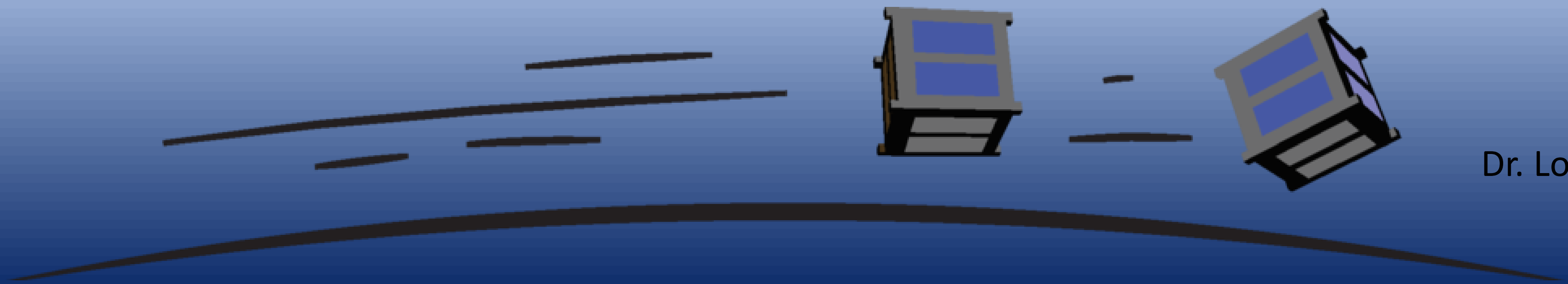
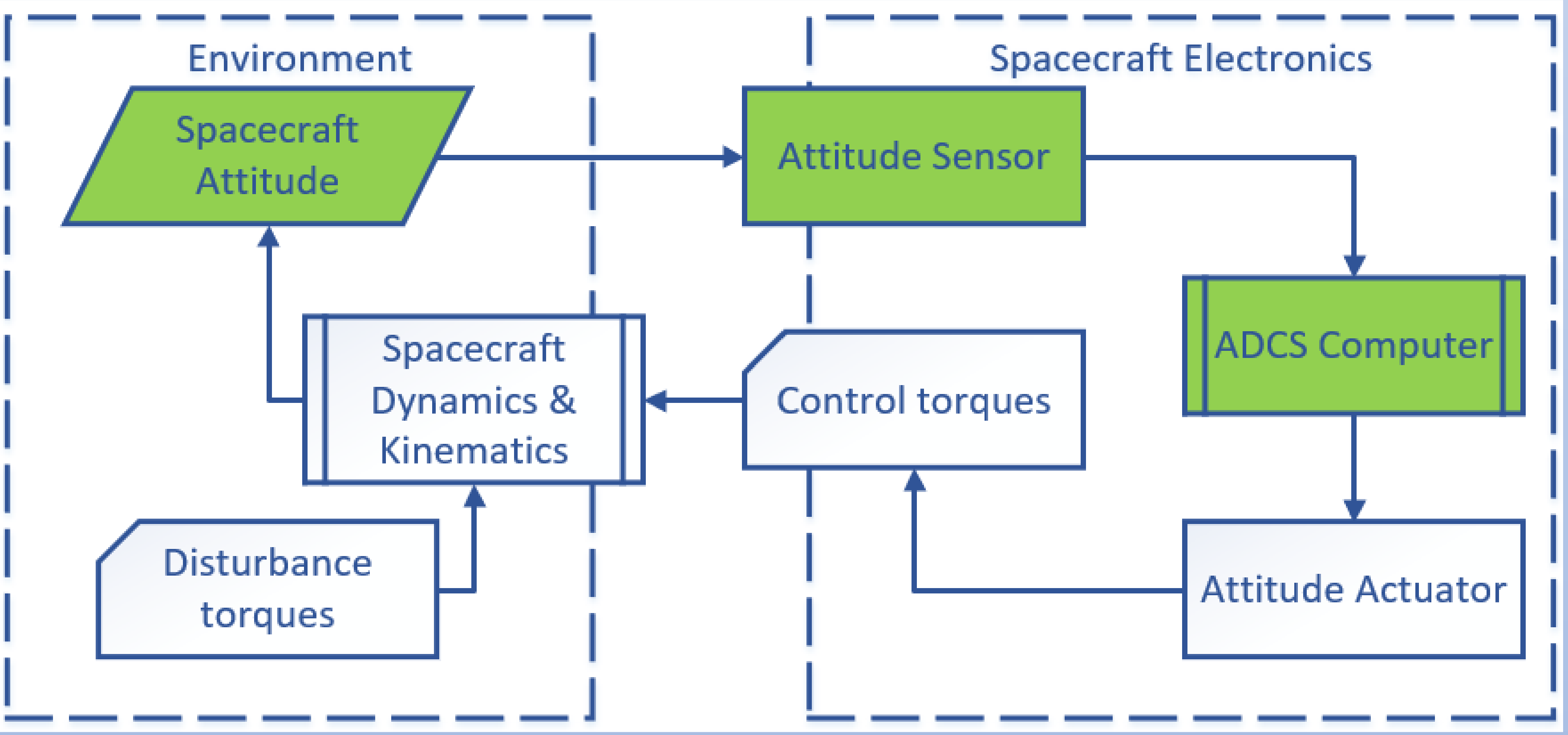
L7. Methods and algorithms for nanosatellite attitude determination and control



Dr. Lomaka Igor

ADCS Structure



ADCS closed-loop control system

Which attitude has CubeSat now?

How to change the attitude of CubeSat?

1. Attitude determination problem definition

The main frames of reference:
- the **body frame** of reference(BFR)
- the **orbital frame** of reference(OFR);
- the **geocentric frame** of reference(GFR).

Attitude matrix:

$$M_{X_1 X_2} = \begin{cases} f_1(\vartheta, \psi, \varphi), \\ f_2(q_0, q_1, q_2, q_3), \\ f_3\left(m_{ij}, \quad i,j = \overline{1,3}\right). \end{cases}$$

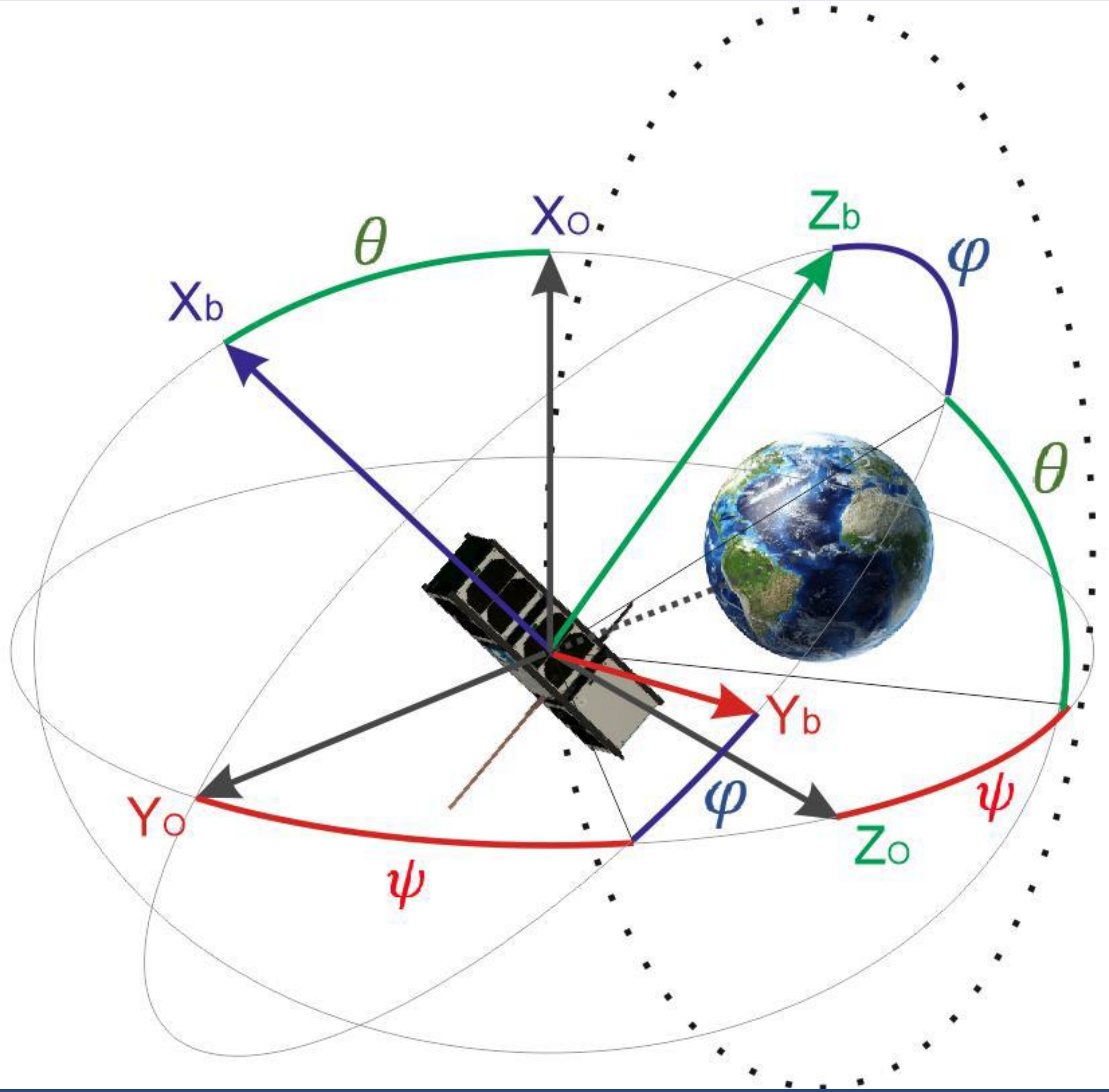


Fig. 1.1–The frames of reference

Representation of Attitude

Representation	Par.	Characteristic	Application
Rotation matrix	9	• Inherently nonsingular • Intuitive representation • Difficult to mantain ortogonality • Expensive to store • Six redundant parameter	Analytical studies and transformation of vectors.
Euler angles	3	• Minimal set • Clear physical interpretation • Trigometric functions in rotation matrix • No simple composition rule • Singular for certain rotations • Trigonometric functions in kinematic relation	Theoretical physics, spinning spacecraft and attitude maneuvers. Used in analytical studies.
Axis-azimuth	3	• Minimal set • Clear physical interpretation • Often computed directly from observations • No simple composition rule • Computation of rotating matrix very difficult • Singular for certain rotation • Trigonometric functions in kinematic relation	Primarily spinning spacecraft.
Rodriguez (Gibbs)	3	• Minimal set • Clear physical interpretation • Singular for rotations near $\theta = \pm\pi$ • Simple kinematic relations	Often interpreted as incremental rotation vector.
Quaternions	4	• Easy orthogonality of rotation matrix • Bilinear composition rule • Not singular at any rotation matrix • Linear kinematic equations • No clear physical interpretation • One redundant parameter • Simple kinematic relation	Widely used in simulations and data processing. Preferred attitude representation for attitude control systems.

Relations of Several Attitude Representations

Rotation matrix depending on the Euler angles

$$A_{yzy} = \begin{bmatrix} \cos\varphi\cos\alpha\cos\psi - \sin\varphi\sin\psi & \cos\varphi\sin\alpha & -\cos\varphi\cos\alpha\sin\psi - \sin\varphi\cos\psi \\ -\sin\alpha\cos\psi & \cos\alpha & \sin\alpha\sin\psi \\ \sin\varphi\cos\alpha\cos\psi + \cos\varphi\sin\psi & \sin\varphi\sin\alpha & -\sin\varphi\cos\alpha\sin\psi + \cos\varphi\cos\psi \end{bmatrix}$$

Rotation matrix depending on the quaternion

$$A = \begin{bmatrix} 1 - 2(q_2^2 + q_3^2) & 2(q_1q_2 - q_3q_0) & 2(q_1q_3 + q_2q_0) \\ 2(q_1q_2 + q_3q_0) & 1 - 2(q_1^2 + q_3^2) & 2(q_2q_3 - q_1q_0) \\ 2(q_1q_3 - q_2q_0) & 2(q_2q_3 + q_1q_0) & 1 - 2(q_1^2 + q_2^2) \end{bmatrix}$$

Quaternion depending on the Euler angles

$$q_0 = \cos\frac{\alpha}{2}\cos\frac{\psi+\varphi}{2}\ ;\ q_1 = \sin\frac{\alpha}{2}\sin\frac{\psi-\varphi}{2}\ ;$$

$$q_3 = \cos\frac{\alpha}{2}\sin\frac{\psi+\varphi}{2}\ ;\ q_4 = \sin\frac{\alpha}{2}\cos\frac{\psi-\varphi}{2}$$

Attitude Dynamics

Vector equation

$$\frac{d\bar{h}_0}{dt} + \bar{\omega} \times \bar{h}_0 = \bar{M}_0^e,$$

where $\bar{h}_0 = I\bar{\omega}$ *- angular momentum vector;*

$\bar{M}_0^e$ *- the main moment of external forces relative to the center of mass;*

$\bar{\omega}$ *- absolute angularvelosity;*

I *– inertia tensor.*

In the projections to the main central axes of inertia of the CS Ox, Oy, Oz, (attitude dynamics equations)

$$I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z = M_{x_g} + M_{x_a} + M_{x_{ctrl}}$$

$$I_y\dot{\omega}_y + (I_x - I_z)\omega_z\omega_x = M_{y_g} + M_{y_a} + M_{y_{ctrl}}$$

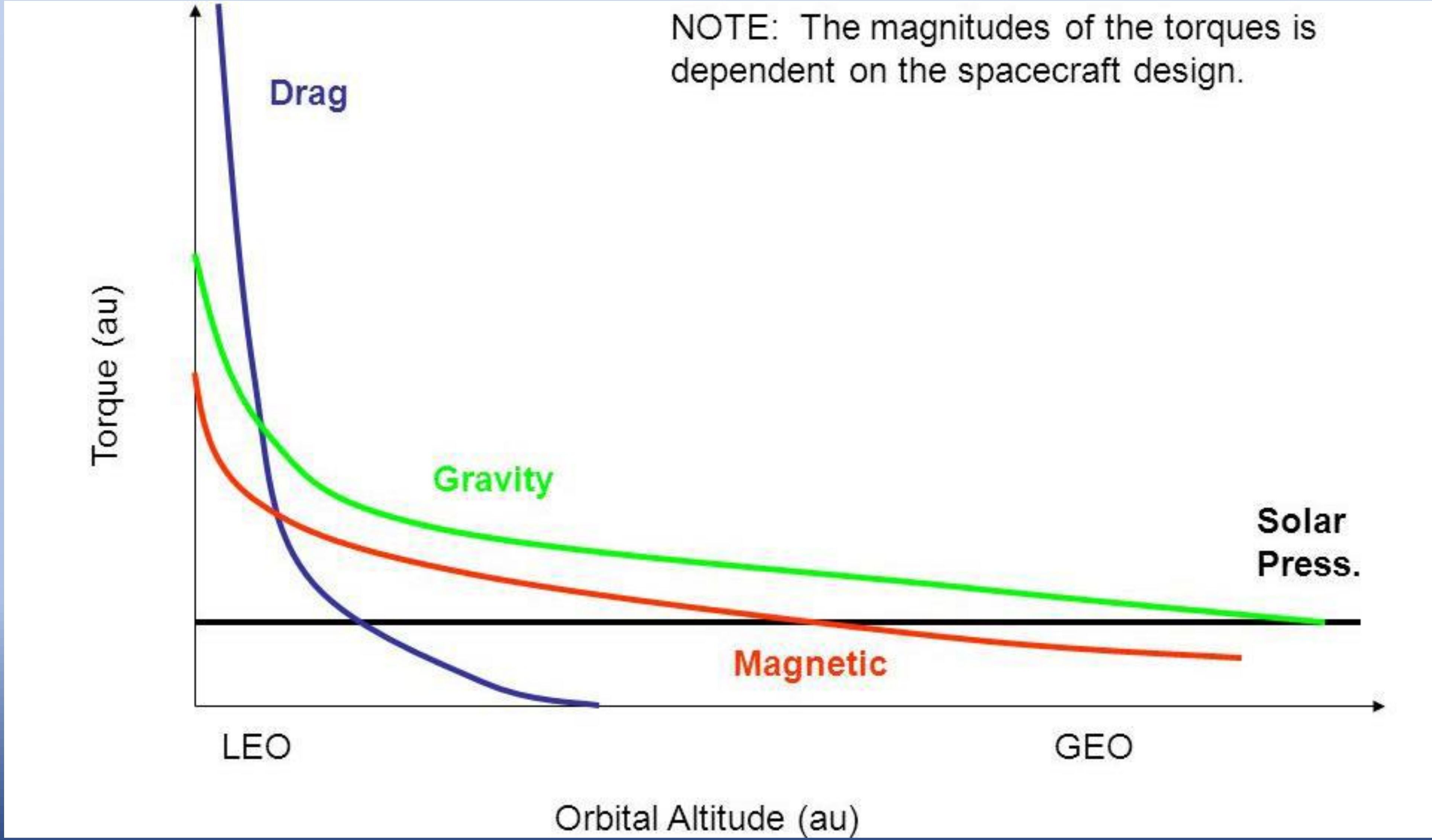
$$I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y = M_{z_g} + M_{z_a} + M_{z_{ctrl}}$$

where $\omega_x, \omega_y, \omega_z$ *– projections of angular velosity vector on the axis Ox, Oy, Oz;*

I_x, I_y, I_z *– main central moments of inertia;*

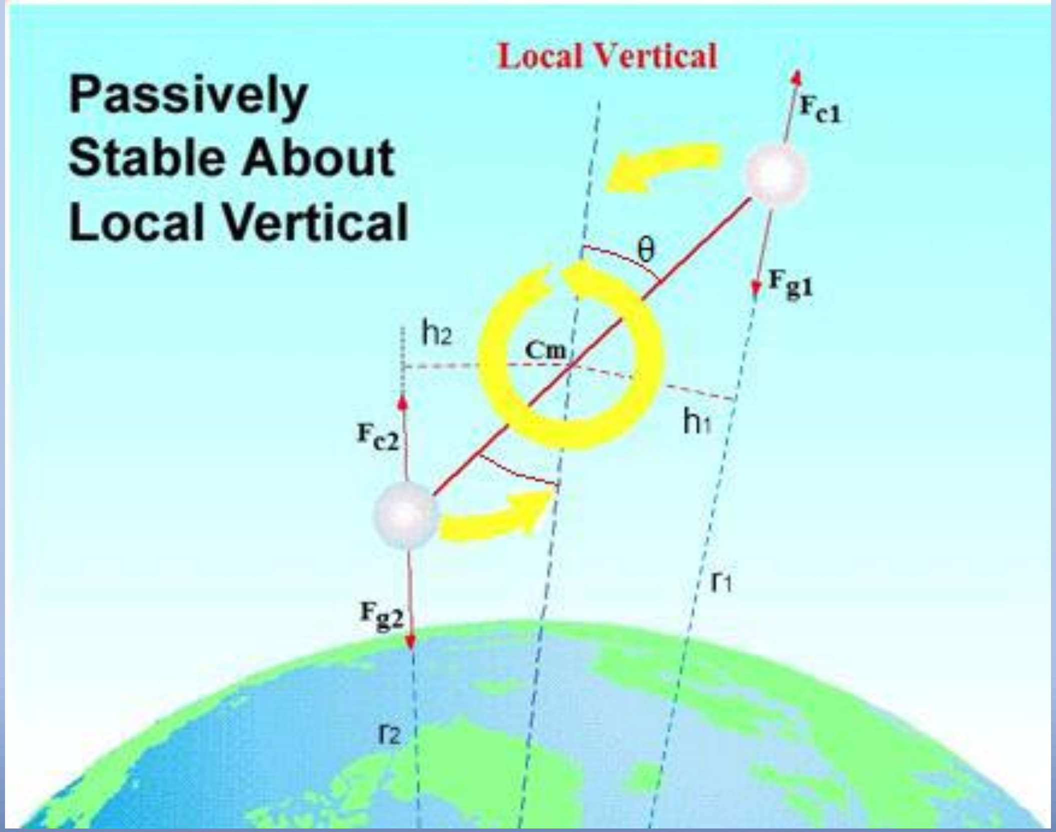
M_x, M_y, M_z *- projections of main moment of extertal forces on the axis Ox,Oy,Oz.*

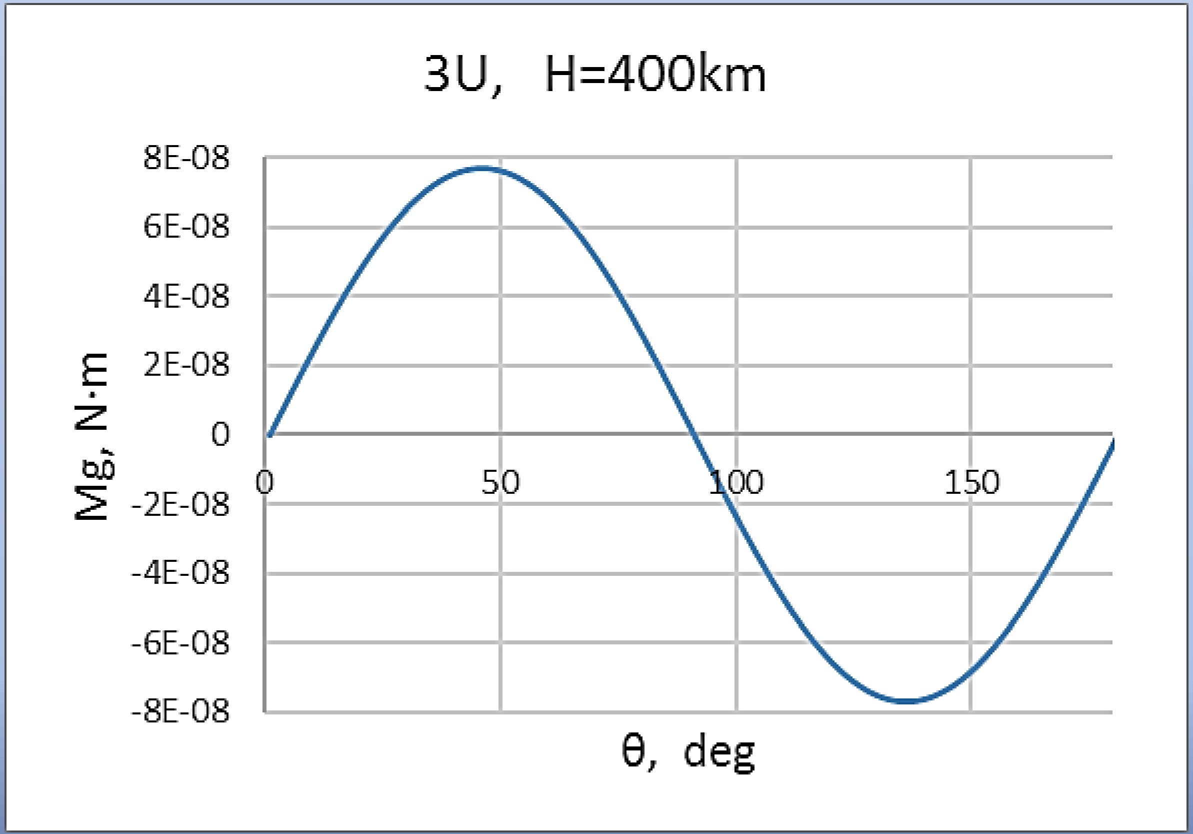
Attitude Dynamics



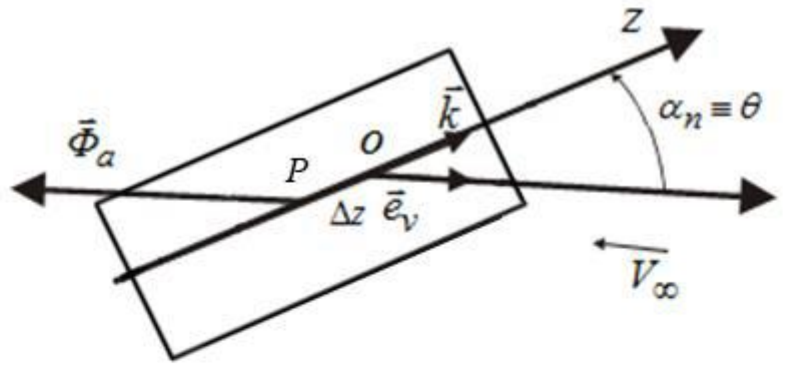
Attitude Dynamics

Gravity Gradient





Attitude Dynamics



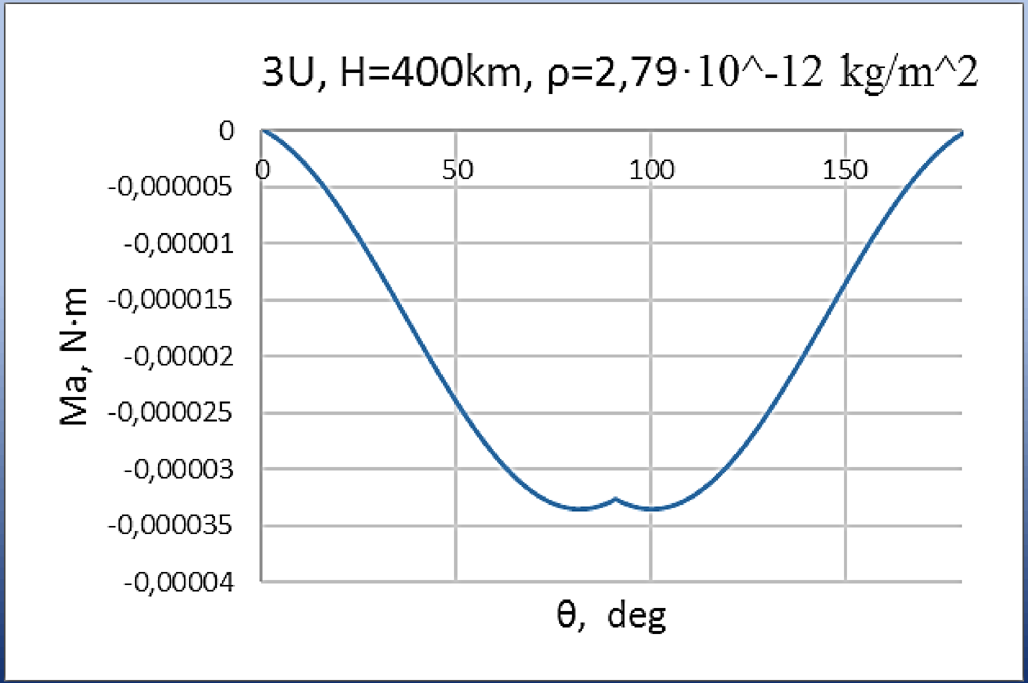
$$M_{x_a} = \frac{1}{2}\rho V^2 C(\alpha_n)\cos\varphi \sin\alpha_n ,$$

$$M_{y_a} = \frac{1}{2}\rho V^2 C(\alpha_n)\sin\varphi \sin\alpha_n ,$$

$$M_{z_a} = 0 .$$

$$\vec{M}_{OA} = \vec{r}_D \times \vec{\Phi}_a = \frac{1}{2}\rho V^2 C(\alpha_n)\vec{e}_v \times \vec{k} ,$$

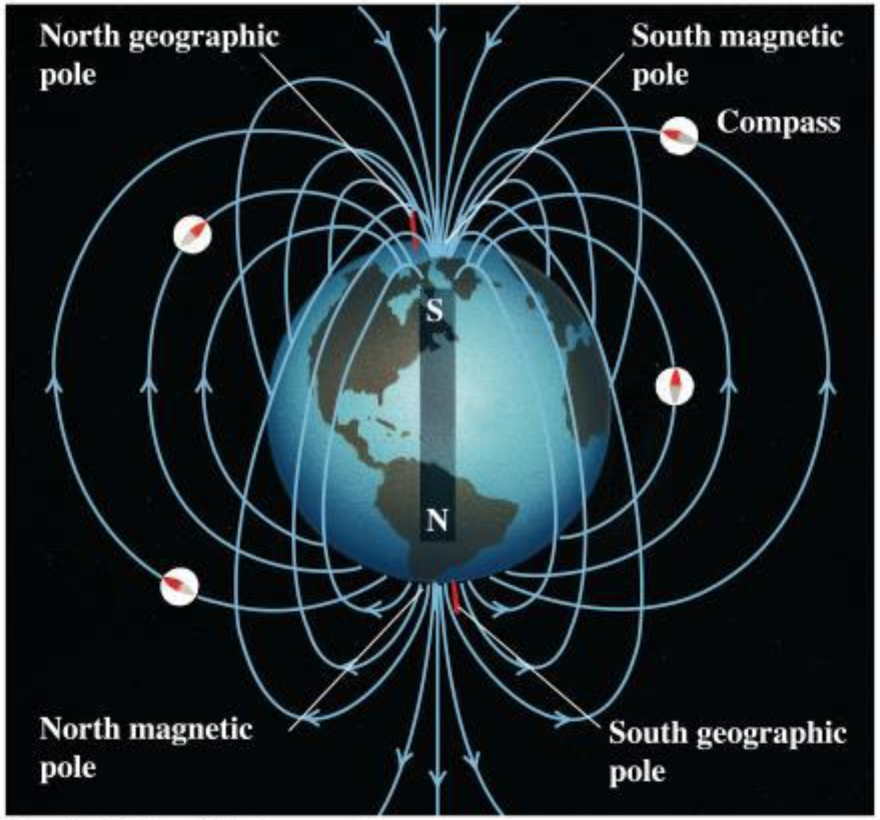
$$C(\alpha_n) = C_{xa} S(\alpha_n) \Delta z(\alpha_n)$$

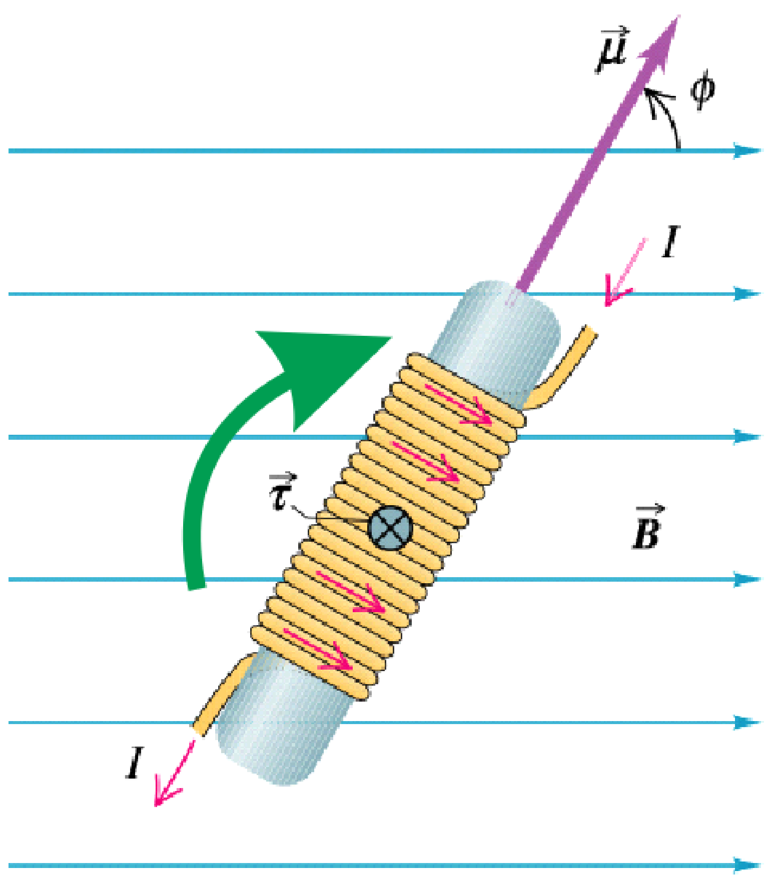


Attitude Dynamics

Magnetic moment

From the right-hand rule we see that the torque vector is directed into the page or screen. The torque tends to rotate the solenoid in a clockwise direction.





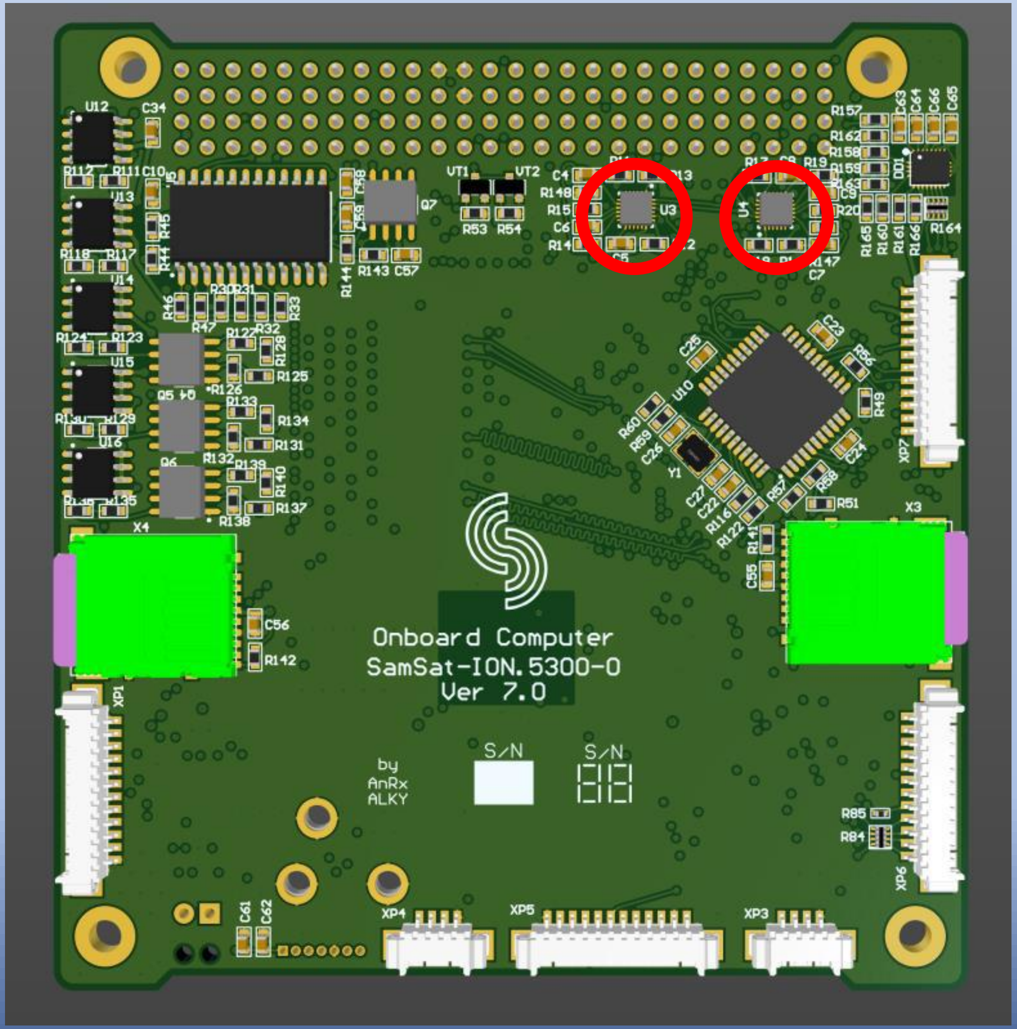
Hardware of ADCS. Attitude Sensors

Two main categories of attitude sensors

Vector sensors	Inertial Sensors
Sun Sensor	Gyroscope
Star Tracker	Accelerometer
Magnetometer	

1. NanoSSOC-D60 Digital Sun Sensor
2. MAI-SS Space Sextant
3. HMR2300R-4853-AXISMagnetometer
4. DSP-1750 Optical Sensor(gyro)

Hardware of ADCS. Attitude Sensors



Gyroscope

Range of measuring±250°/s

Sensitivity scale factor131LSB/(º/s)

digitally-programmable low-pass filter

Total RMS Noise 0.1º/s-rms

Rate noise spectral density0.01º/s/√Hz

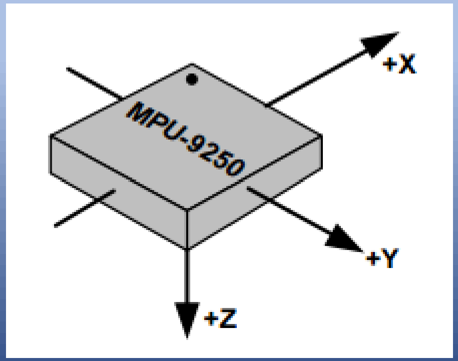
Zero shift of the gyroscope measurements has a nonlinear temperature characteristic.

Magnetometer

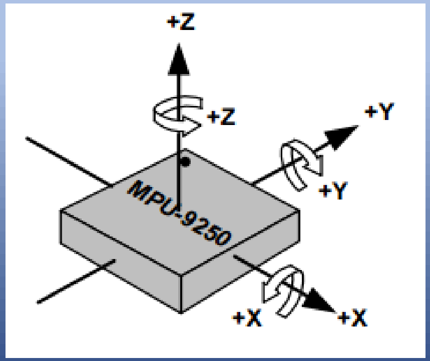
Range of measuring±4800μT

Sensitivity scale factor 0.6μT/LSB

MPU-9250coordinate system

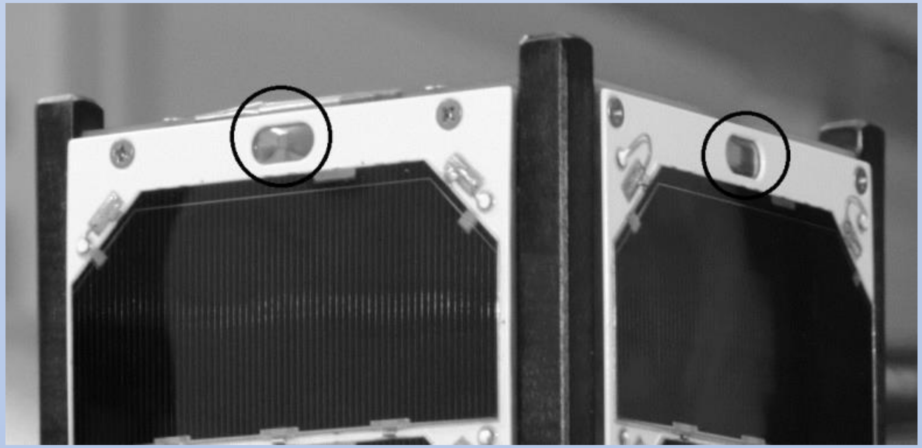


Magnetometer



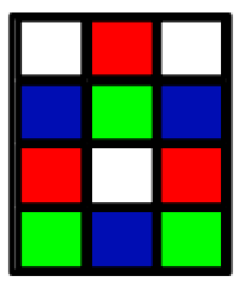
Accelerometer & Gyro

Hardware of ADCS. Attitude Sensors

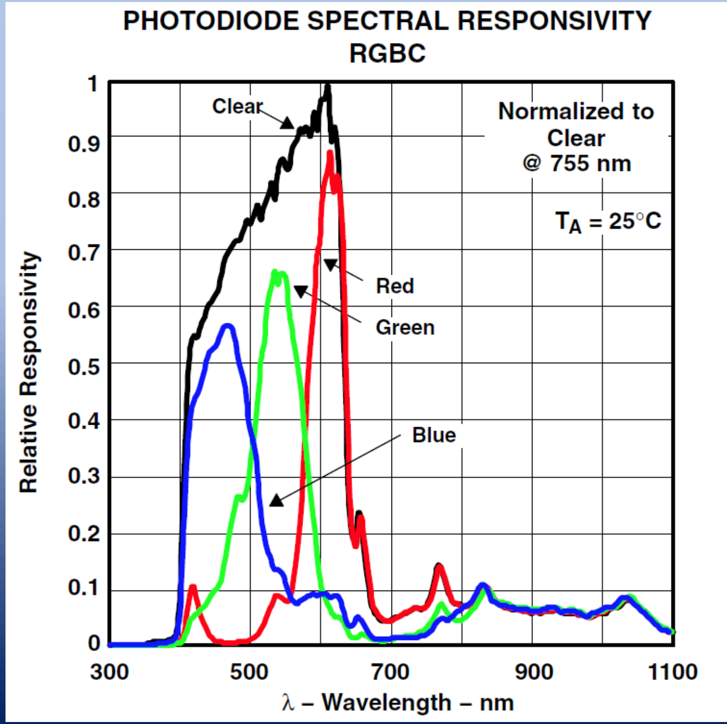


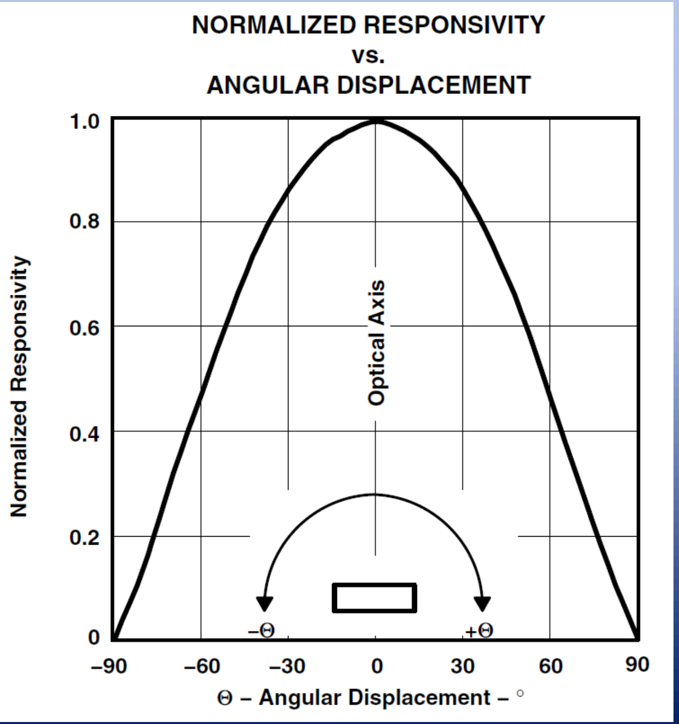
Location of light sensors on SamSat platform

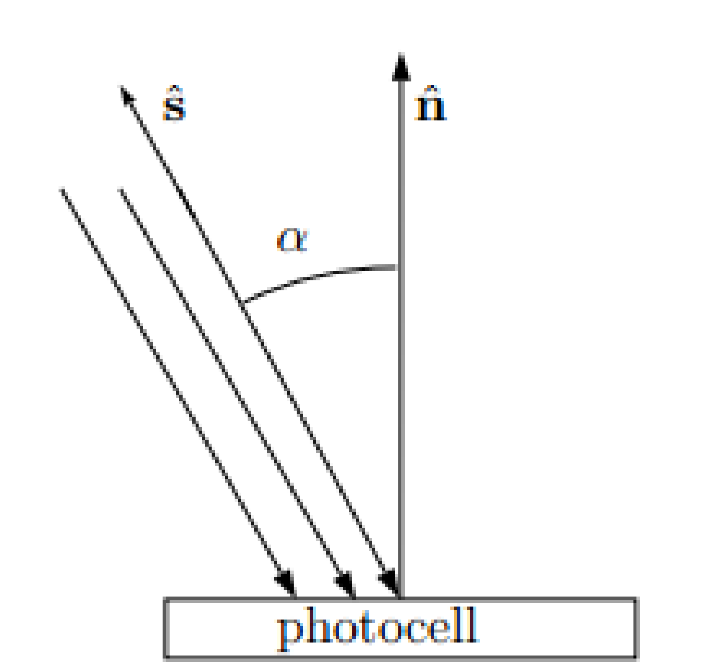
TCS34725Color (Sun) Sensor



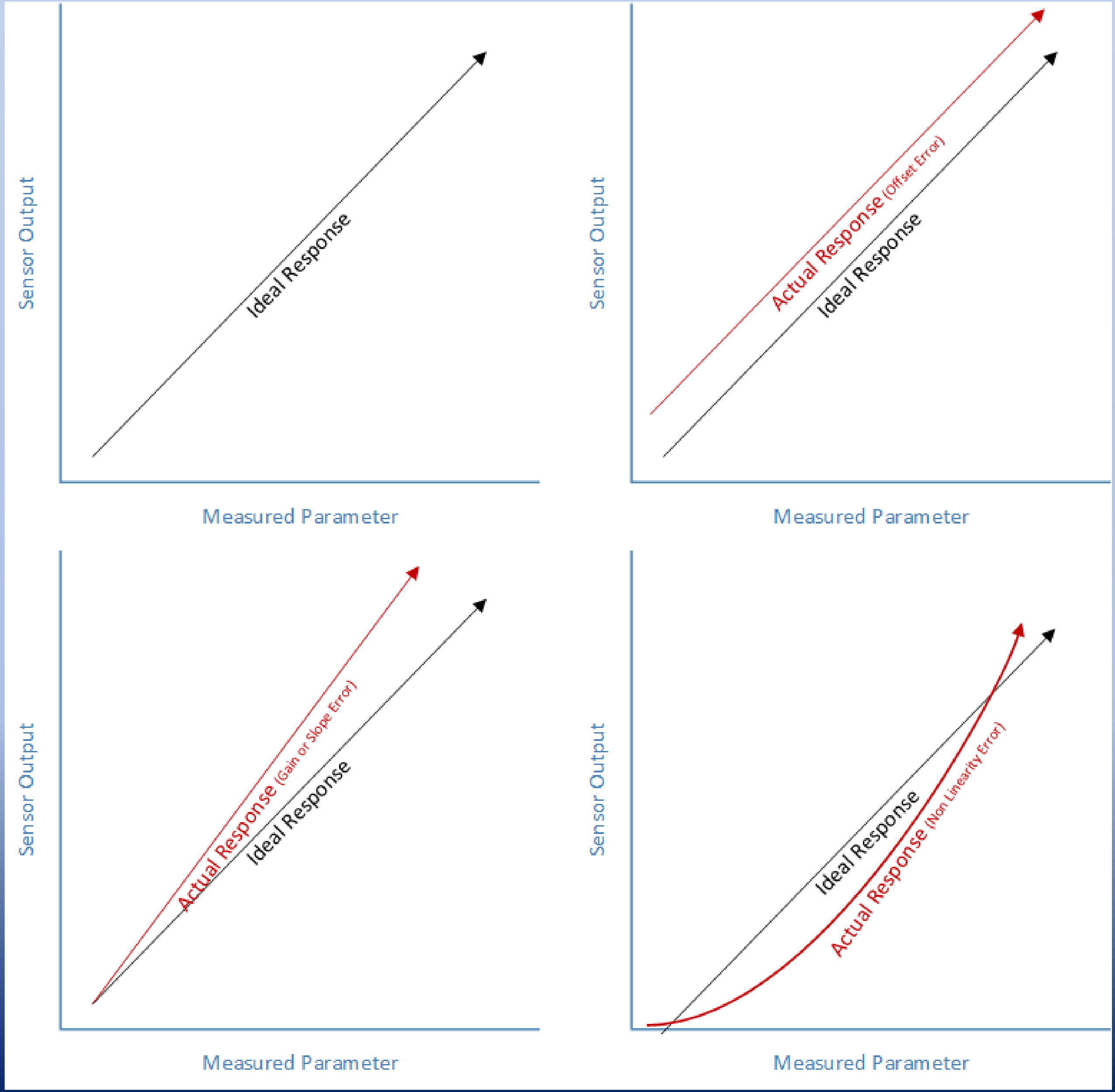
Red-filtered,
green-filtered,
blue-filtered,
and clear (unfiltered) photodiodes







Sensor Deviations

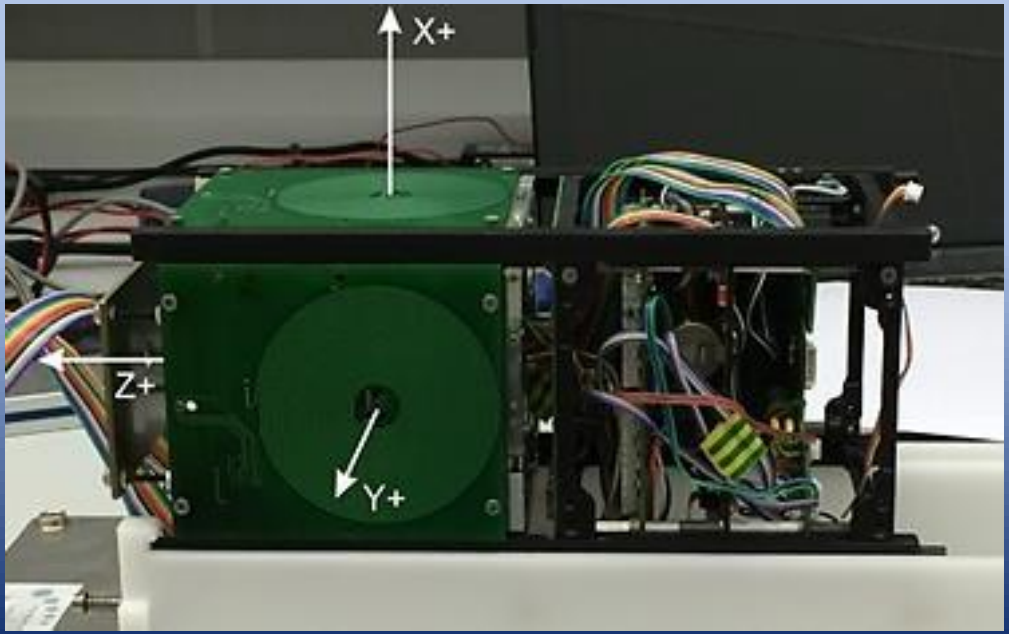


Types of errors:

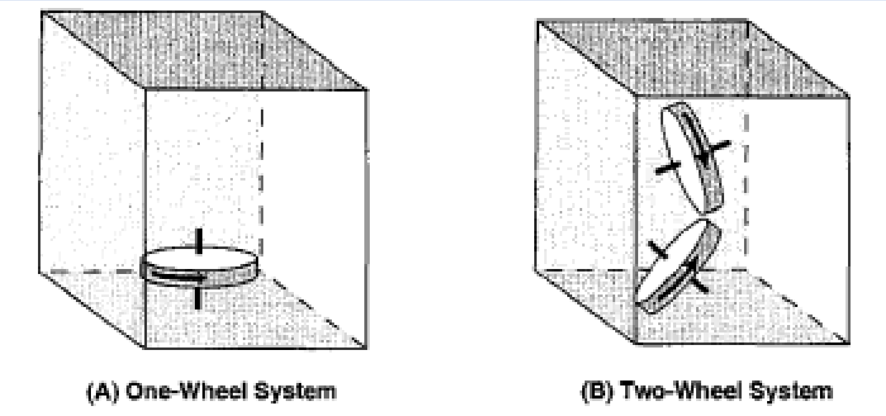
- Bias;
- Scale factor;
- Nonlinearity;
- Noise;
- Depending from temperature etc.

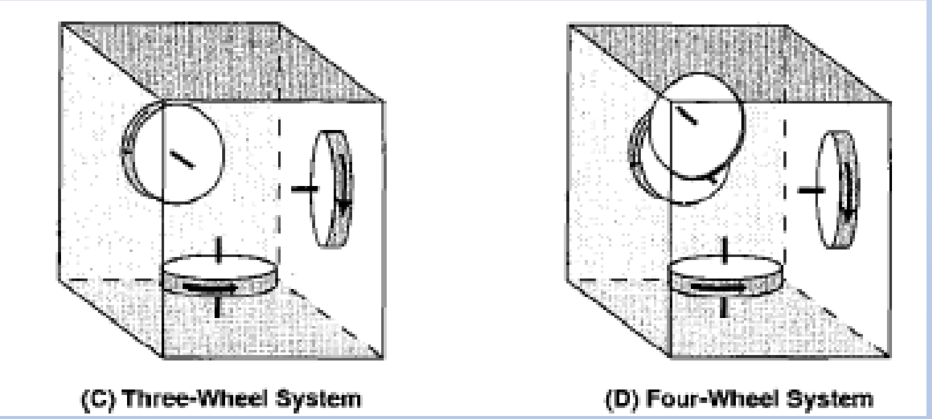
(a) Ideal Response;
(b) Actual Response (bias error);
(c) Actual Response (scale factor error);
(d) Actual Response (non linearity error).

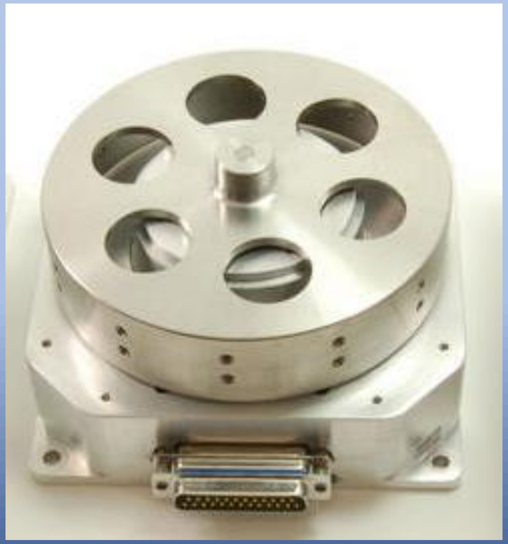
Hardware of ADCS. Attitude Sensors

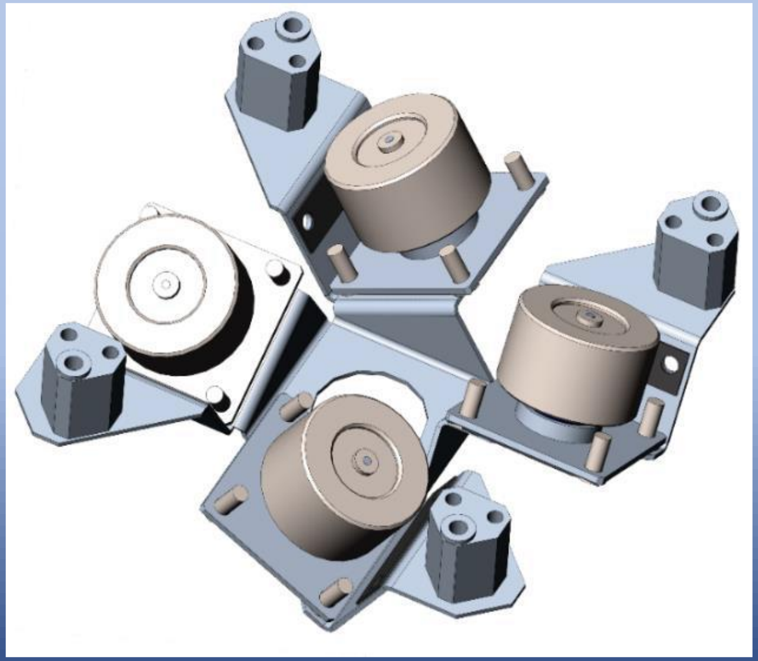


Hardware of ADCS. Attitude Sensors









Attitude Determination Algorithms. The Basic Idea

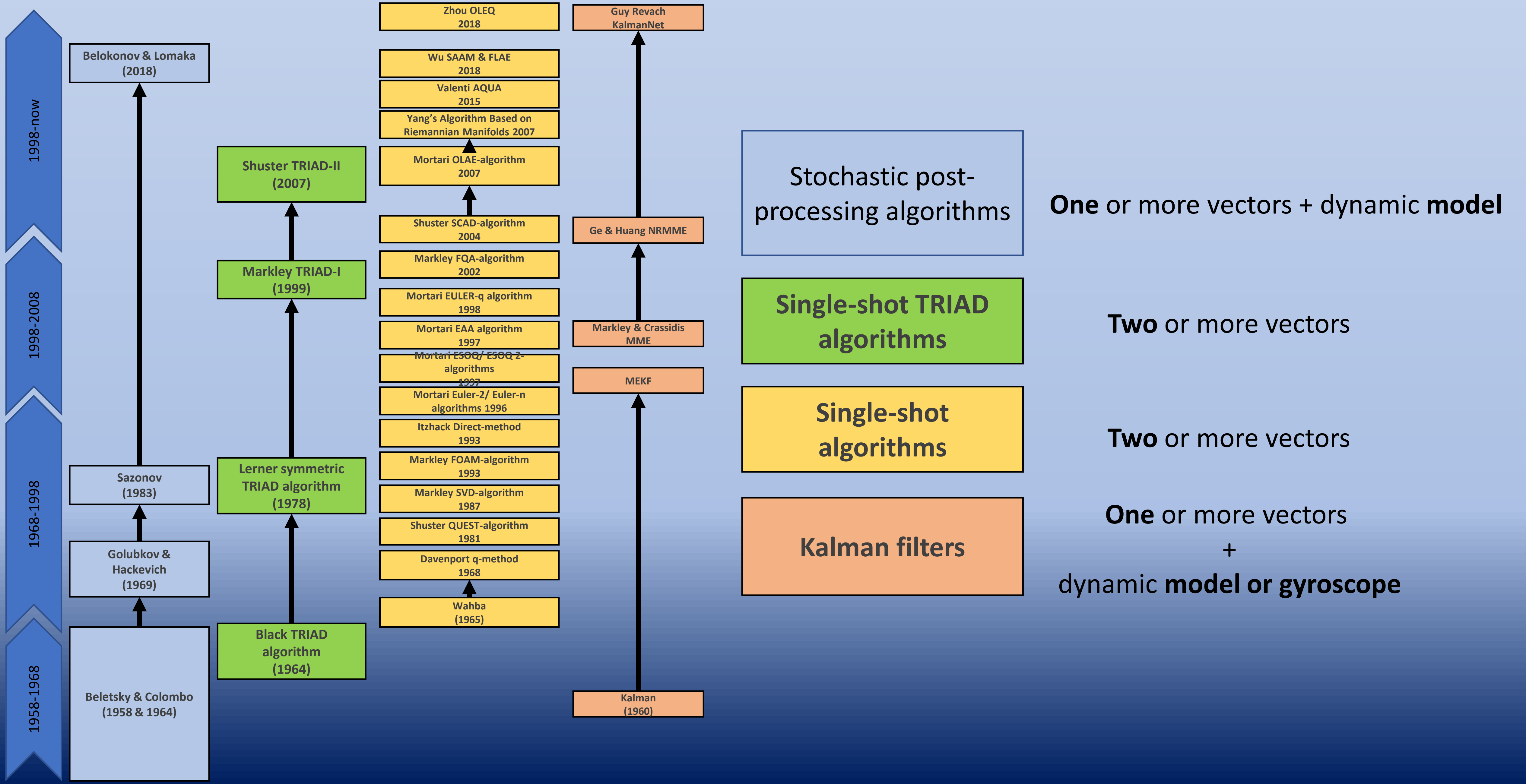
Attitude determination uses a combination of sensors and mathematical models to collect vector components in the body and inertial reference frames. These components are used in one of several different algorithms to determine the attitude, typically in the form of a quaternion, Euler angles, or a rotation matrix. It takes at least two vectors to estimate the attitude.

In general, the attitude determination solutions fall into two groups:

–**Deterministic (point-by-point)** solutions, where the attitude is found based on two or more vector observations from a single point in time,

–**Filters**, **recursive stochastic estimators** that statistically combine measurements from several sensors and often dynamic and/or kinematic models in order to achieve an estimate of the attitude.

Attitude Determination Algorithms. The Basic Idea



Attitude determination on single-shot measurements (Wahba problem)

Example

Measurements

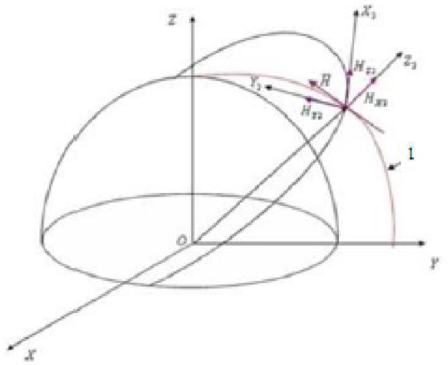
BFR

Earth magnetizing force

Current from solar battery panels

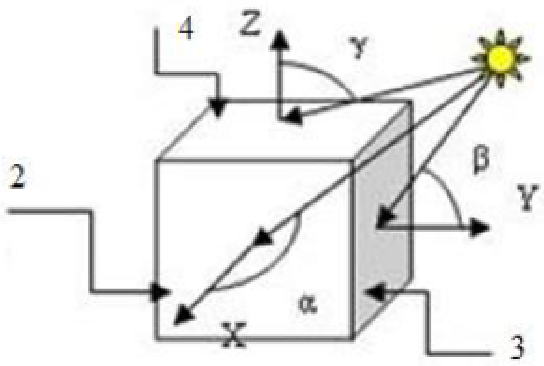
- measured Earth magnetic vector in the body frame of reference

- current in the body frame of reference



Algorithm QUEST

1 – power line of the Earth magnetic field;
2,3,4 – solar battery panels



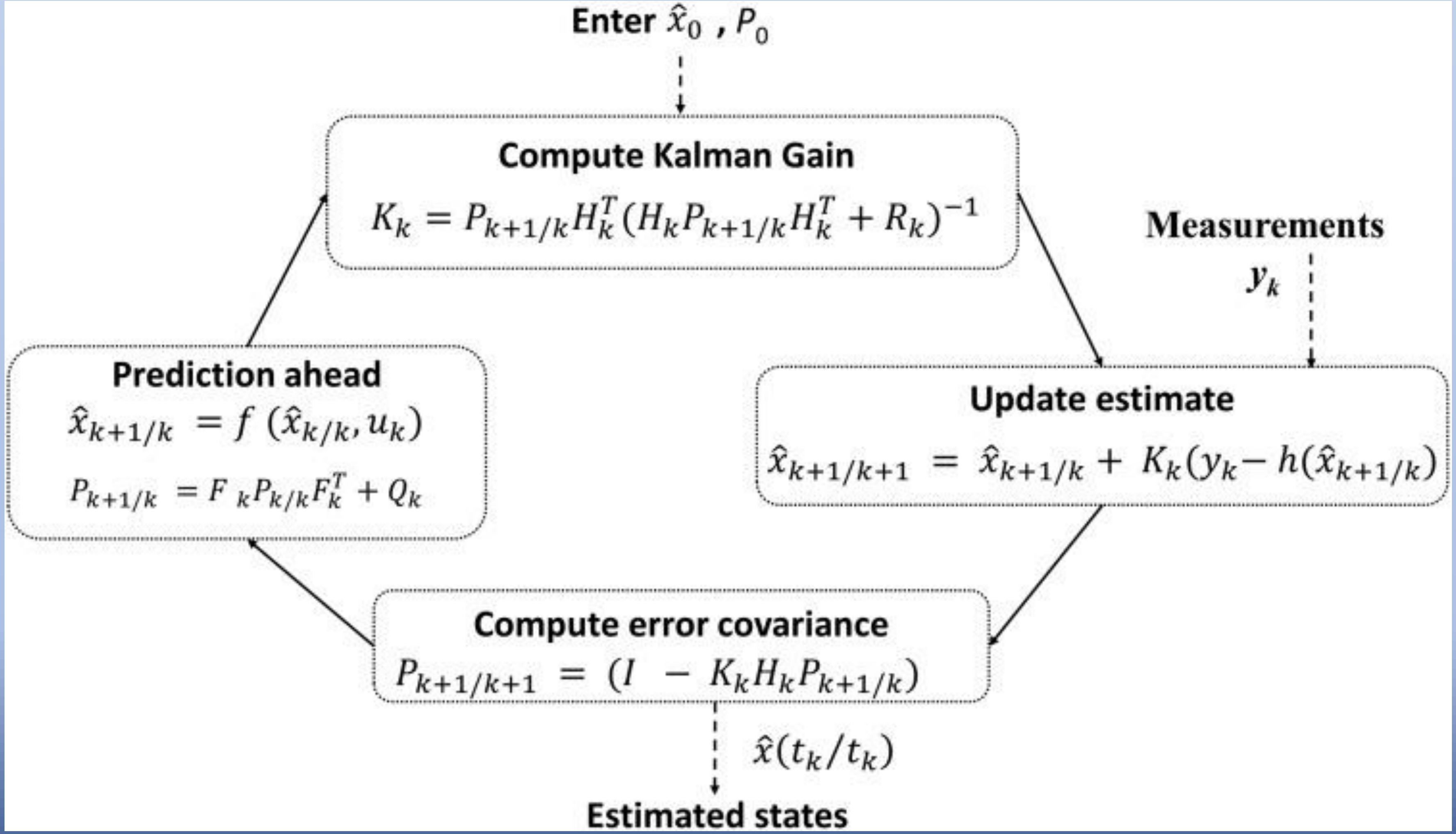
OFR

Model of the Earth magnetic field in the orbital frame of reference

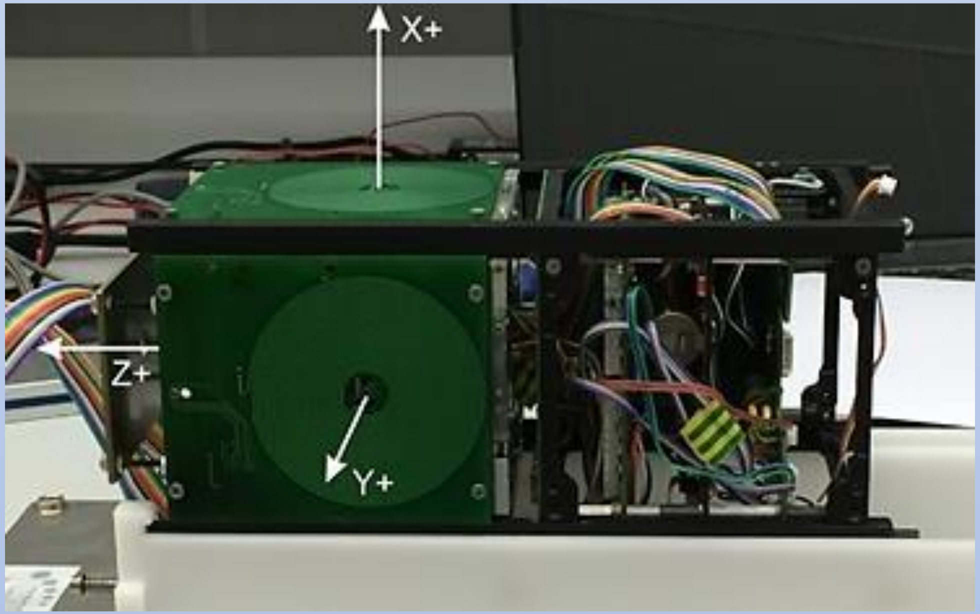
Model of the Sun motion in the orbital frame of reference

Models

The Kalman filter theory elements



Damping Control



B-dot method

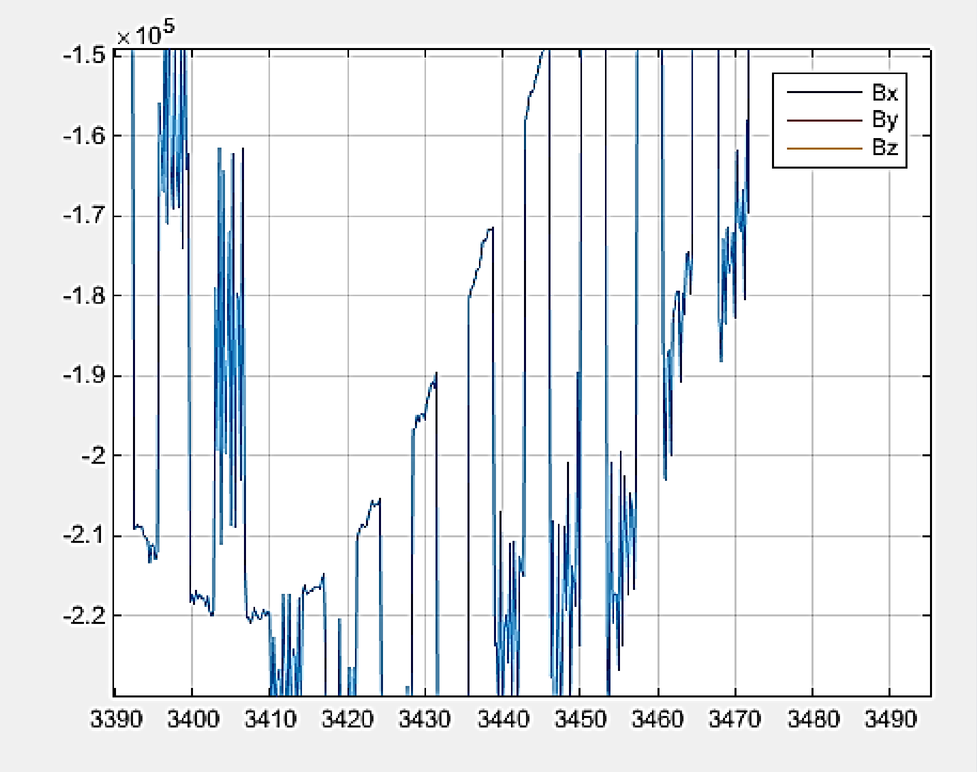
$$\bar{m} = -k\dot{\bar{B}}$$

$$\bar{m} = -JS\bar{n}$$

$$J\bar{n} = -\frac{k}{S}\dot{\bar{B}}$$

B-dot method has a low amount of calculation required and fast convergence speed.

B-dot method is severely affected by the magnetometer measurement noise.



Damping Control

Computing of Bdot phase:
Second-degree polynomial

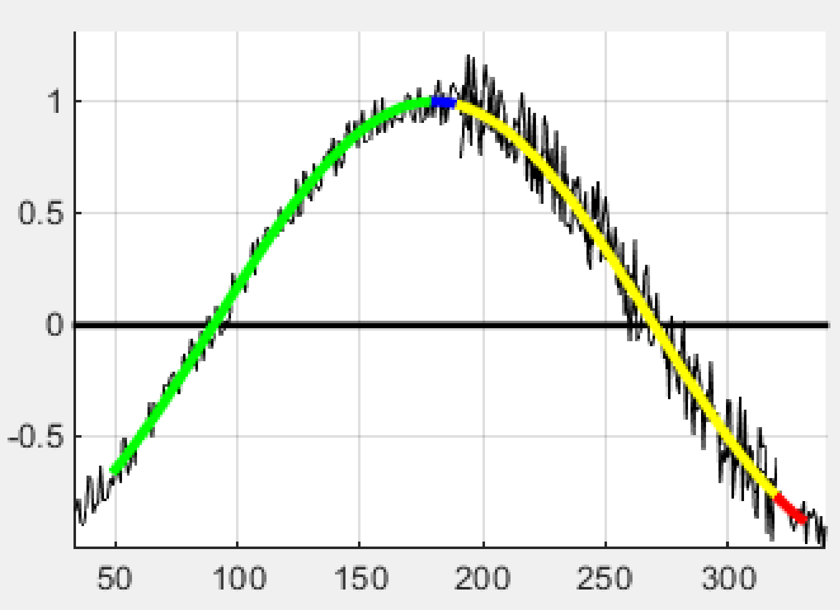
$$B(x) = a_0 + a_1 x + a_2 x^2$$

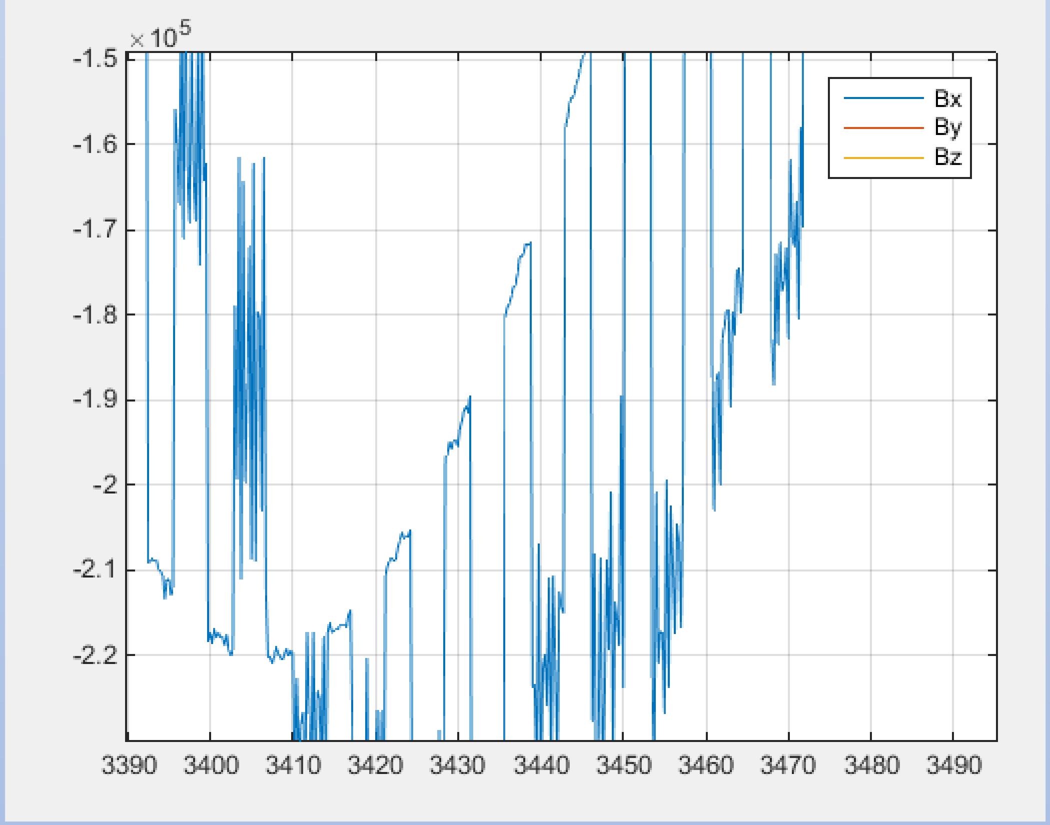
Derivative at the point

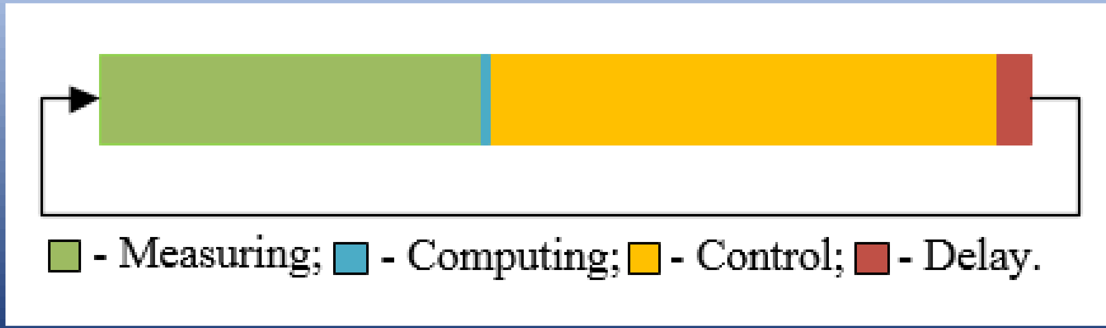
$$\dot{B}(x) = a_1 + 2a_2 x$$

Required control current

$$J = -k^* \cdot (a_1 + 2a_2 x_n)$$

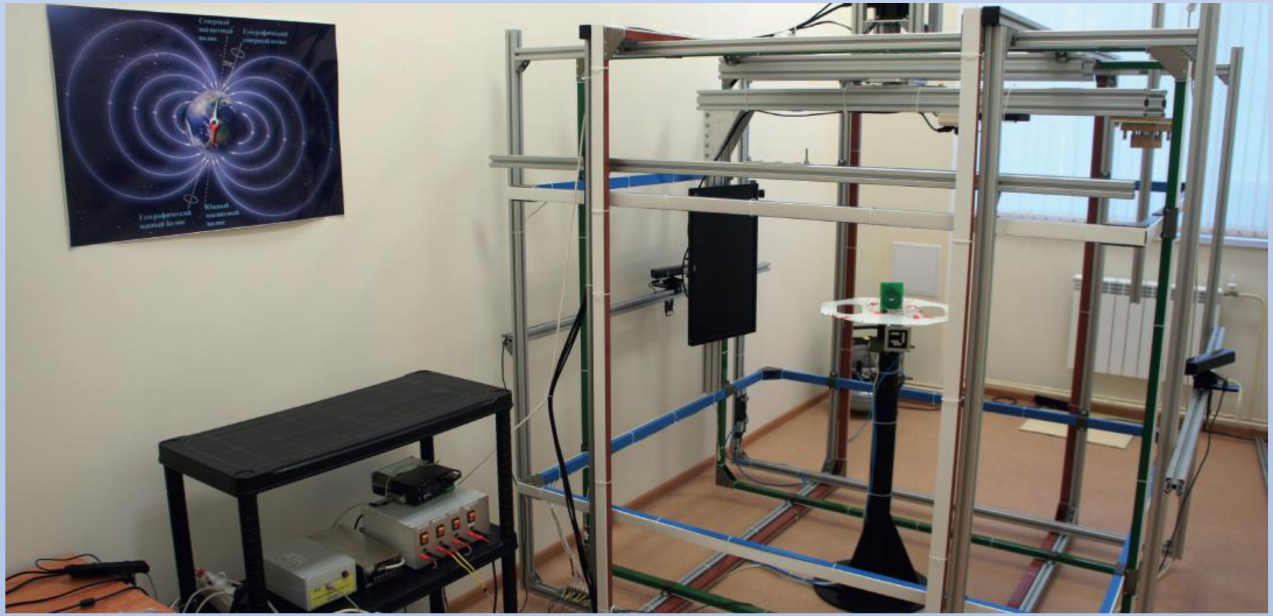


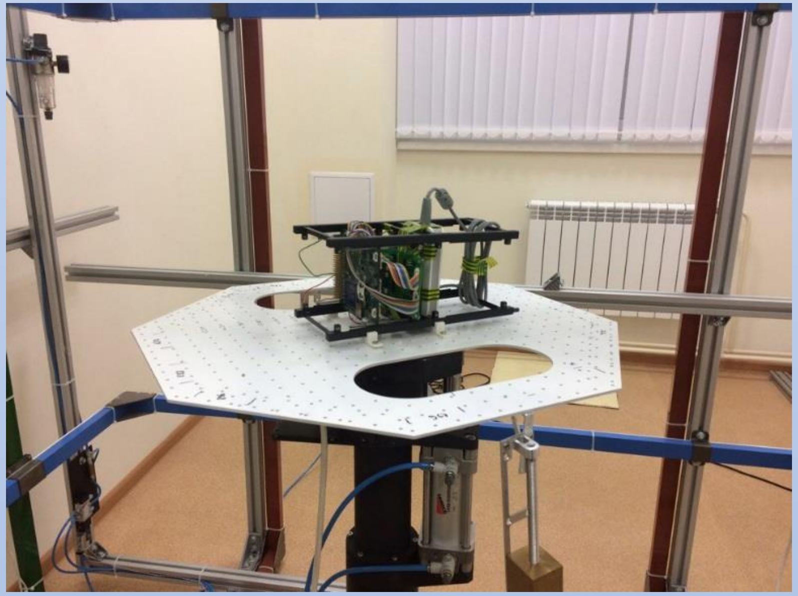


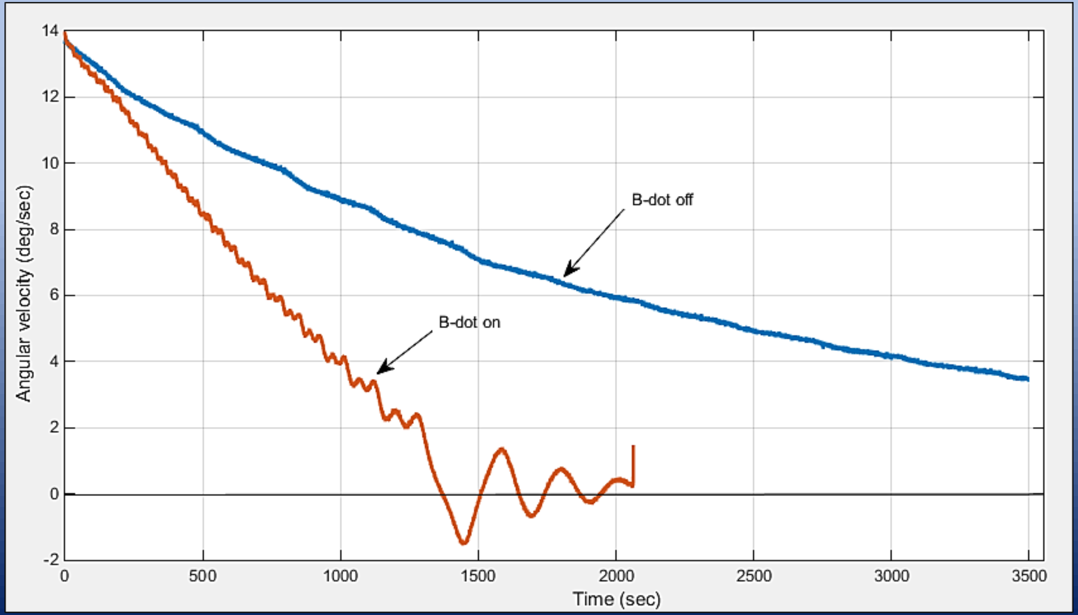


Algorithm work cycle

Damping Control







1. The Laboratory of the Nanosatellite Motion Control System Testing
2. The engineering model of the satellite mounted on the rotating platform (B=250nT)
3. Plots of the angular velocities of the engineering model for the cases:(blue) no damping;(red) damping is performed

Thanks for attention!