6x_0 + \frac{2f_{px}}{n^2}\right)sin(nt) - \frac{3}{2}f_{py}t^2,$$

$$z = \left(z_0 - \frac{f_{pz}}{n^2}\right)cos(nt) + \frac{\dot{z}_0}{n}sin(nt) + \frac{f_{pz}}{n^2},$$

$$\dot{x} = \left(2\dot{y}_0 + 3x_0 n + \frac{f_{px}}{n}\right)sin(nt) + \left(\dot{x}_0 - \frac{2f_{py}}{n}\right)cos(nt) + \frac{2f_{py}}{n},$$

$$\dot{y} = -3\dot{y}_0 - 6x_0 n - \frac{2f_{px}}{n} - \left(2\dot{x}_0 - \frac{4f_{py}}{n}\right)sin(nt) + \left(4\dot{y}_0 + 6x_0 n + \frac{2f_{px}}{n}\right) cos(nt) - 3f_{py}t$$

$$\dot{z} = -\left(z_0 n - \frac{f_{pz}}{n}\right)sin(nt) + \dot{z}_0 cos(nt).$$

The analytical solution of the system of equations has the simplest form of perturbing acceleration is zero

$$x = 4x_0 + \frac{2\dot{y}_0}{n} - \left(\frac{2\dot{y}_0}{n} + 3x_0\right)cos(nt) + \frac{\dot{x}_0}{n}sin(nt),$$

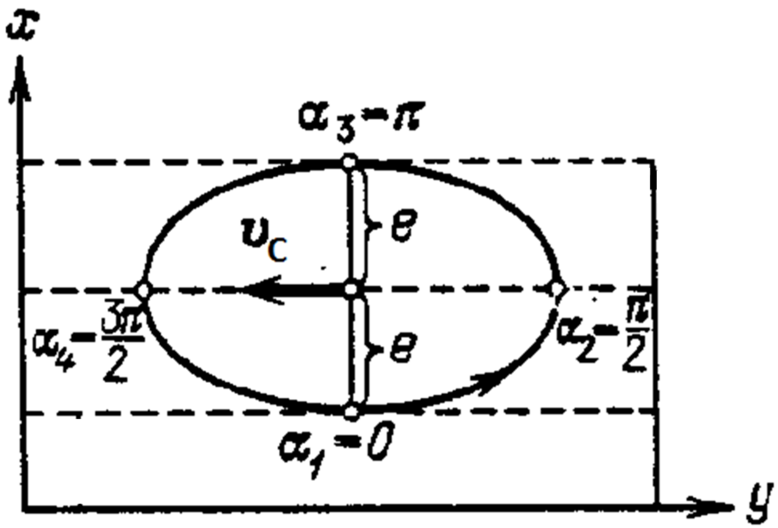
$$y = y_0 - \frac{2\dot{x}_0}{n} - 3\dot{y}_0 t - 6x_0 nt + \frac{2\dot{x}_0}{\omega}cos(nt) + \left(\frac{4\dot{y}_0}{n} + 6x_0\right)sin(nt),$$

$$z = z_0 cos(nt) + \frac{\dot{z}_0}{n}sin(nt),$$

$$\dot{x} = (2\dot{y}_0 + 3x_0 n)sin(nt) + \dot{x}_0 cos(nt),$$

$$\dot{y} = -3\dot{y}_0 - 6x_0 n - 2\dot{x}_0 sin(nt) + (4\dot{y}_0 + 6x_0\omega)n\, cos(nt),$$

$$\dot{z} = -z_0 n sin(nt) + \dot{z}_0 cos(nt).$$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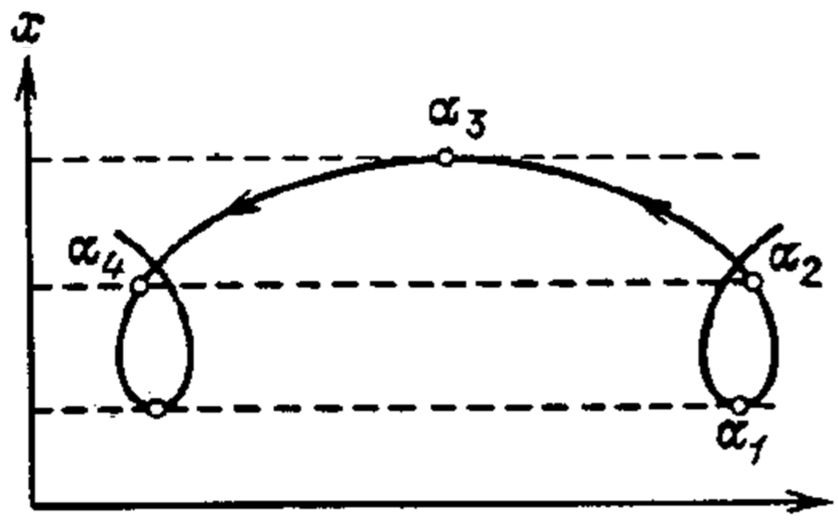
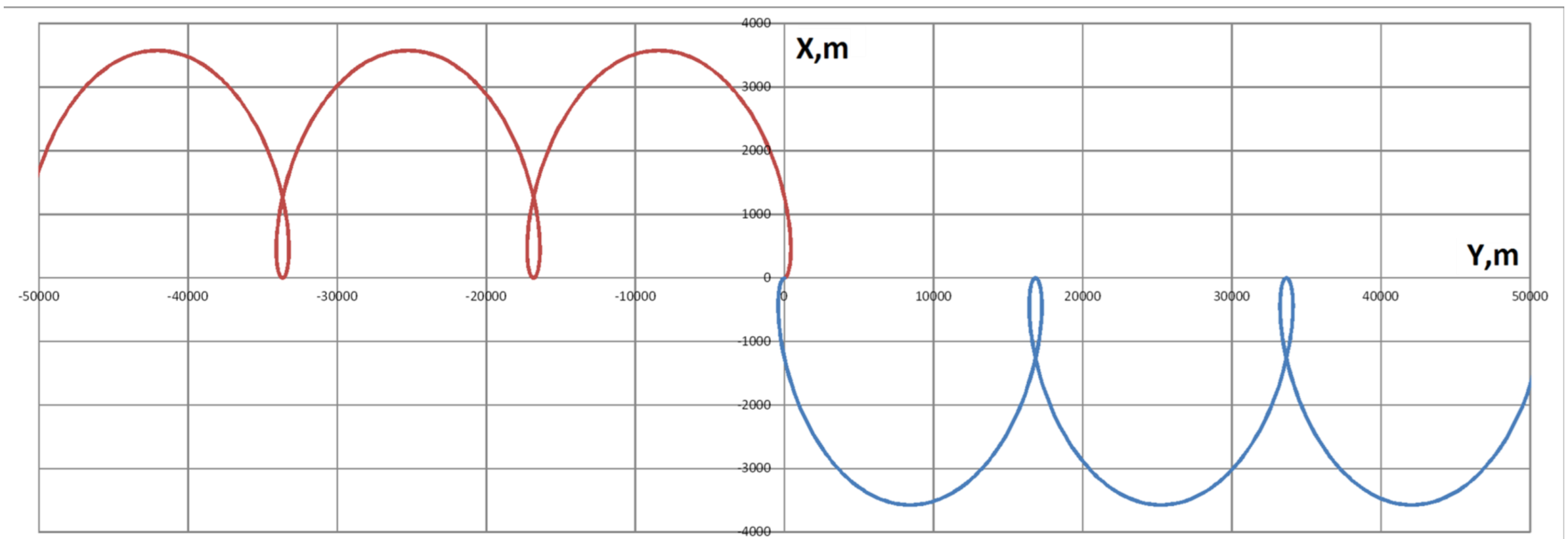
α3 = π
vC
в
в
α4 = 3π/2
α2 = π/2
α1 = 0
y


x
α3
α4
α2
α1
y


X,m
Y,m
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2000
1000
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-10000
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10000
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