2021, SAMARA

«Tether System Technologies for Space Applications»

Zabolotnov Yuriy M.

Yumz @yandex.ru

MAIN DIRECTION OF APPLICATION SPACE TETHER SYSTEMS (STS)

The words of the academician S.P.Korolev:

"Cosmonautics has an unlimited future, its prospects are infinite like the Universe"

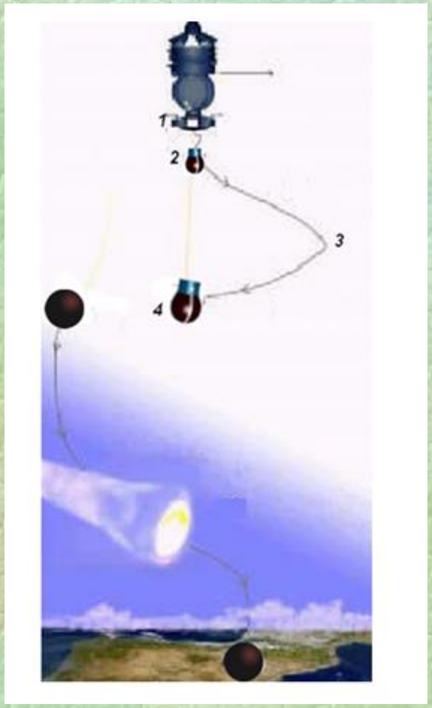
1. Transport operations in space
   1.1 Descent of payloads to the surface of the Earth and planets with minimal costs
   1.2 Launch of small spacecraft and nano-satellites from the base spacecraft to other orbits.
   1.3 Moving cargo between spacecraft and space stations.
   1.4 Space lift between spacecraft and planets, Moon, etc.
2. Long-range measuring systems and surveillance systems
   2.1 Monitoring systems for gravitational, magnetic fields, etc.
   2.2 Space observatories
   2.3 Long range interferometers
   2.4 Atmospheric monitoring systems
   2.5 Surface monitoring systems for planets with higher resolution
   2.6 Long geostationary systems

MAIN DIRECTION OF APPLICATION SPACE TETHER SYSTEMS (STS)

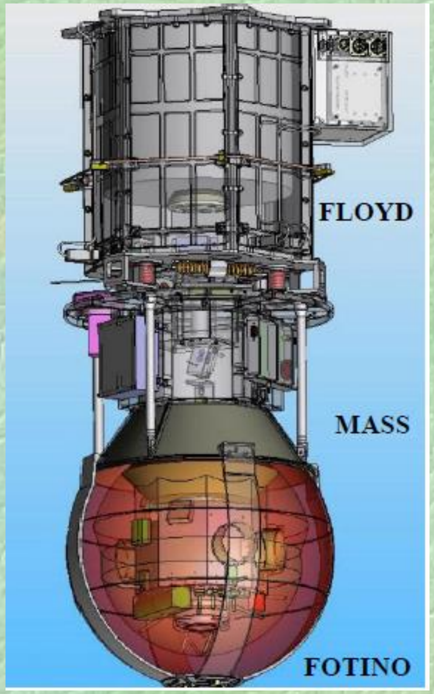
3. Rotating STS
    - 3.1 Interplanetary flights with rotating STS
    - 3.2 Creation of artificial gravity in interplanetary flights
4. Collection and disposal of space debris
    - 4.1 Space Tugboats
    - 4.2 Aerodynamic tether systems
    - 4.3 Network technologies
    - 4.4 Robotic technologies
5. Electrodynamic tether systems (EDTS)
    - 5.1 Changes in the orbital parameters of satellites and space stations
    - 5.2. Utilization of nano-satellites
6. Stabilization of motion of spacecraft and space stations
    - 6.1 Gravitational stabilization
    - 6.2. Aerodynamic stabilization
    - 6.3 Magnetic Stabilization

TETHER EXPERIMENT YES -2 (Young Engineers Satellite)

Delivery of payloads to the Earth without use of engines



The stages of the experiment



Equipment composition

The equipment consists of following parts:

1. **Module "Floyd" where the control mechanism is located**
2. **Module "Mass" where communication devices and the navigation equipment is located**
3. **Descent capsule "FOTINO"**

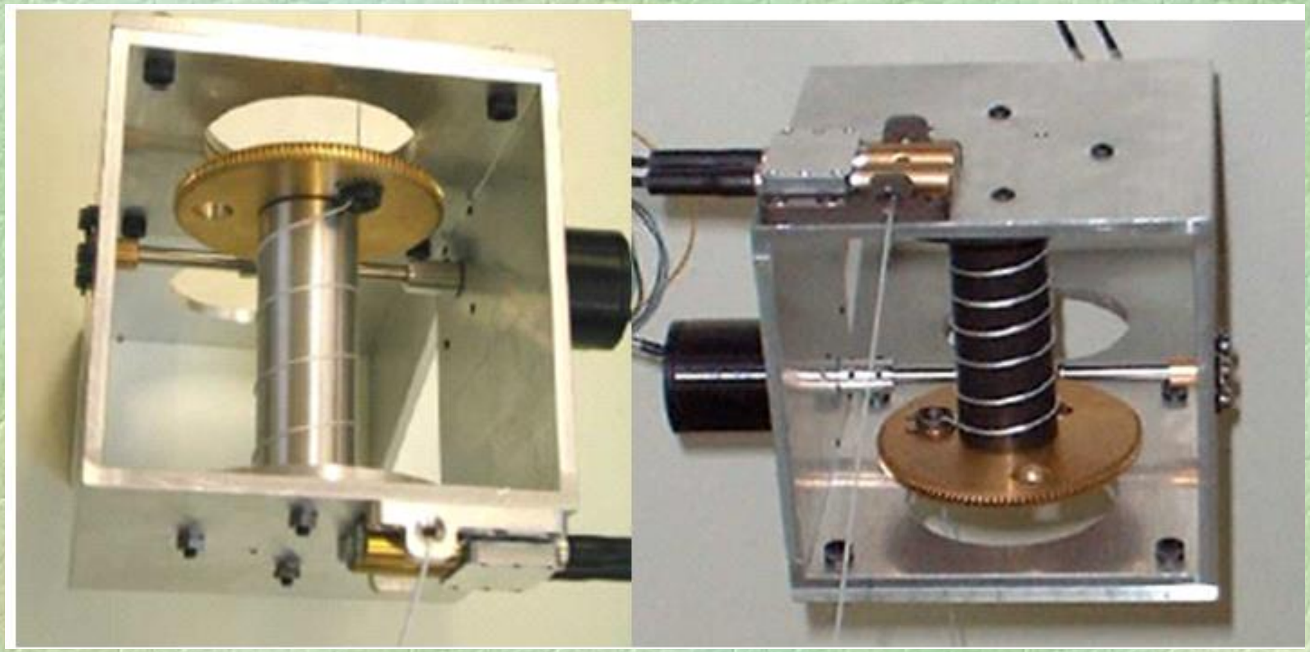
Deployment process consists of following stages:

1. Branch “Fotino+Mass” downwards in a direction of a local vertical. Movement in a vicinity of the base space vehicle. Exit on a local vertical and stabilization (6000 sec).
2. Deviation on the maximum angle from a local vertical in a direction of orbital movement of the base space vehicle (2300 sec)
3. Passive movements. Trimming of a tether and the block "Mass" at passage of a local vertical (3200 sec)
4. Free movement and re-entry “Fotino” (1700 sec).

Duration of experiment : 220 Minutes

Cycle time on an orbit of the base space vehicle: 90 Minutes

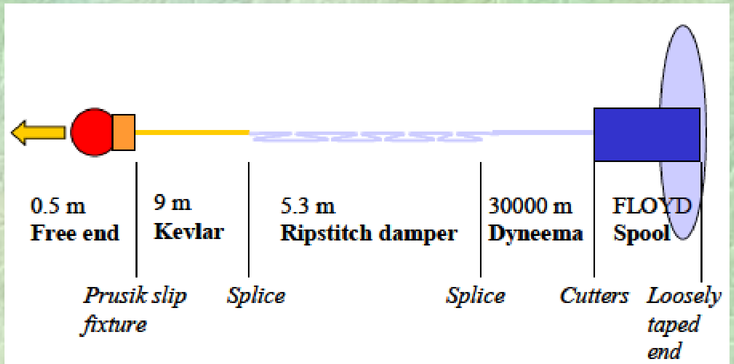
Control mechanism



Composition:
1) the braking cylinder, 2) the control disk, 3) optical sensors for measurement of length and speed of a tether

The tether from the coil passes through the control disk which regulates quantity of orbits on the braking cylinder.

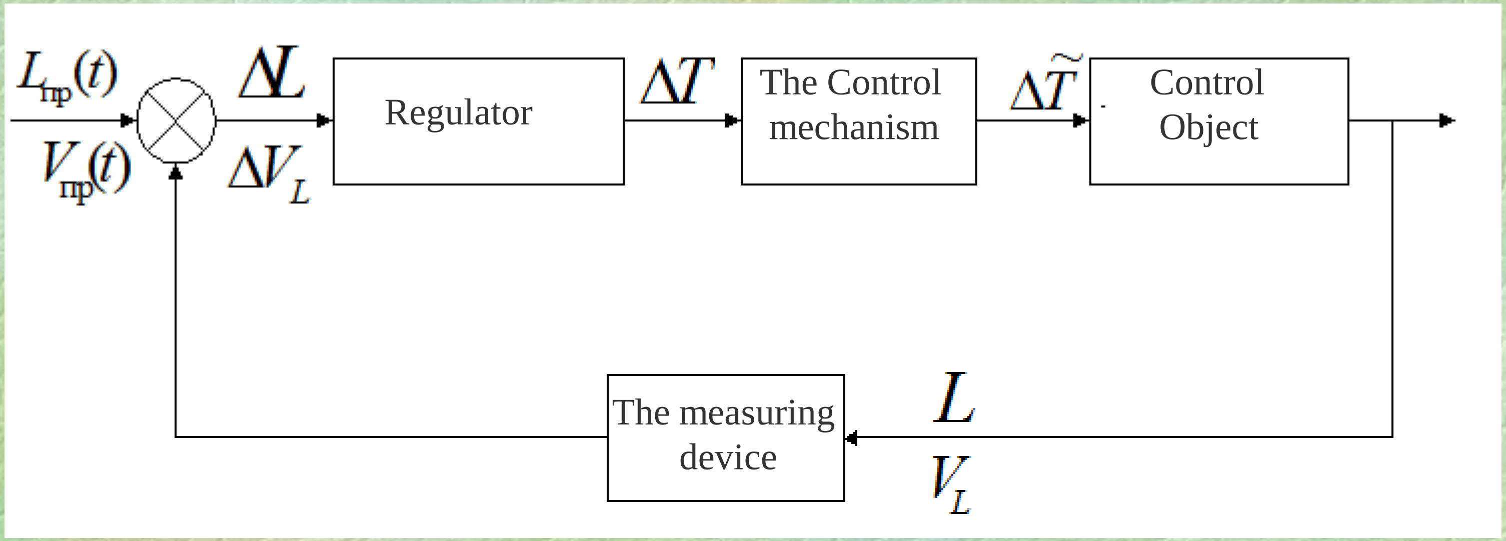
Sections of the tether



The tether consists of several section:

1. **Synthetic material Kelvar**
2. **Damper**
3. **Main section - dyneema material**

Principal scheme of regulation system



The basic units of system:

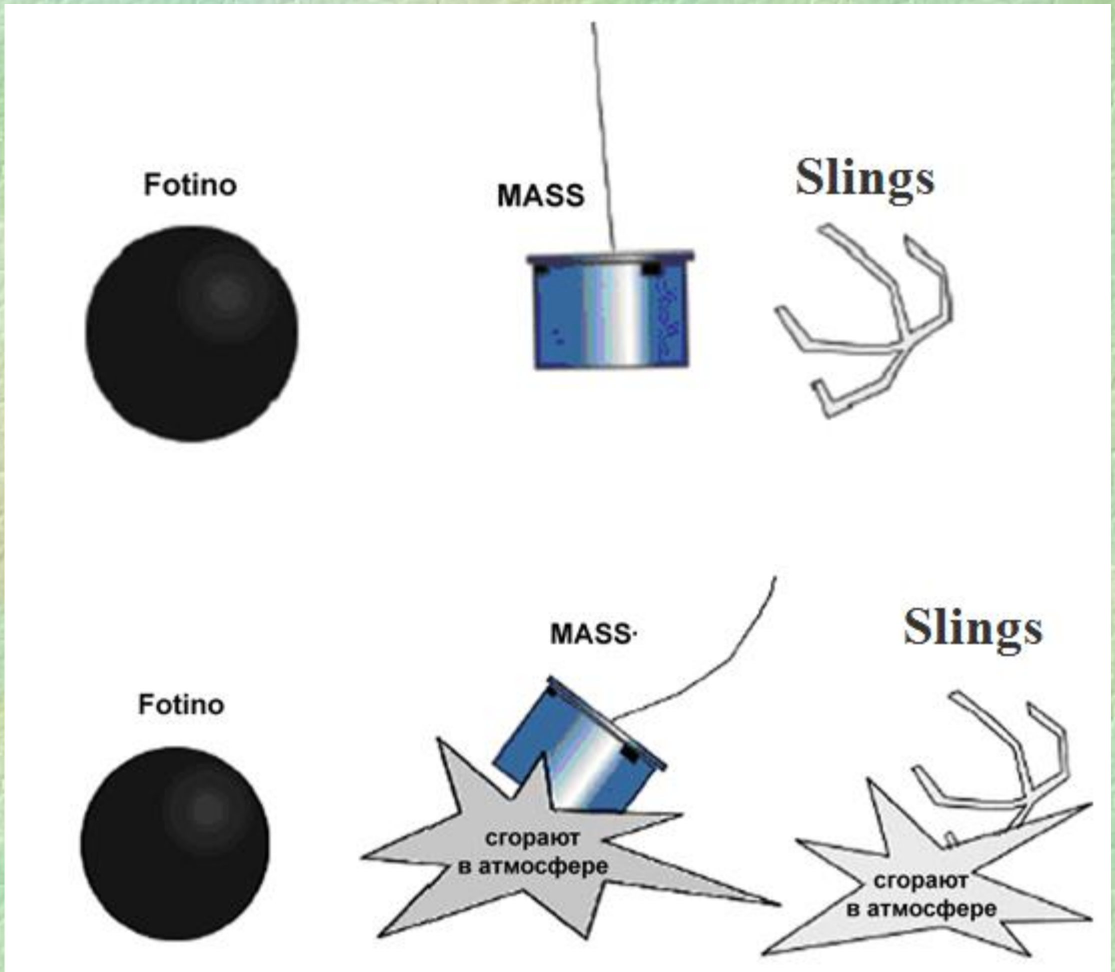
1. **A control mechanism**
2. **The measuring device**
3. **The regulator or controller**

You can use two types of control mechanism:

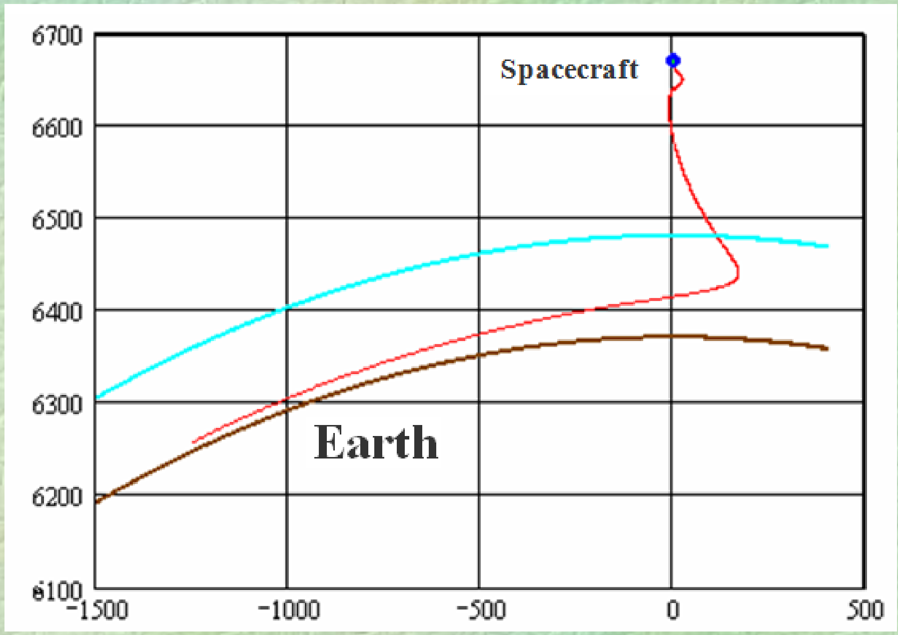
1. **The mechanism in which the tether can back be retracted**
2. **The mechanism in which the tether cannot back be retracted**

Separation of the capsule “Fotino" and block “MASS"

Block “MASS” and slings burn up in the atmosphere



The trajectory of the descent capsule to Earth

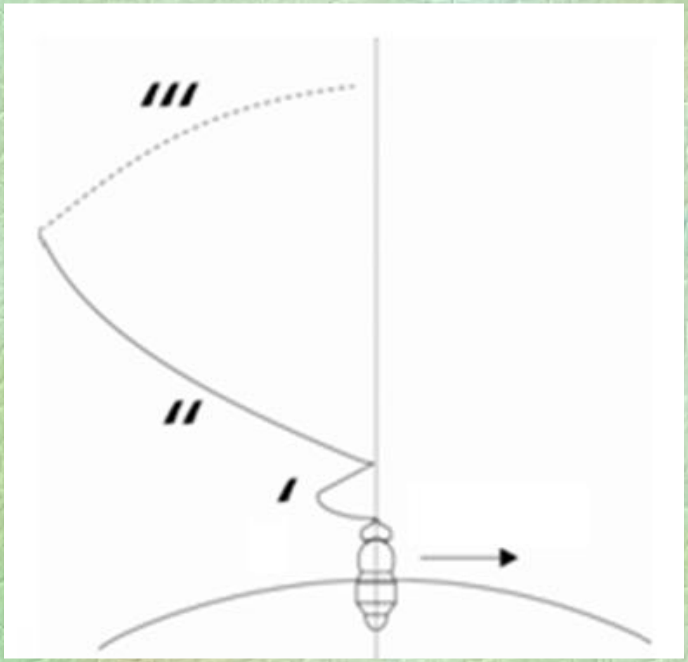


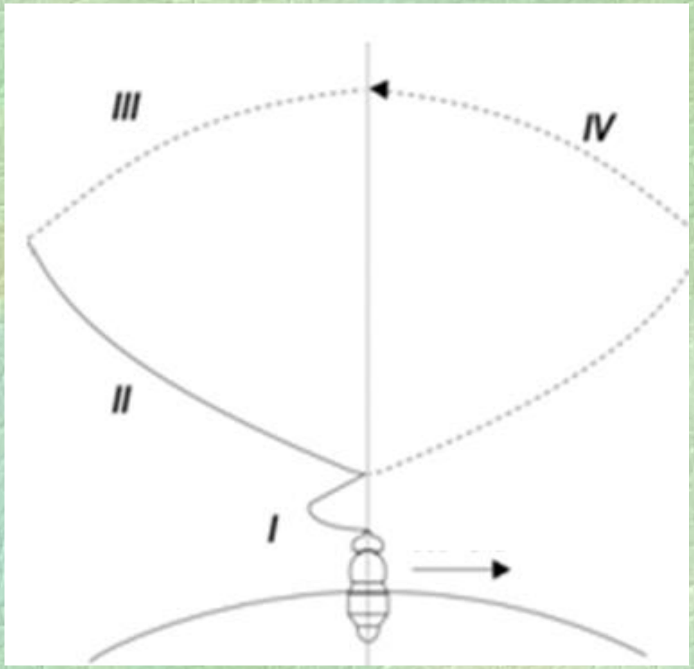
The calculated trajectory of the capsule concerning the base space vehicle.
The capsule makes atmospheric entry.
Conditional border of atmosphere (approximately hundred kilometers).
The basic space vehicle moves to the right.

Start small spacecraft into a higher orbit

Two possible schemes

.



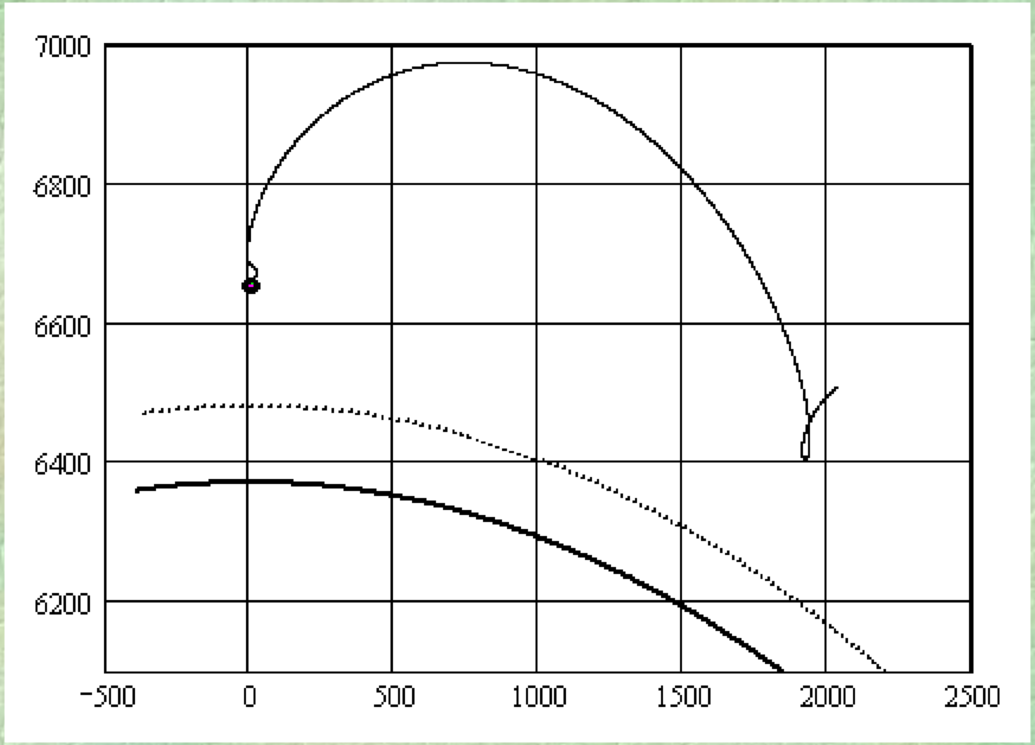


1. The scheme of launching a small satellite on an elliptic orbit (on the left)
2. The scheme of launching a small satellite into a circular orbit (right)

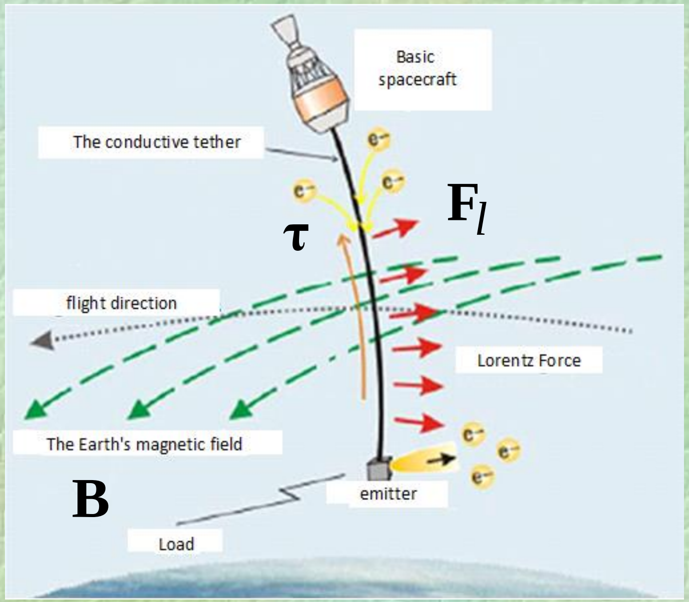
Schemes differ in that the cutoff of the tether occurs at different times.

In the scheme on the left, the velocities of the base spacecraft and the small spacecraft are summarized. In the scheme on the right – subtracted.

Trajectories of the small spacecraft relative to the base spacecraft



Electrodynamics tether systems (EDTS)



Recently, a very promising application of the electrodynamic tether systems. More than ten real tether experiments have already been conducted in different countries of the world.

In the interaction of the conducting tether with the magnetic field of the Earth, the Lorentz force arises, with which you can control the movement of the system

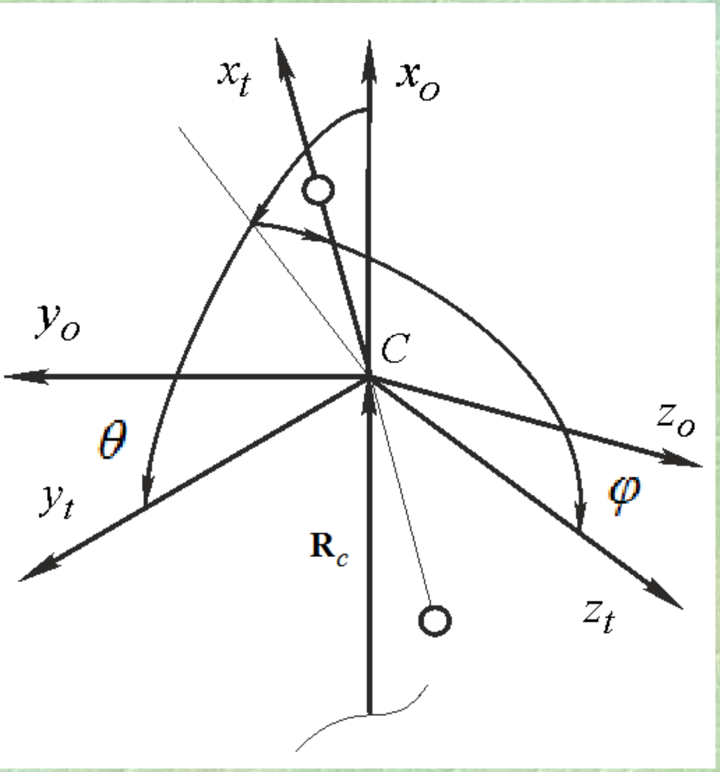
$$\mathbf{F}_l = I\,\boldsymbol{\tau} \times \mathbf{B}$$

Where $\mathbf{B}$ **- magnetic induction vector,** I **- current**

$\boldsymbol{\tau}$ **- the unit vector defines the direction of the current**

Electrodynamics tether systems

Coordinate Systems



Equations of motion around the center of mass of the system

$$\ddot{r} - r\left[\dot{\varphi}^2 + \left(\dot{\theta} + \omega\right)^2 \cos^2\varphi + \nu^{-1}\omega^2\left(3\cos^2\theta\cos^2\varphi - 1\right)\right] = Q_1 / m_e$$

$$\ddot{\theta} + \dot{\omega} + 2\left(\dot{\theta} + \omega\right)\left(\dot{r}/r - \dot{\varphi}\,\mathrm{tg}\varphi\right) + 1.5\nu^{-1}\omega^2\sin 2\theta = Q_2 / m_e r^2\cos^2\varphi$$

$$\ddot{\varphi} + 2\dot{\varphi}\dot{r}/r + \left[0.5\left(\dot{\theta} + \omega\right)^2 + 1.5\nu^{-1}\omega^2\cos^2\theta\right]\sin 2\varphi = Q_3 / m_e r^2$$

r – distance between space vehicles

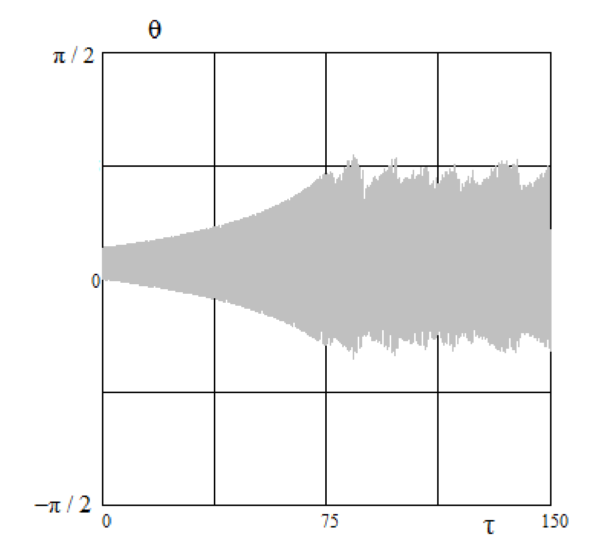
$Q_{1,2,3}$ – generalized forces

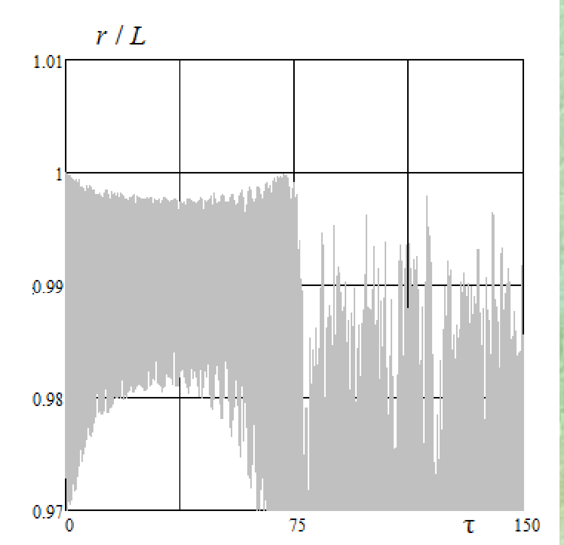
θ, φ – angles of deviation from the vertical

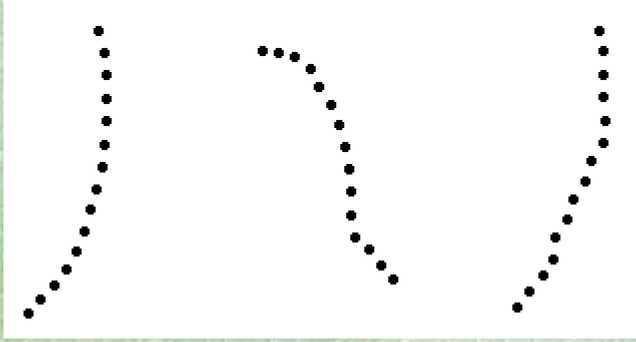
Electrodynamics tether systems

Electrodynamic tether systems (EDTS) can be used in active and passive modes of operation. The active mode is used to decelerate the satellite and to raise the altitude of the orbit. Passive mode is only used for braking.

Problem: with a constant current motion of the system is unstable.







The angle of deviation of the tether from the vertical increases and the tether weakens (not stretched)
The necessary algorithms to stabilize the system motion

Electrodynamics tether systems

The equilibrium position of the system is determined from the conditions:

$$\ddot{r} = \ddot{\varphi} = \ddot{\theta} = \dot{\theta} = \dot{r} = \dot{\varphi} = 0$$

Equilibrium position:

$$\theta_1 = \frac{1}{2}\arcsin(\sigma), \quad \theta_2 = \theta_1 + \pi, \quad r_k = L\gamma_t \frac{\sin\psi_k}{\psi_k}, \quad \psi_k = \operatorname{arctg}\left(\frac{\mu_m |I_n|}{6\mu\, m_e \cos^2\theta_k}\right), \quad k = 1, 2$$

Linearized system with respect to equilibrium positions:

$$dy / d\tau = Ay$$

Where A **- the matrix of partial derivatives (the Jacobian)**

For stabilization, it is necessary $I = I_n + \Delta I$

Where $u = \Delta I$ **- stabilizing control**

Electrodynamics tether systems

Linearized system with control:

$$dy / d\tau = Ay + Mu$$

The method of dynamic Bellman programming with quadratic optimality criterion is used for control synthesis:

$$J = \int_0^\infty y^T Dy + hu^2 dt$$

Where $\sum D_k + h = 1$ **- sum of weight coefficients**

Optimal control:

$$u = q^T y$$

Where $q = -BM / h$

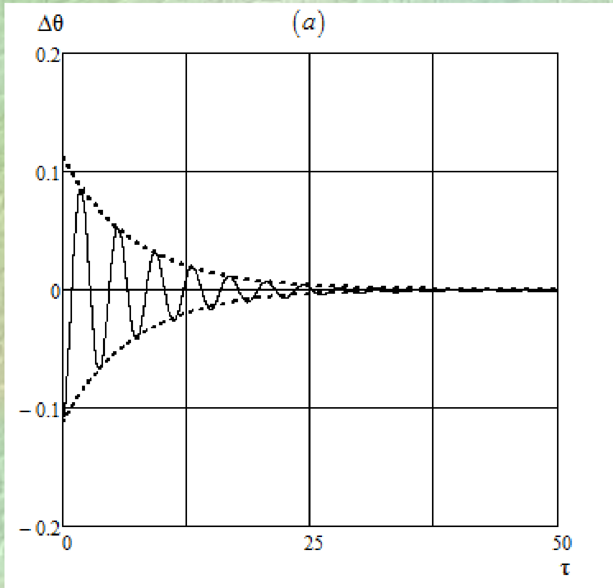
The matrix is determined from the Riccati equation:

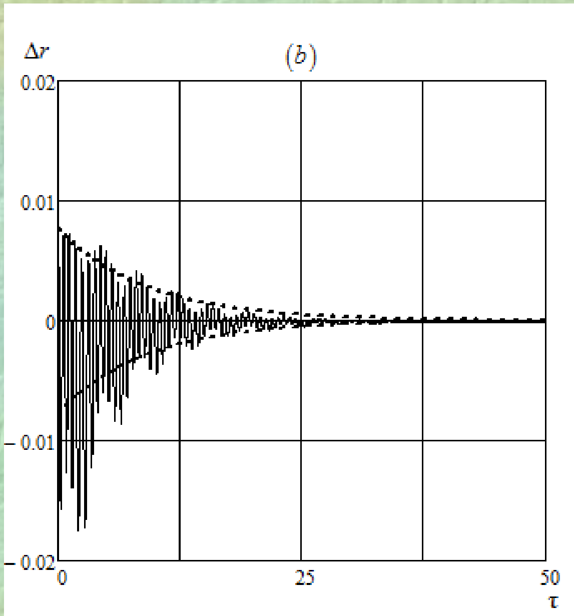
$$BA + A^T B + D - BMM^T B = 0$$

Electrodynamics tether systems

Nonlinear system with control (Numerical results)

For the equatorial orbit, the transients are considered





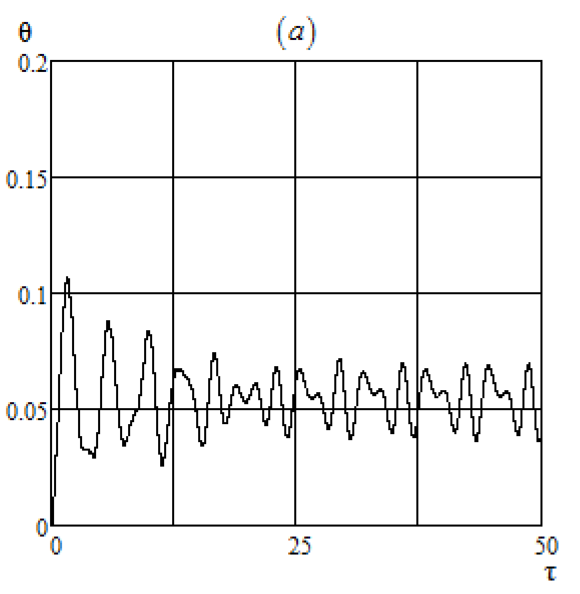
The oscillation amplitudes are determined by the averaging method (analytical solution)

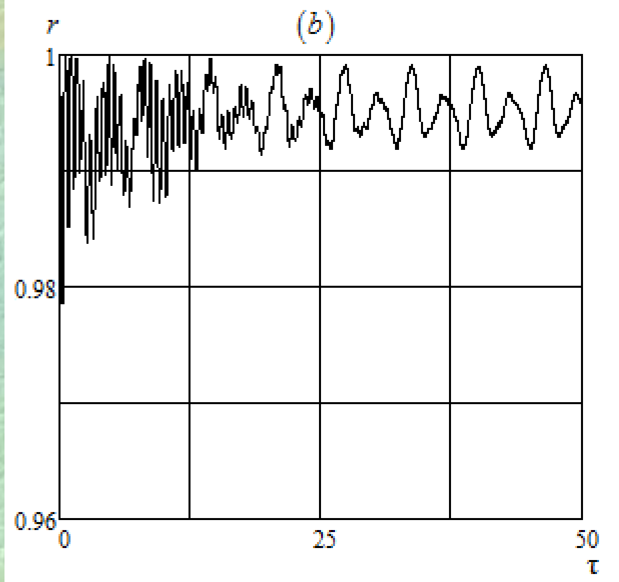
$$K_k(\tau) = K_k(0)e^{\eta_k \tau}, \qquad k = 1,2$$

Electrodynamics tether systems

Nonlinear system with control (Numerical results)

For orbits with inclination $i = \pi / 3$ **:**





Periodic perturbations act on inclined orbits

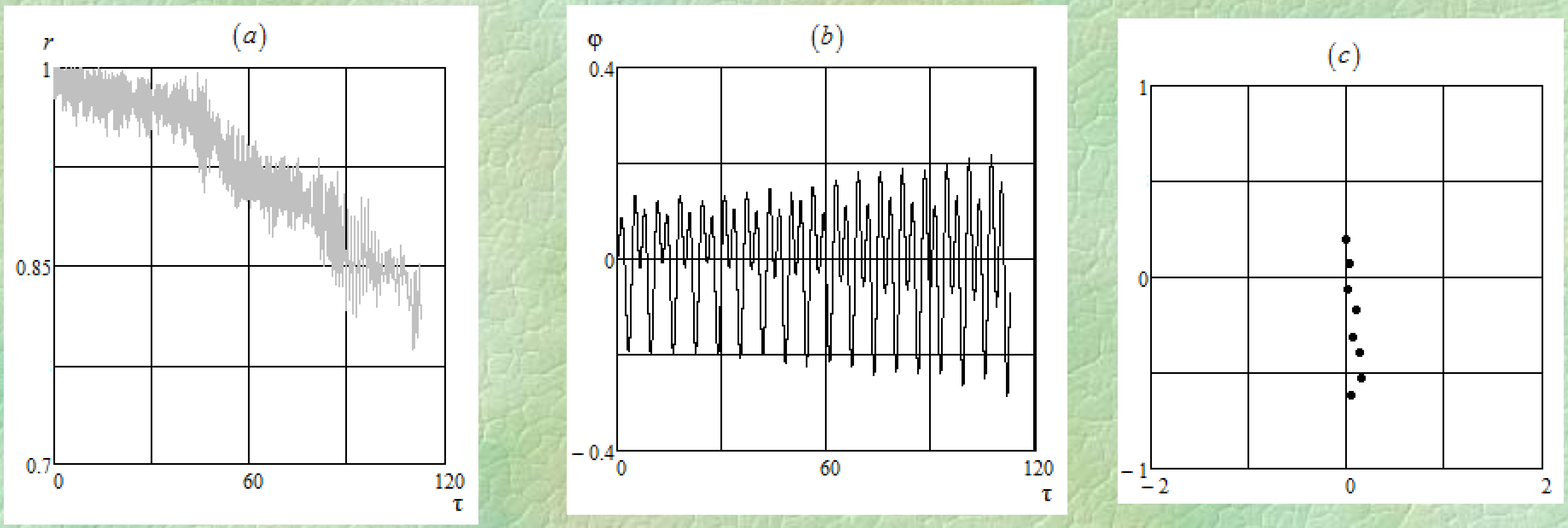
Controller is used $u = q_1\dot{\theta} + q_2\dot{r}$

$q_{1,2}$ **- feedback ratios**

Electrodynamics tether systems

Nonlinear system with control (Numerical results)

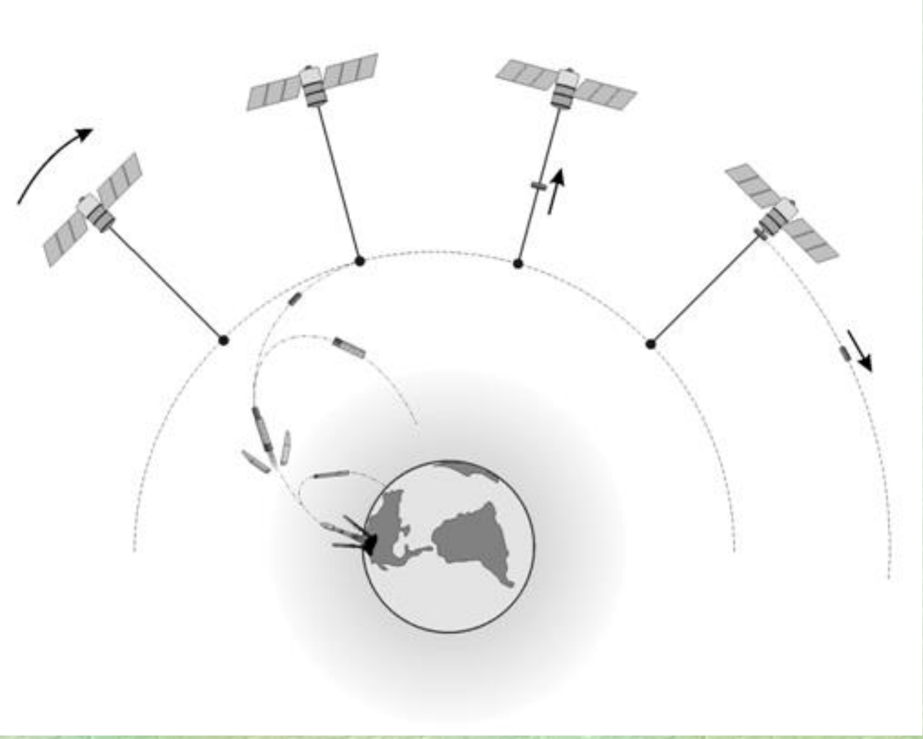
If $k_r = 0$, then the movement of the system is unstable



The tether is weakened (the tether is not stretched)

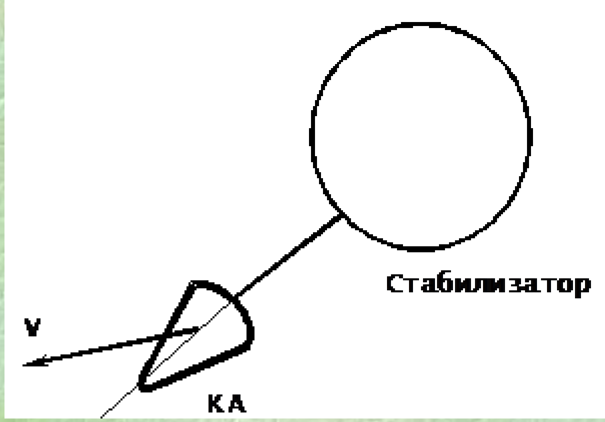
Transport operations in space

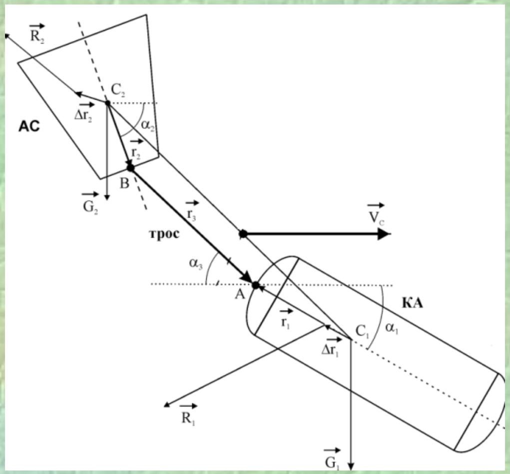
The space escalator



An example of the use of a tether system for transport operations.
This is a space escalator.
The small rocket payload moves to the lower end of the tether.
Then the cargo is lifted along the tether.

System with a tethered aerodynamic stabilizer





Types of aerodynamic stabilizers:

1. Deployable solid and flexible designs
2. Inflatable designs

Use:

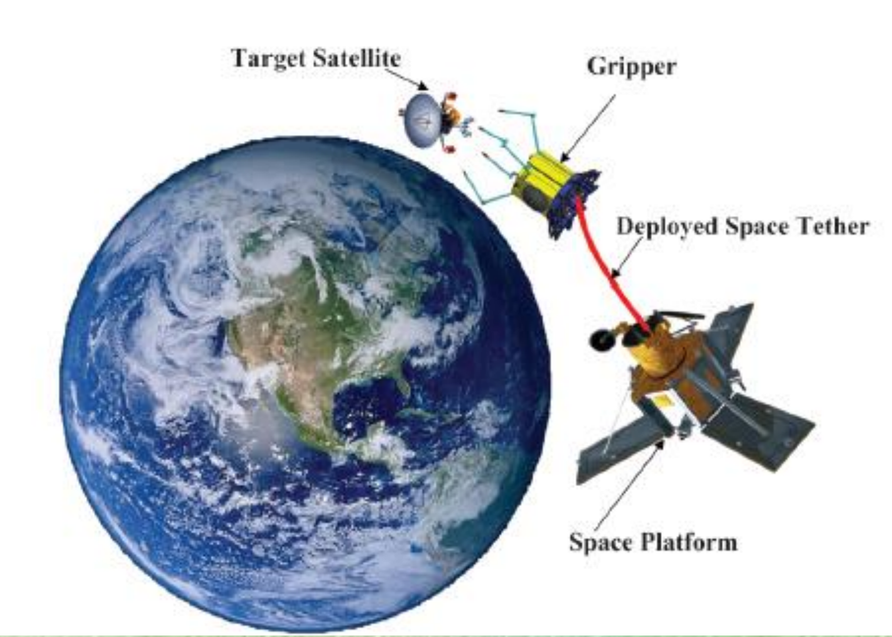
1. For stabilize the motion of spacecraft and nanosatellites
2. For faster removal from space of spacecraft and upper stages of multistage missiles

Problems:

1. Ensuring the stability of the system movement
2. Thermal deformation of inflatable designs

Removal of a nanosatellite from orbit

Using a robot on a tether to remove nano-satellites from orbit



Main task:
control of approach of gripper on a flexible tether with target satellite.

This task is very complex and requires detailed study.

Control movement of tether system

Management problems:

1. Control at tether system expansion:

a) Construction of the nominal program of expansion;

b) Regulation (stabilization) of the program

the control law at action of indignations..

2. Damping of oscillations in the system

Nominal programs of deployment:

.

a) The dynamic;

b) The kinematic. .

Dynamic laws:

Force of a tension of a tether is set $F_T(t, L, V_L)$

.

Kinematic laws:

The law of change of length of a tether or its speed is set

$$L(t),\ V_L(t)$$

Examples of nominal programs of deployment of tether system

1. Deployment for vertical final position of a body

The law of change of force of a tension:

$$F_T = m\Omega^2 (aL^0 + b\frac{V_L^0}{\Omega} - cL_K) \quad , \qquad (10)$$

Where m - Mass of a body, Ω - Angular speed of orbital movement of the base space vehicle,

a, b, c - Parameters of the law, L_K - Final length of a tether.

Trajectory of movement of a body concerning a local vertical

.

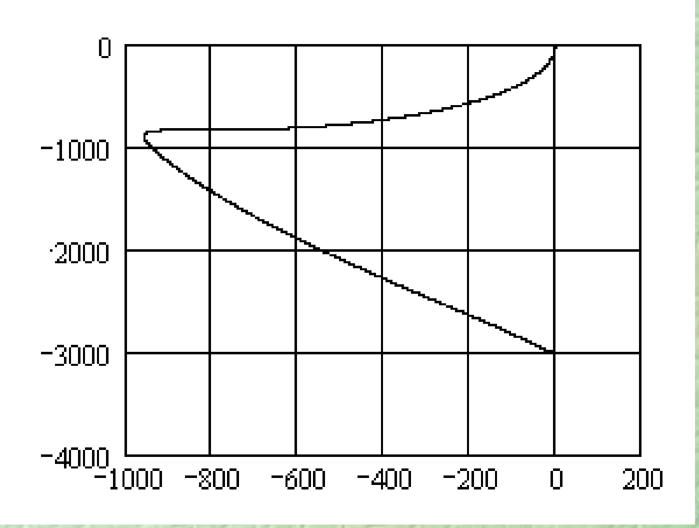


Figure 1

Laws of change of length of a tether and its speed of deployment

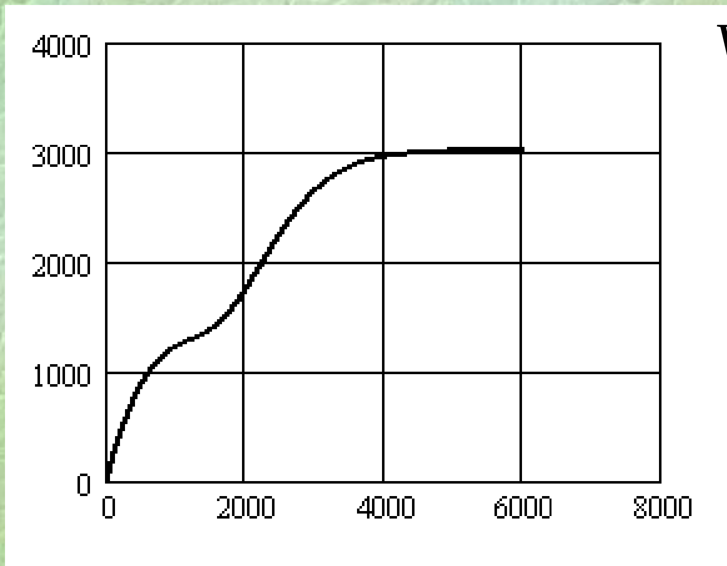


Figure 2

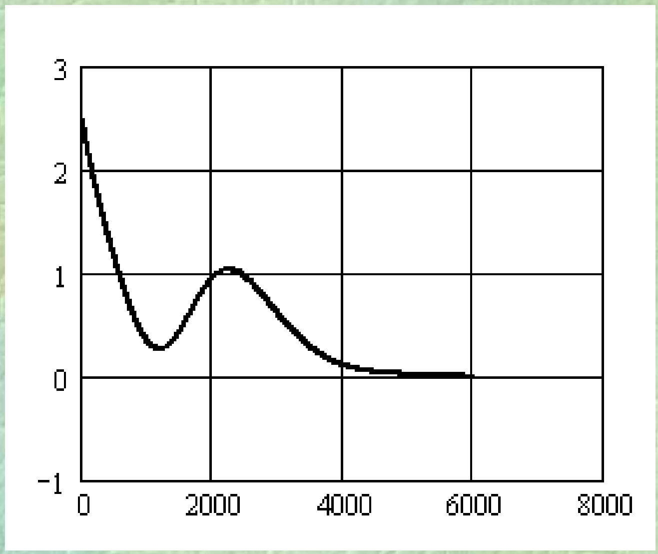


Figure 3

The **law of change of force of a tension**

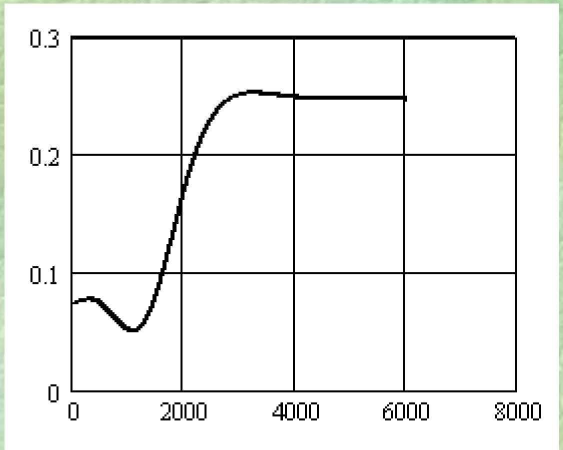


Figure 4

2. Deployment with a deviation from a vertical (fast deployment)

$$F_T = \begin{cases} T_{\min}, \; if \;\; L^0 < L_n \\ T_{\max}, \; if \;\; L^0 \geq L_n \end{cases} , \qquad (11)$$

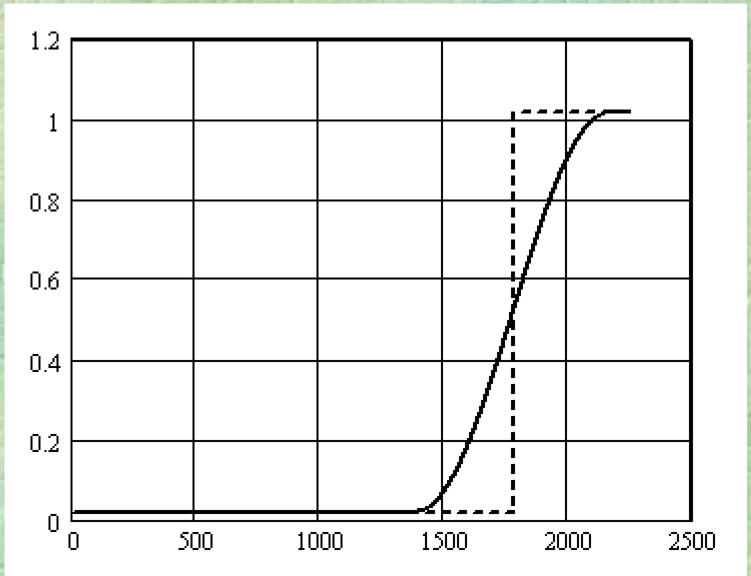
Where $T_{\min}, T_{\max}, L_n$ - Parameters of the law.

The smoothed law

$$F_T = \begin{cases} T_{\min}, \; if \;\; t < t_1 \\ T_{\min} + (T_{\max} - T_{\min}) \sin^2\left[k_p (t - t_1)\right], \; if \;\; t_1 \leq t \leq t_2 \\ T_{\max}, \; if \;\; t > t_2 \end{cases} , \qquad (12)$$

Where t_1, t_2, k_p - Parameters of the law.

F_T, N



t, s

Figure 5

Trajectory of movement of body concerning a local vertical

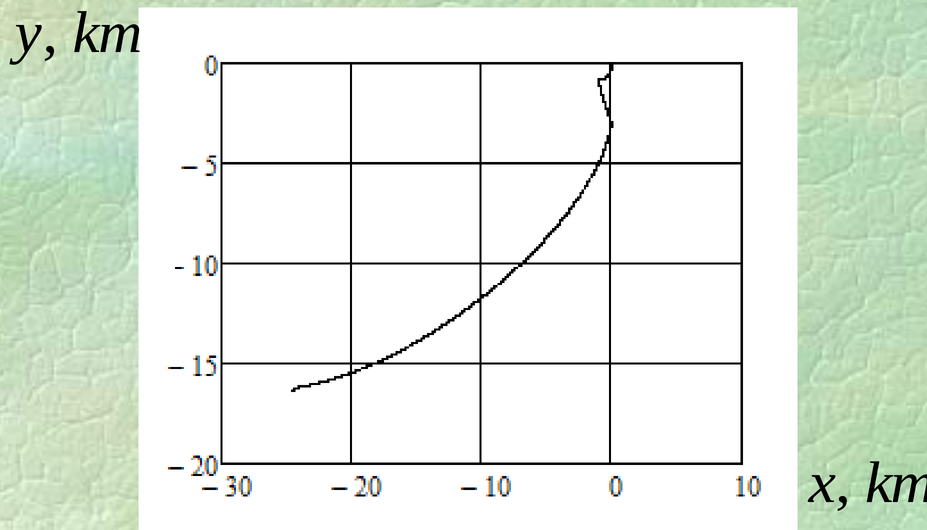


Figure 6

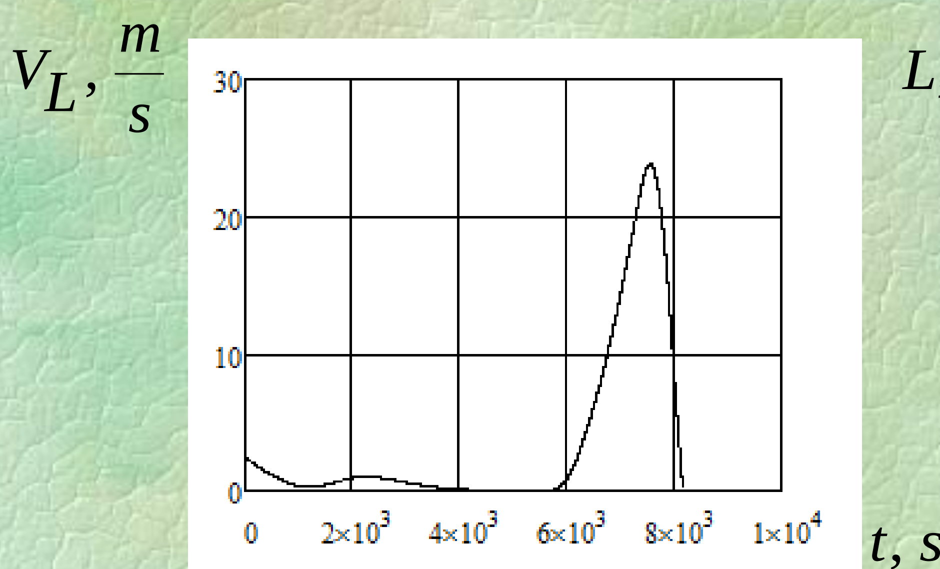


Figure 7

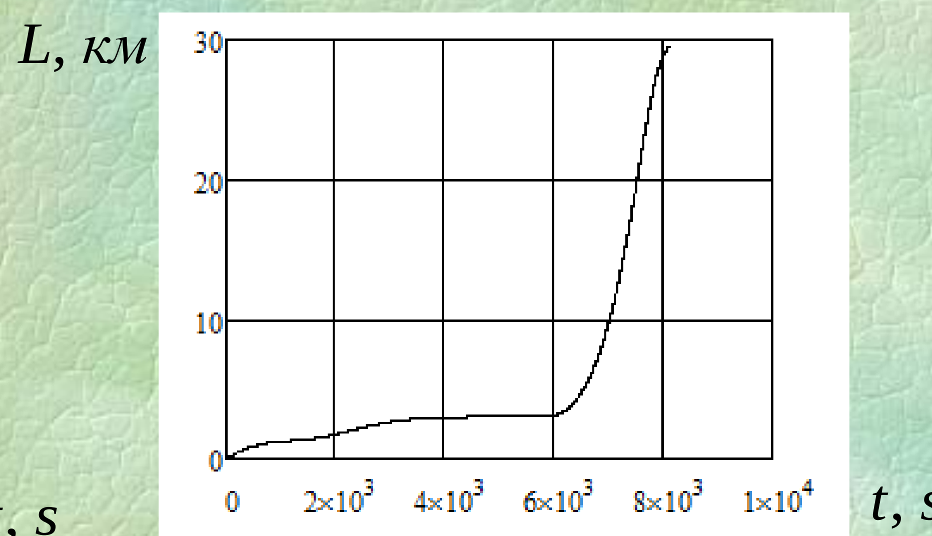


Figure 8

End of the first part of the lecture

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

THE ANALYSIS OF THE DYNAMICS OF A DEPLOYED TETHER SYSTEM CONSISTING OF TWO NANOSATELLITES

2021

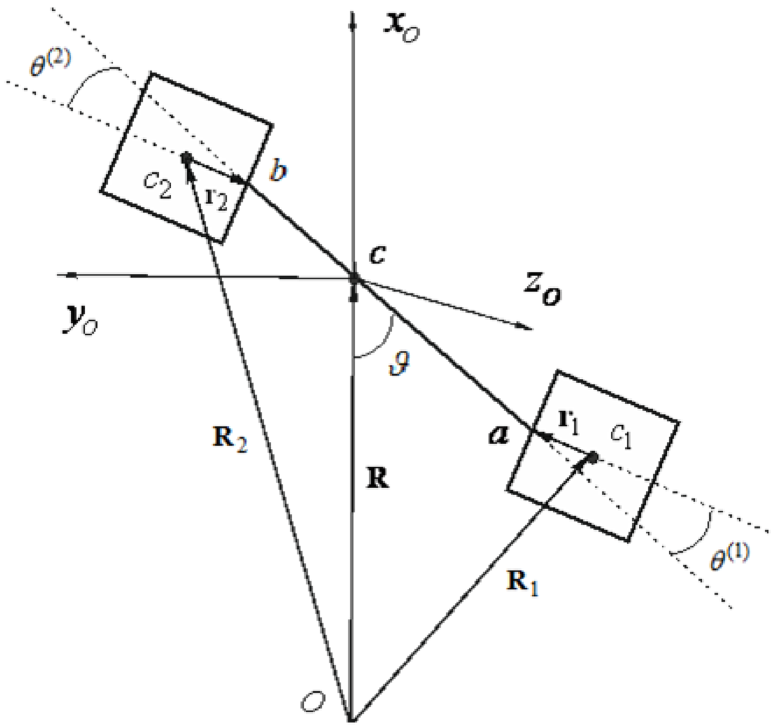


Figure 1

The formation of the satellite tether system will allow:

1. To reduce the angular speed of the system
2. Provide gravitational stabilization of the system
3. Provide stabilization of angular motion of satellites relative to their centers of mass

Main stages of deployment:

1. Separation from the base spacecraft (SC)
2. Division of nano-satellites
3. Controlled system deployment
4. Stabilization of the system with respect to the vertical

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

MATHEMATICAL MODEL OF THE MOTION OF THE TWO NS CONNECTED BY A TETHER

The equations of motion for the mass centers of the NS in the geocentric fixed coordinate system:

$$m_k \ddot{\mathbf{R}}_k = \mathbf{G}_k + \mathbf{T}_k , \quad k = 1,2 \qquad (1)$$

where $\mathbf{G}_k = -Km\mathbf{R}_k / R_k^3$ - gravitational force, $\mathbf{T}_k$ - tension force of the tether,

$\mathbf{R}_k$ - vectors for the center of mass of NS, $\mathbf{T}_1 = -\mathbf{T}_2$

The tension force is calculated Hooke's law :

$$T = \begin{cases} C\dfrac{R_t - l}{l}, & if \quad R_t - l \geq 0 \\ 0 , & if \quad R_t - l < 0 \end{cases} \qquad (2)$$

where $C = EA$ - the stiffness of the tether, E - elastic modulus,

A - tether cross-sectional area, l - undeformed length of the tether,

$R_t = |\mathbf{R}_b - \mathbf{R}_a|, \quad \mathbf{R}_a = \mathbf{R}_1 + \mathbf{r}_1, \; \mathbf{R}_b = \mathbf{R}_2 + \mathbf{r}_2,$

$\mathbf{r}_{1,2}$ - the vectors of attachment points of the cable relative to the centers of mass of the NS (Figure 1).

Dynamic Euler equations :

$$J_x^{(k)}\frac{d\omega_x^{(k)}}{dt}+\omega_y^{(k)}\omega_z^{(k)}\left(J_z^{(k)}-J_y^{(k)}\right)=M_x^{(k)}$$

$$J_y^{(k)}\frac{d\omega_y^{(k)}}{dt}+\omega_x^{(k)}\omega_z^{(k)}\left(J_x^{(k)}-J_z^{(k)}\right)=M_y^{(k)} \qquad (3)$$

$$J_z^{(k)}\frac{d\omega_z^{(k)}}{dt}+\omega_x^{(k)}\omega_y^{(k)}\left(J_y^{(k)}-J_x^{(k)}\right)=M_z^{(k)}$$

where $\omega_x^{(k)},\omega_y^{(k)},\omega_z^{(k)}$ - angular velocities of the NS,

$J_x^{(k)},J_y^{(k)},J_z^{(k)}$ and $M_x^{(k)},M_y^{(k)},M_z^{(k)}$ - moments of inertia and forces of NS in the main coupled coordinate systems.

Kinematic Poisson equations:

$$\dot{\mathbf{e}}_{xk}=\boldsymbol{\omega}_k\times\mathbf{e}_{xk},\quad \dot{\mathbf{e}}_{yk}=\boldsymbol{\omega}_k\times\mathbf{e}_{yk},\quad \dot{\mathbf{e}}_{zk}=\boldsymbol{\omega}_k\times\mathbf{e}_{zk} \qquad (4)$$

where $\boldsymbol{\omega}_k=\left(\omega_x^{(k)},\omega_y^{(k)},\omega_z^{(k)}\right)$, $\mathbf{e}_{xk},\mathbf{e}_{yk},\mathbf{e}_{zk}$ - the unit vectors directed along the main axes of the related coordinate systems.

Gravitational moments:

$$M_{xg} = 3\frac{K}{R^3}(J_z - J_y)\gamma_y\gamma_z,\ M_{yg} = 3\frac{K}{R^3}(J_x - J_z)\gamma_x\gamma_z,\ M_{zg} = 3\frac{K}{R^3}(J_y - J_x)\gamma_x\gamma_y \qquad (5)$$

where R - module of the radius of the center of mass of the system,

$\gamma_x, \gamma_y, \gamma_z$ - guiding cosines of the mass center vector in the orbital coordinate system

The moments of forces of the tether tension:

$$\mathbf{m}_k = \mathbf{r}_k \times \mathbf{T}_k \qquad (6)$$

where $\mathbf{r}_{1,2}$ - the vectors of attachment points of the cable relative to the centers of mass of the NS (Figure 1).

The moments of the forces in the main connected systems of coordinates:

$$\mathbf{M}_k = U_k^* \mathbf{m}_k$$

U_k^* - matrixes of transition from the connected systems to the main connected coordinate systems

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

The calculation of the speeds of the NS after their separation with a relative speed $\mathbf{V}_r$

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{V}_r, \quad \mathbf{V}_2 = \mathbf{V}_c + \frac{m_1}{m_1 + m_2}\mathbf{V}_r \tag{7}$$

where $\mathbf{V}_c$ - the speed of the center of mass of the system to the separation

The calculation of the angular speeds of the satellites after their separation

$$\omega_{x,y,z}^{(1,2)} = \omega_{x,y,z}^{(0)} + \frac{1}{J_{x,y,z}^{(1,2)}}\left(\mathbf{r}_{1,2} \times \mathbf{S}_{1,2}\right)_{x,y,z} \tag{8}$$

$\omega_{x,y,z}^{(0)}$ and $\omega_{x,y,z}^{(1,2)}$ - angular velocities of satellites before and after their separation,

$|\mathbf{S}_{1,2}| = S$ - the impulse in the separation, $\mathbf{S}_2 = -\mathbf{S}_1, \ \mathbf{S}_2 = -\frac{m_1 m_2}{m_1 + m_2}\mathbf{V}_r.$

<u>When calculating the linear and angular velocities of nanosatellites, the laws of conservation of impulse and angular impulse are used</u>

The equations that model the release tether:

$$m_u \dot{V}_l = T - F_u, \quad \dot{l} = V_l \qquad (9)$$

where $F_u = k_v V_l$ - force in the control mechanism,
k_v - management program ratio, V_l - speed of tether

For smooth braking of the tether the program is used:

$$k_v = \begin{cases} k_{v1} & if \quad l \le l_1 \\ k_{v1} + \dfrac{l - l_1}{l_{end} - l_1}(k_{v2} - k_{v1}) & if \quad l_1 < l \le l_{end} \end{cases} \qquad (10)$$

k_{v12} - deployment program settings.

if $l \le l_1$ - fast system deployment

if $l_1 < l \le l_{end}$ - braking and provision at the end of deployment $V_l \approx 0 \ (V_l > 0)$.

Data: $m_1 = m_2 = 2\,\text{kg}$, $L_{end} = 1\,\text{km}$, $C = 7000\,\text{H}$, $H = 400\,\text{km}$, $V_r = 0.25\,\text{m/s}$,

$\omega_x = \omega_y = \omega_z = 0.05\,\text{s}^{-1}$

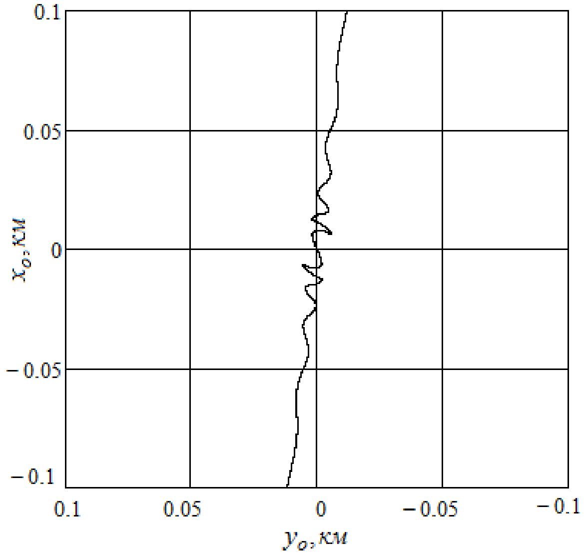


Fig. 2 The trajectories of the satellites with respect to the vertical

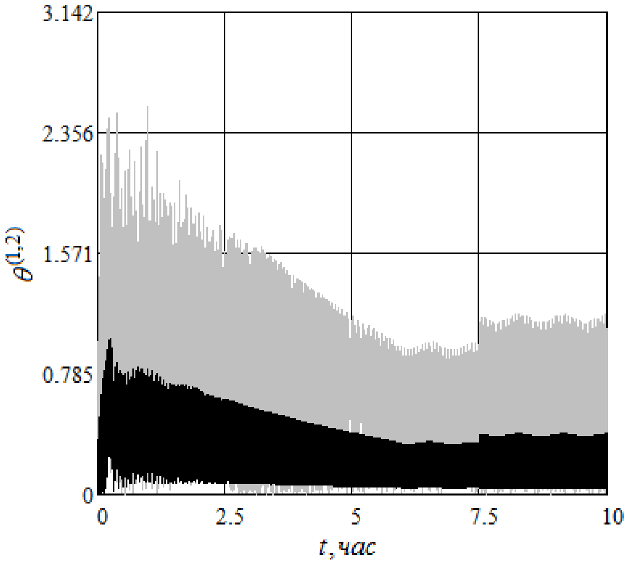


Fig. 3 The deflection angles of the satellites relative to the tether

One satellite with an offset center of mass relative to its longitudinal axis

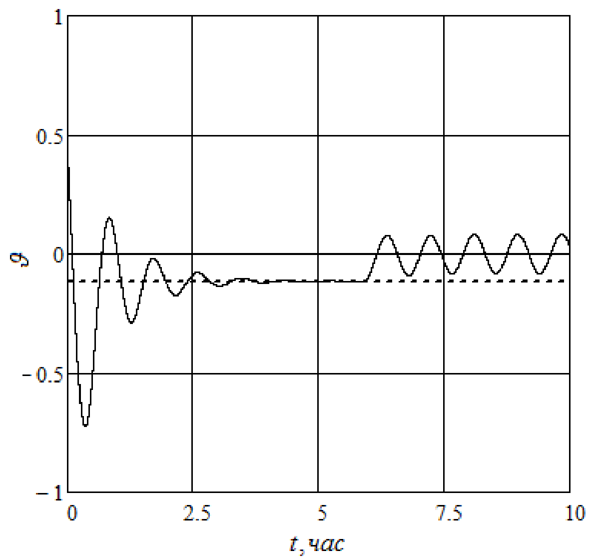


Fig. 3 The deviation angle of tether from vertical

Analytical formula for the angle of deviation of the tether from the vertical

$$\operatorname{tg} \vartheta_s = -2 m_e \frac{\omega_h}{k_{v1}} \tag{11}$$

The main problems of the tether system formation:

1. The choice of a simple program control without feedback
2. Providing restrictions on the tension force

   $T > 0$

3. Providing restrictions on the angular movement of satellites
4. Providing restrictions on the speed of the tether $V_{end} \approx 0 \ (V_l > 0)$

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

End of the second part of the lecture

ул. Московское шоссе, д. 34, г. Самара, 443086
Тел.: +7 (846) 335-18-26 , факс: +7 (846) 335-18-36
Сайт: www.ssau.ru, e-mail: ssau@ssau.ru

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

Dynamics and deployment control of a triangular tethered formation system with attitudes of microsatellites

2021

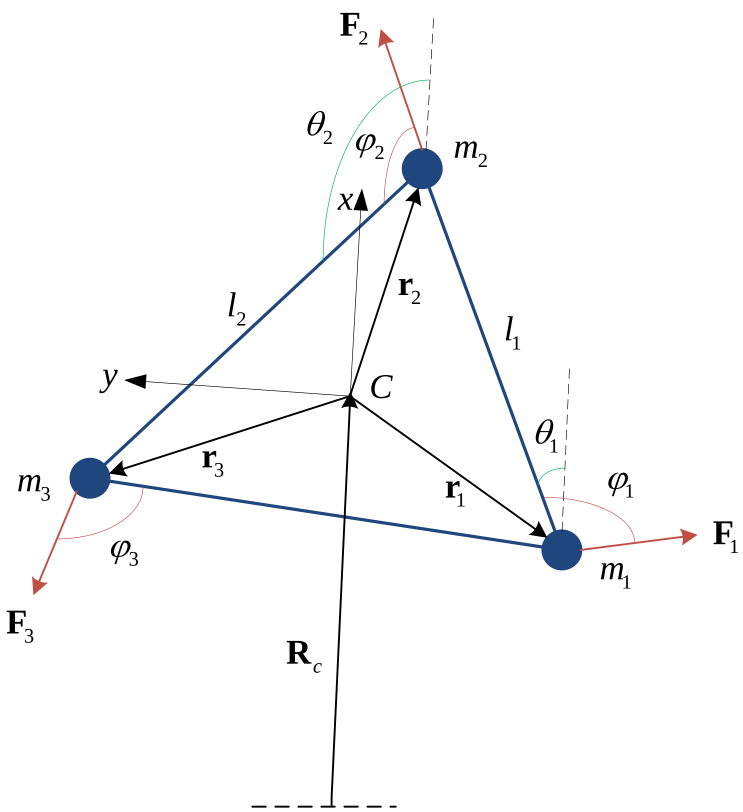


Fig. 1 Tether system

Problem statement:
it is necessary to form a triangular rotating tethered system

Devices for controlling:
1) low-thrust engines;
2) tether deployment mechanisms

In the initial state, the MS form a regular triangle and rotate with some initial angular velocity

The problem is considered taking into account the rotation of the MS relative to the center of mass

Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial \dot{\mathbf{q}}}\right)-\frac{\partial T_c}{\partial \mathbf{q}}=-\frac{\partial \Pi}{\partial \mathbf{q}}+\mathbf{Q} \qquad (1)$$

where T_c and Π represent the kinetic and potential energy, respectively. In order to describe the motion of the formation system, the generalized coordinates are chosen as the lengths of the tethers l_1, l_2 and the corresponding libration angles θ_1, θ_2 . The tethers l_1, l_2 connect the masses m_1 and m_2 and masses m_2 and m_3, respectively. Thus, $\mathbf{q}=(q_1, q_2, q_3, q_4)^T=(l_1, l_2, \theta_1, \theta_2)^T$ denotes the vector of the generalized coordinates; $\dot{\mathbf{q}}$ denotes the vector of corresponding generalized velocities; and $\mathbf{Q}=\left(Q_{l_1}, Q_{l_2}, Q_{\theta_1}, Q_{\theta_2}\right)^T$ is the vector of the generalized forces.

Kinetic and potential energies of the system:

$$T_c = \frac{1}{2}\sum_{i=1}^{3} m_i \left(\dot{\mathbf{R}}_c + \dot{\mathbf{r}}_i\right)^2, \quad \Pi = -\sum_{i=1}^{3} \frac{\mu m_i}{\left|\mathbf{R}_c + \mathbf{r}_i\right|} \tag{2}$$

where $\mathbf{R}_c$ is the vector from the center of the Earth to the CM of the formation system; $\mathbf{r}_i, i = 1,2,3$ represents the radius-vector of the ith microsatellite relative to the CM of the system. Considering that the modulus of $\mathbf{r}_i$ is much smaller than the distance between the Earth and the CM of the system, the expression of potential energy can be approximated by expanding into a binomial series and preserving the terms up to $O\left(1/R_c^3\right)$:

$$\begin{aligned}\Pi = -\sum_{i=1}^{3} m_j \omega^2 R_c^2 + \frac{1}{2} m\omega^2 \{ & \mu_1(\mu_2 + \mu_3) l_1^2 \left(1 - 3\cos^2\theta_1\right) + \\ & \mu_3(\mu_1 + \mu_2) l_2^2 \left(1 - 3\cos^2\theta_2\right) + \\ & 2\mu_1\mu_3 l_1 l_2 \left[\cos(\theta_1 - \theta_2) - 3\cos\theta_1 \cos\theta_2\right]\}\end{aligned} \tag{3}$$

where $m = \sum_{i=1}^{3} m_i$ is the total mass of the three microsatellites and $\mu_i = m_i/m$, $i = 1,2,3$ denotes the mass parameters; $\omega = \sqrt{\mu/R_c^3}$ is the orbital velocity of the CM of the system.

The nominal law

The deployment control law of the tether tensions is proposed as a kind of simple feedback linearization of the tether length dynamics, aiming to ensure that the tether tensions are more robust to length tracking.

$$T_i = T_i^0 + \mathrm{k}_i \cdot (l_i - L_\mathrm{d}) + \mathrm{w}_i \cdot \dot{l}_i, \quad i = 1, 2, 3 \tag{4}$$

where L_d is the expected final length of tethers; $\mathrm{k}_i, \mathrm{w}_i$ are the constant control gains; $T_i^0 = ml_i\left(\dot{\theta}_i + \omega\right)^2 / 9$ represents the tensions determined in the case of zero-length acceleration of the tethers in the equilateral triangular configuration.

Subsequently, the thrust forces follow a relay law according to the working characteristics of low-thrust engines:

$$F_i = \begin{cases} F, & \text{if } t < t_\mathrm{e} \\ 0, & \text{otherwise} \end{cases}, \quad i = 1, 2, 3 \tag{5}$$

where F denotes constant thrust force; t_e is the acting time of the thrust forces.

Analytical solution

For the nominal case, when the thrust forces $F_{1,2,3} = 0$, the equations of motion shown could be reduced to a linear system:

$$\Delta\ddot{l} = -9\left[\mathrm{k}\cdot\Delta l + \mathrm{w}\cdot\Delta\dot{l}\right]/\,m \qquad (6)$$

$$\ddot{\theta} = -2\Delta\dot{l}\,/\,\dot{\theta}l \qquad (7)$$

where $\Delta l = l - L_\mathrm{d}$, $l = l_{1,2,3}$, $\theta = \theta_{1,2,3}$, $\mathrm{k} = \mathrm{k}_{1,2,3}$, $\mathrm{w} = \mathrm{w}_{1,2,3}$.

The analytical solutions for (6-7):

$$\Delta l(t) = C_1 e^{\lambda_1(t-t_\mathrm{e})} + C_2 e^{\lambda_2(t-t_\mathrm{e})} \qquad (8)$$

$$\dot{\theta}(t) = \dot{\theta}(t_\mathrm{e})\exp\left(-2\int_{t_\mathrm{e}}^{t}\frac{\Delta\dot{l}(t)}{\Delta l(t)+l_d}dt\right) = \dot{\theta}(t_\mathrm{e})\left(\frac{C_1+C_2+L_\mathrm{d}}{\Delta l(t)+L_\mathrm{d}}\right)^2 \qquad (9)$$

where $\lambda_{1,2} = -\dfrac{9\mathrm{w}}{2m}\left(1\pm\sqrt{1-\dfrac{4m\mathrm{k}}{9\mathrm{w}^2}}\right)$, and the constants $C_{1,2}$ are determined by the values of the system states at the moment $t = t_\mathrm{e}$. If

$$\mathrm{w} > 0,\quad 0 < \frac{4m\mathrm{k}}{9\mathrm{w}^2} < 1 \qquad (10)$$

Then in this case, the limit value of the angular velocity can be found:

$$\dot{\theta}_\mathrm{d} = \lim_{t\to\infty}\dot{\theta}(t) = \dot{\theta}(t_\mathrm{e})(C_1+C_2+L_\mathrm{d})^2/L_\mathrm{d}^2,\quad l_i\to L_\mathrm{d},\ \dot{l}_i\to 0 \qquad (11)$$

Numerical results (nominal movement)

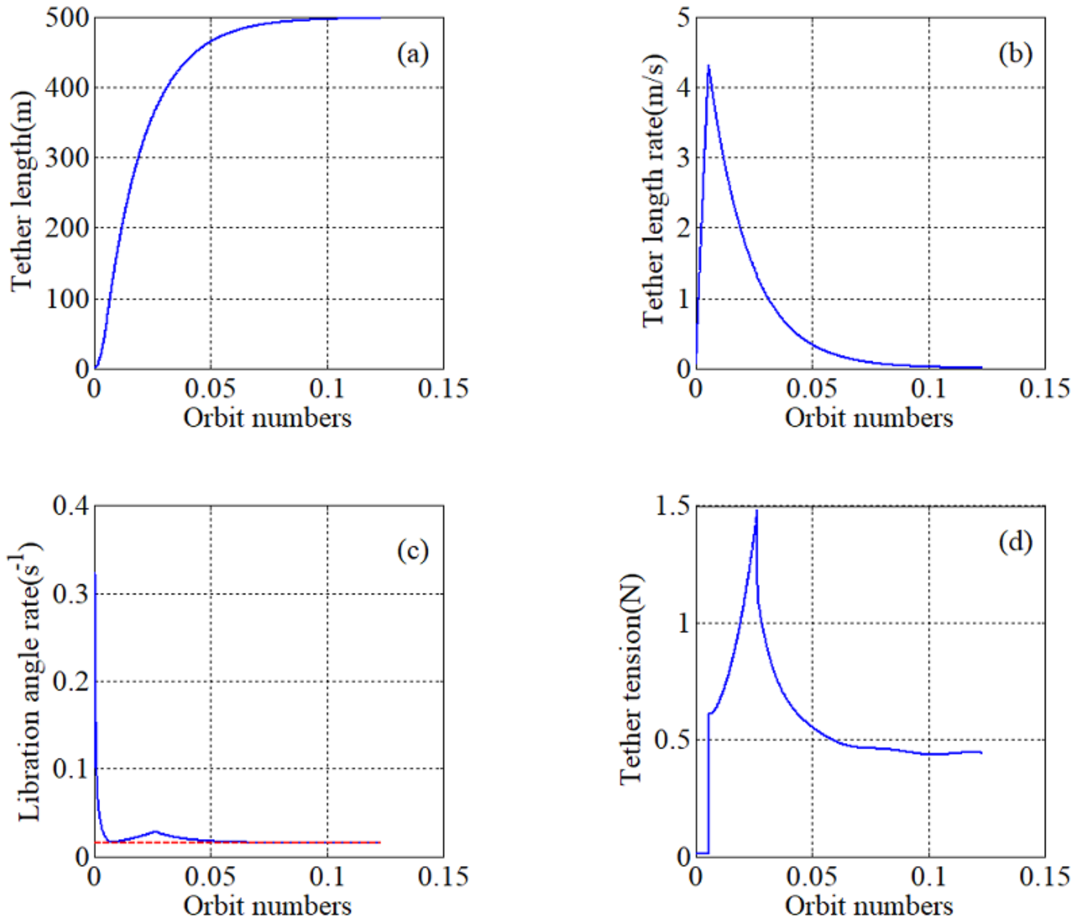


Fig. 2 The system states under the nominal deployment law

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

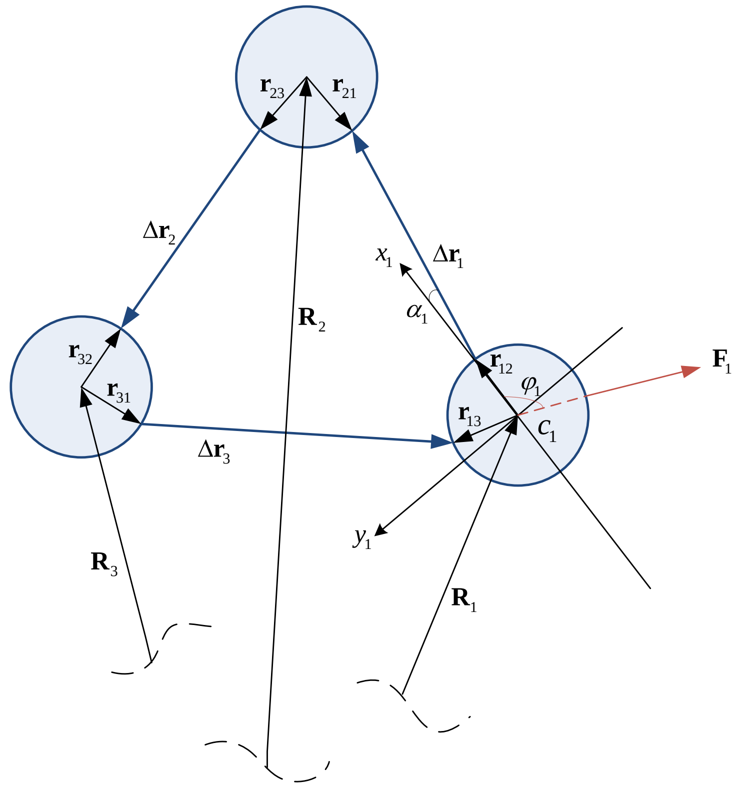


Fig. 4

The equations of motion are written in a fixed geocentric coordinate system

$$\begin{aligned} m_1\ddot{\mathbf{R}}_1 &= \mathbf{G}_1 + \mathbf{T}_1 + \mathbf{T}'_3 + \mathbf{F}_1 \\ m_k\ddot{\mathbf{R}}_k &= \mathbf{G}_k + \mathbf{T}_k + \mathbf{T}'_{k-1} + \mathbf{F}_k \quad (k = 2,3) \end{aligned} \qquad (12)$$

where $\mathbf{G}_k = -\mu m_k \mathbf{R}_k / R_k^3$ are the gravitational forces;

μ is the gravitational parameter of the Earth;

$\mathbf{T}_k = -\mathbf{T}'_k \ (k = 1,2,3)$ represents the tether tension vectors;

$\mathbf{R}_k \ (k = 1,2,3)$ are the position vectors of the centers of mass of microsatellites; $\mathbf{F}_k \ (k = 1,2,3)$ are the thrust force vectors.

The tether tension vectors describe the one-way mechanical connections between the adjacent microsatellites and have the following form according to the Hooke's law:

$$\mathbf{T}_k = T_k \, \Delta\mathbf{r}_k / \Delta r_k , \quad T_k = \begin{cases} C \dfrac{\Delta r_k - L_k}{L_k}, & if \quad \Delta r_k - L_k \geq 0 \\ 0 \quad , & if \quad \Delta r_k - L_k < 0 \end{cases} \tag{13}$$

where $\Delta\mathbf{r}_k \, (k = 1,2,3)$ are the vectors connecting the adjacent attachment points of tethers; L_k represents the undeformed tether lengths; $C = ES$ denotes the stiffness of tethers, E is Young's modulus of elasticity, and S is the cross-sectional area of tethers.

The vectors $\Delta\mathbf{r}_k \, (k = 1,2,3)$ are given by:

$$\begin{aligned} \Delta\mathbf{r}_1 &= (\mathbf{R}_2 + \mathbf{r}_{21}) - (\mathbf{R}_1 + \mathbf{r}_{12}) \\ \Delta\mathbf{r}_2 &= (\mathbf{R}_3 + \mathbf{r}_{32}) - (\mathbf{R}_2 + \mathbf{r}_{23}) \\ \Delta\mathbf{r}_3 &= (\mathbf{R}_1 + \mathbf{r}_{13}) - (\mathbf{R}_3 + \mathbf{r}_{31}) \end{aligned} \tag{14}$$

where $\mathbf{r}_{12}, \mathbf{r}_{21}, \mathbf{r}_{23}, \mathbf{r}_{32}, \mathbf{r}_{13}, \mathbf{r}_{31}$ represent the position of the tether attachment points relative to the centers of mass of the microsatellites, respectively.

It is assumed that the release of the tether L_k is carried out from the *k*th microsatellite. The deployment process of tethers is modeled using the presented dynamic equations in:

$$m_i \ddot{L}_k = T_k - U_k \tag{15}$$

where the coefficient m_i represents the inertia of the control mechanisms (the tethers are considered massless); U_k are the control forces and are given by:

$$U_k = \mathrm{k_g}\left(L_k - l_k\right) + \mathrm{w_g}\left(\dot{L}_k - \dot{l}_k\right) \tag{16}$$

where l_k and $\dot{l}_k$ are the nominal values; $\mathrm{k_g}, \mathrm{w_g}$ are coefficients of the feedback law.

Since the tether deployment mechanisms only work on the braking of tethers, if $T_k - F_k \le 0, T_k \le 0$ or $L_k > L_d$, then let $\dot{L}_k = \ddot{L}_k = 0$.

The classical Euler dynamic equations are applied to describe the angular motions of microsatellites relative to their centers of mass:

$$\dot{\boldsymbol{\omega}}_k = J_k^{-1}\left(\mathbf{M}_k - \boldsymbol{\omega}_k \times J_k \boldsymbol{\omega}_k\right) \tag{17}$$

where $\boldsymbol{\omega}_k$ and J_k are the angular velocity vector and inertia tensor of the k th microsatellite; $\mathbf{M}_k$ is the vector of moment of tether tensions, acting on the microsatellites. Here the gravitational and aerodynamic torques are not considered. The inertia tensor J_k is defined in the inertia principal coordinate system fixed on each microsatellite, and it is assumed that the ellipsoid of inertia of each microsatellite is close to a sphere.

Then, the kinematic equations are written in the form of the Euler–Poisson equations:

$$\dot{\mathbf{e}}_{xk} = \boldsymbol{\omega}^{(k)} \times \mathbf{e}_{xk},\ \dot{\mathbf{e}}_{yk} = \boldsymbol{\omega}^{(k)} \times \mathbf{e}_{yk},\ \dot{\mathbf{e}}_{zk} = \boldsymbol{\omega}^{(k)} \times \mathbf{e}_{zk} \tag{18}$$

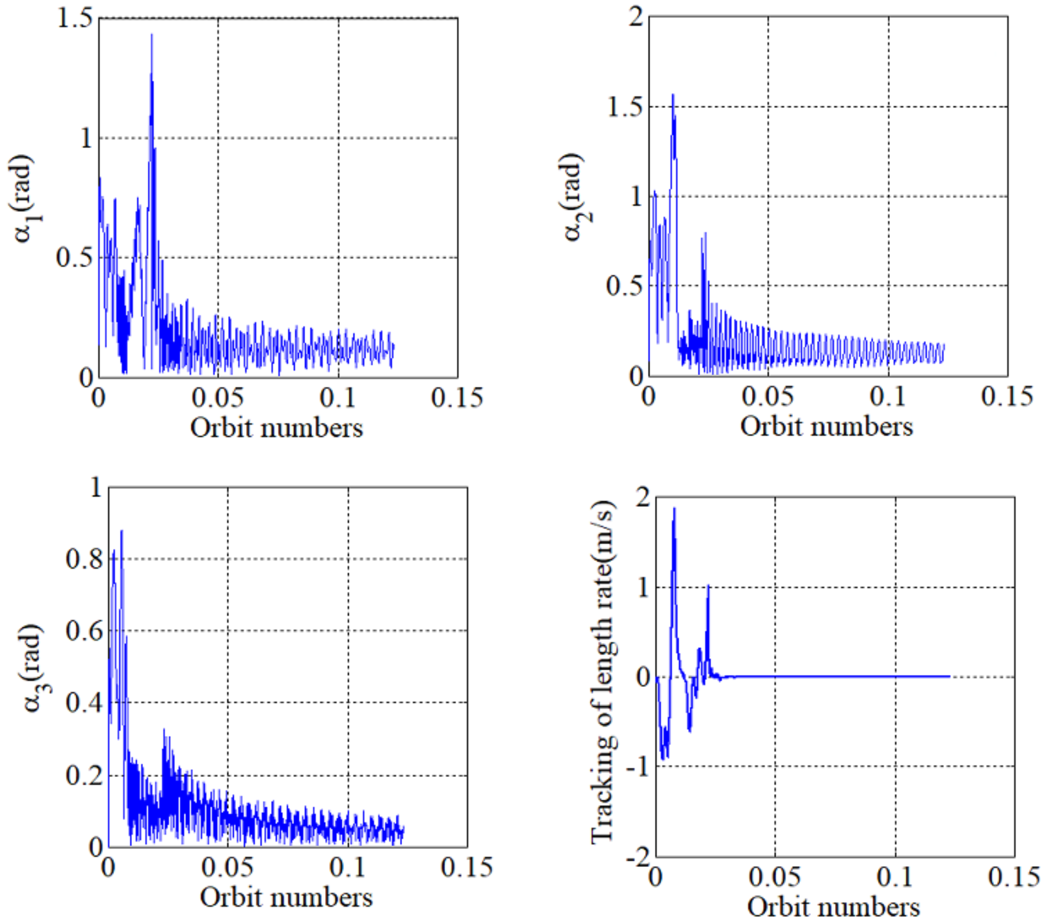
where $\mathbf{e}_{xk}, \mathbf{e}_{yk}, \mathbf{e}_{zk}$ are the unit vectors of the inertia principal coordinate systems $c_k x_k y_k z_k$, $k = 1,2,3$.

Disturbed motion of the tether system

The masses of the microsatellites are chosen as $m_1 = 8\text{kg}$, $m_2 = 10\text{kg}$ and $m_3 = 12\text{kg}$. The components of the initial spinning velocity of the configuration before the separation of microsatellites are $\omega_x = \omega_z = 0.1\text{s}^{-1}$ and $\omega_y = -0.1\text{s}^{-1}$, which means the modulus of ω_x, ω_y are the same as that of the nominal rotation velocity $\omega_z = 0.1\text{s}^{-1}$. In addition, the static and dynamic asymmetries of the microsatellites are also taken into account: 1) the centers of mass of the microsatellites are shifted relative to their geometric centers by an amount $\Delta D / D = 0.1$, where D represents the diameter of the microsatellites and ΔD is the displacement of the centers of mass; 2) the difference of the moments of inertia is set as $|J_{zk} - J_{yk}| / J_s = 0.1$, where J_s is the moment of inertia of a homogeneous sphere, and J_{yk}, J_{zk} are the components of the moments of inertia of the microsatellites relative to the principle coordinate systems $c_k x_k y_k z_k$. The presence of the static and dynamic asymmetries results in a corresponding change in the initial angular velocities of the microsatellites after separation since the impulses do not pass through their centers of mass, in addition, these asymmetries affect the subsequent motion of the microsatellites relative to their centers of mass.

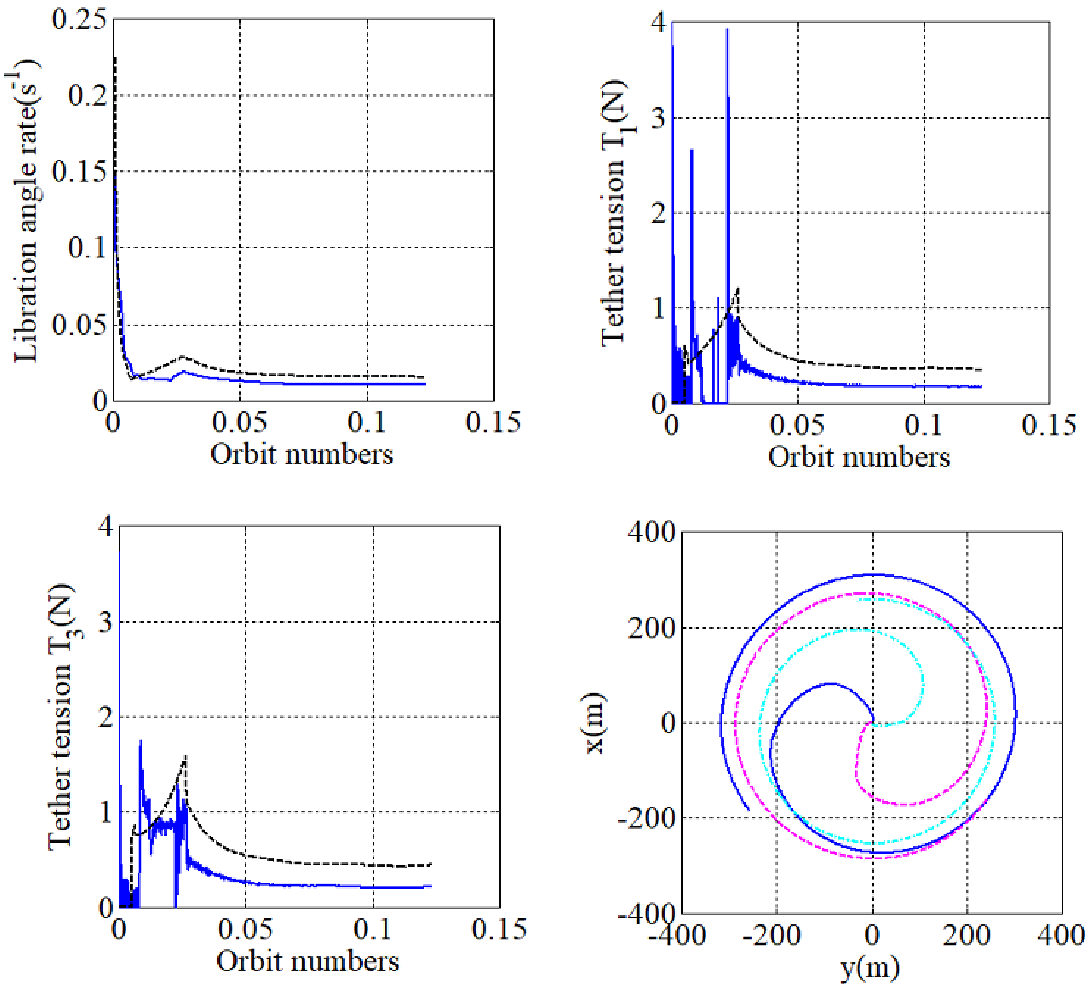
Dynamics of the formation of the system taking into account the motion of the MS relative to the center of mass

Disturbed motion of the tether system (numerical results)



Dynamics of the formation of the system taking into account the motion of the MS relative to the center of mass

Disturbed motion of the tether system (numerical results)



САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

THANK YOU FOR YOUR ATTENTION

ул. Московское шоссе, д. 34, г. Самара, 443086
Тел.: +7 (846) 335-18-26 , факс: +7 (846) 335-18-36
Сайт: www.ssau.ru, e-mail: ssau@ssau.ru

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

THE DEPLOYMENT OF THE TETHER GROUPS OF NANOSATELLITES

2021

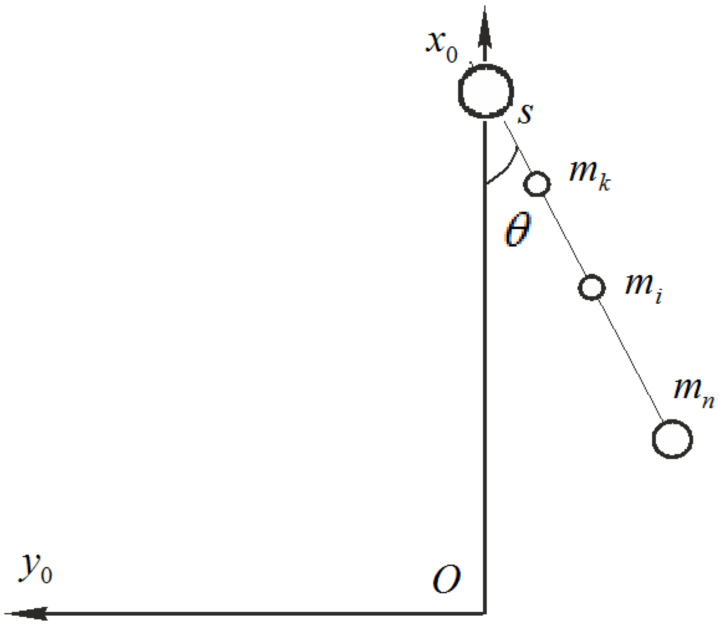


Fig. 1 Tether system

The system includes:
a base spacecraft and a few nanosatellites (NS)

After deployment,
the system is located near the vertical

The system is formed sequentially by separating one NS from the base spacecraft

Methods of formation: "slow" and "fast" deployment

Lagrange equations:

$$\frac{d}{dt}\left(\frac{\partial T_c}{\partial \dot{q}_j}\right)-\frac{\partial T_c}{\partial q_j}=-\frac{\partial P}{\partial q_j}+Q_j \qquad (1)$$

Generalized coordinate $q_1 = s$, $q_2 = \theta$

T_c and P - kinetic and potential energy.

$$T_c=\frac{1}{2}\sum_{i=k}^{n}\left(\dot{x}_i^2+\dot{y}_i^2\right),\ P=\sum_{i=k}^{n}P_i \qquad (2)$$

$$P_i=-Km_i/r_i,\ r_i=\sqrt{r_1^2+\left[s+(i-k)\Delta L\right]^2-2r_1\left[s+(i-k)\Delta L\right]\cos\theta}$$

$$x_i=x_{oi}\cos u-y_{oi}\sin u,\ y_i=x_{oi}\sin u+y_{oi}\cos u,$$

$$x_{oi}=r_1-\left[s+(i-k)\Delta L\right]\cos\theta,\ y_{oi}=-\left[s+(i-k)\Delta L\right]\sin\theta$$

$$\Delta L=L_{\text{end}}/(n-1),\ i=k,k+1,\dots n$$

Model for building a nominal program:

$$\ddot{s} = \left[m_k(s)\left(\Omega + \dot{\theta}\right)^2 + m_k(s)\Omega^2\left(3\cos^2\theta - 1\right) - T \right] / M_k$$

$$\ddot{\theta} = -2m_s(s)\dot{s}\left(\Omega + \dot{\theta}\right) / J_k(s) - 1.5\Omega^2 \sin 2\theta \tag{3}$$

Where $M_k = m(n-k) + m_n$

$$m_k(s) = m_n\left[s + (n-k)\Delta L\right] + m\sum_{i=k}^{n-1}\left[s + (i-k)\Delta L\right]$$

$$J_k(s) = m_n\left[s + (n-k)\Delta L\right]^2 + m\sum_{i=k}^{n-1}\left[s + (i-k)\Delta L\right]^2$$

Dynamic deployment program:

$$T = \nu_e\Omega^2\left[a(s - \Delta L) + b\dot{s}/\Omega + 3\Delta L\right] \tag{4}$$

Where

$$\nu_e = m_k(\Delta L) = m_n(n + 1 - k) + m\sum_{i=k}^{n-1}(i + 1 - k)$$

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

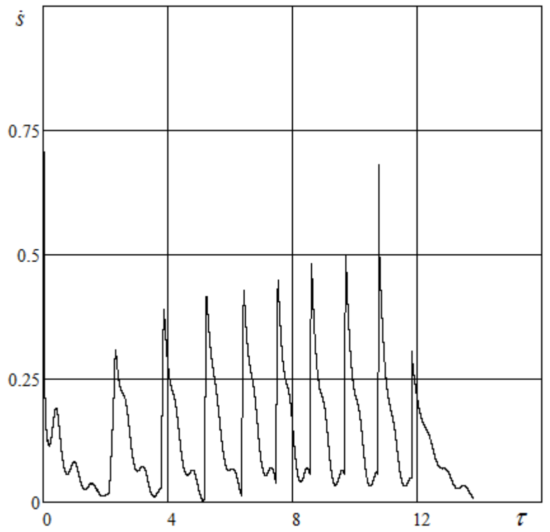


Fig.2 Tether speed

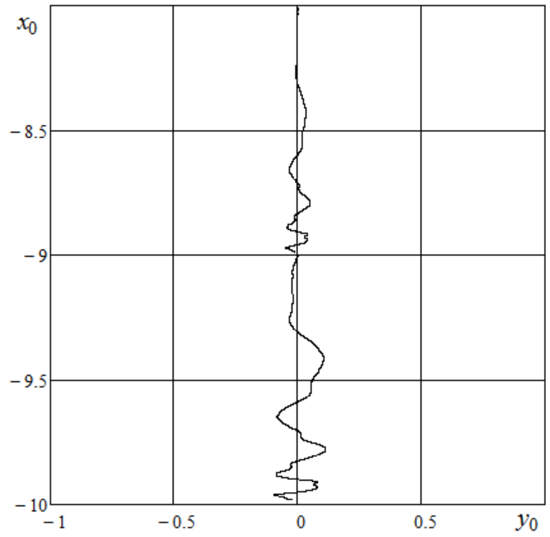


Fig.3 Trajectory of the two lower NS at the last stage of deployment

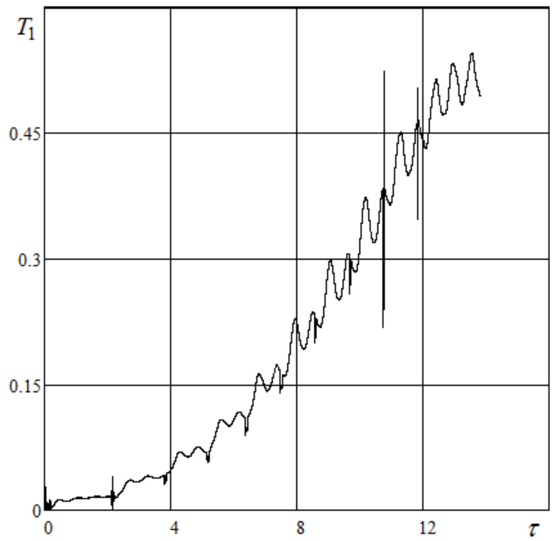


Fig. 4 Tension force

1. Deployment consists of n stages
2. Each stage consists of acceleration and deceleration sections
3. After completion of each stage, the system is located near the vertical
4. All satellites are located almost on the same line
5. After the last stage, the system is located near the vertical and has zero speed

Equivalent tether density: the mass of satellites is distributed evenly over the tether

$$\rho = m(n-2)/L_{\mathrm{end}} \qquad (5)$$

Where n - number of satellites, including the base; m - mass of nanosatellites; L_{end} - total tether length

Equations of motion of the tether system:

$$(m_n + \rho L)\ddot{L} = (m_n + \rho L/2)L F_{11} - T - \rho \dot{L}^2$$
$$(m_n + \rho L/3)L^2\ddot{\theta} = -2(m_n + \rho L/2)L\dot{L} F_{21} + (m_n + \rho L/3)L^2 F_{22} \qquad (6)$$

Where

$$F_{11} = \dot{\theta}^2 + 2\Omega\dot{\theta} + 3\Omega^2\cos^2\theta,\ F_{21} = \dot{\theta} + \Omega,\ F_{22} = -1.5\,\Omega^2\sin 2\theta$$

Nominal deployment program:

$$T = (m_n + \rho L/2)\Omega^2\left[a(L - L_{\mathrm{end}}) + b\dot{L}/\Omega + 3L_{\mathrm{end}}\right] \qquad (7)$$

Where m_n - mass of the lower nanosatellite, a, b - program parameters

THE EQUATIONS OF MOTION IN A GEOCENTRIC COORDINATE SYSTEM FOR TENSILE TETHER

The capabilities of the "fast" deployment program are tested using the following model

$$\frac{d\mathbf{r}_i}{dt} = \mathbf{V}_i, \quad m_i \frac{d\mathbf{V}_i}{dt} = \mathbf{G}_i + \mathbf{T}_i - \mathbf{T}_{i+1}, \; i = 1, 2, \ldots, n \qquad (8)$$

Where

$\mathbf{r}_i$, $\mathbf{V}_i$, m_i - radiuses, velocities, masses of satellites,

$\mathbf{G}_i$, $\mathbf{T}_i$ - the gravitational forces and the tension forces of the tether

The tension forces of the tether

$$\mathbf{T}_i = \begin{cases} c(\gamma_i - 1)\Delta\mathbf{L}_i / \Delta L_{0i}, & if \; \gamma_i \geq 1 \\ 0, & if \; \gamma_i < 0 \end{cases}, \qquad \gamma_i = \frac{\Delta L_i}{\Delta L_{0i}} \qquad (9)$$

Where

c - the stiffness of the tether, γ_i - relative deformation of the tether section

$$\Delta\mathbf{L}_i = \mathbf{r}_{i+1} - \mathbf{r}_i$$

Equations of operation of the control mechanism :

$$m_e \frac{dV}{dt} = T_1 - F_c, \quad \frac{dl}{dt} = V \qquad (10)$$

where the parameter m_e takes into account the inertia of the control mechanism,

V, l - speed and undeformed length of the tether, F_c - control force,

T_1 - tension force on the first section of the tether, counting from the base spacecraft

Control force:

$$F_c = T + p_L (l - L) + p_V (V - \dot{L}) \qquad (11)$$

Where

T - nominal tension force, p_l, p_V - feedback ratios,

$L, \dot{L}$ - program values of speed and tether length

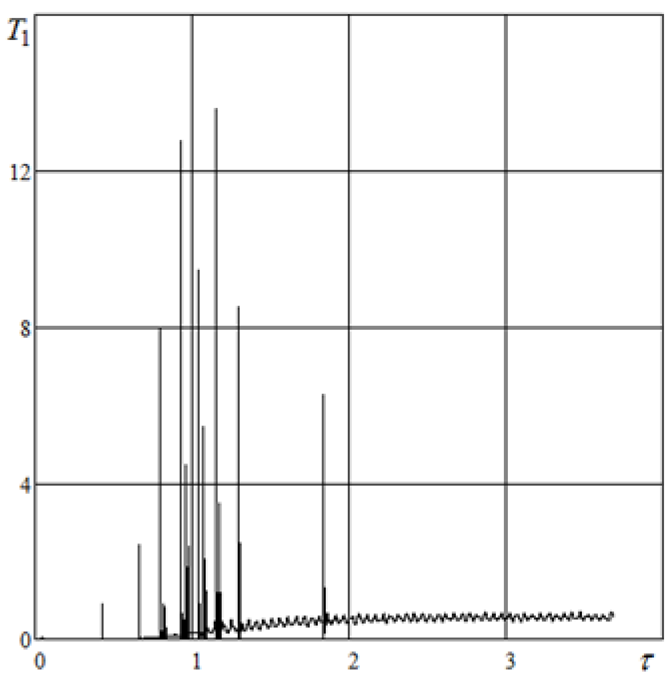


Fig. 5 The tension force of the tether

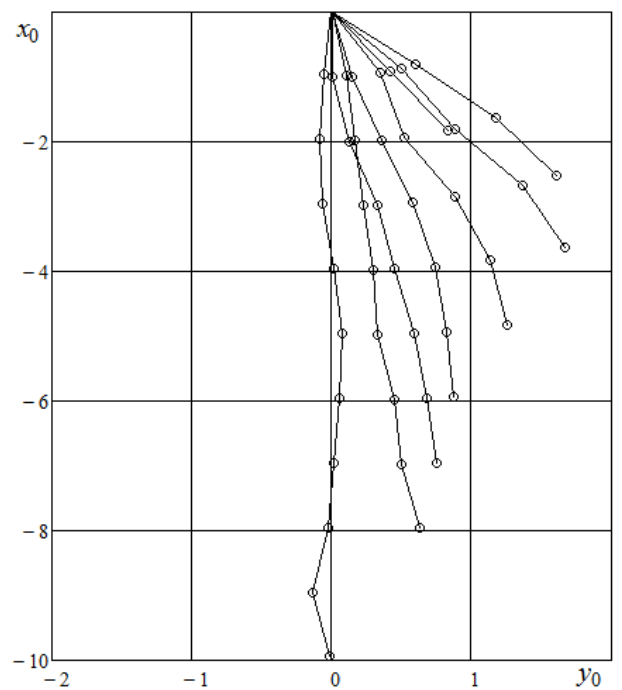


Fig.6 Positions of system in various point in time

Problems of "fast" deployment:

1. Need a rotary mechanism for the separation of the NS
2. It is necessary to ensure given the speed of separation of the satellite

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

The end of the third part of the lecture

ул. Московское шоссе, д. 34, г. Самара, 443086
Тел.: +7 (846) 335-18-26 , факс: +7 (846) 335-18-36
Сайт: www.ssau.ru, e-mail: ssau@ssau.ru